Modeling and Simulating Vortex Pinning and Transport Currents for High Temperature Superconductors

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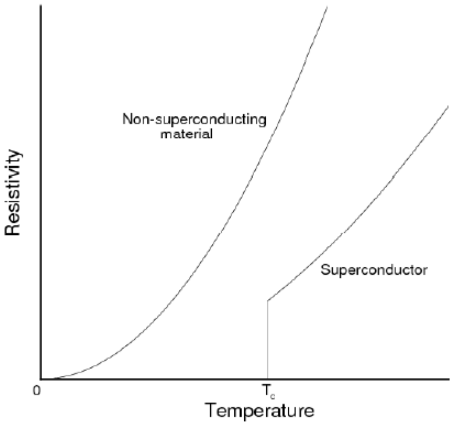
Superconductivity and Motivation

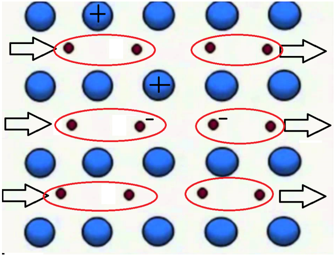
What is Superconductivity?

- Two Hallmark Properties:
  - 1. Zero Electrical Resistance
  - 2. The Meissner Effect
- The first property was discovered by Onnes in 1911.
- Only occurs below critical temperature T_c.
- Normal Metal Vs. Superconductor:



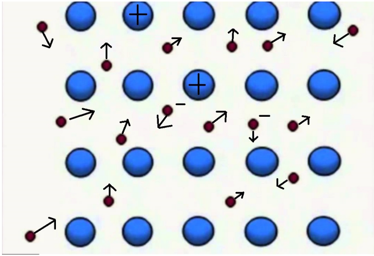
How does this Occur

- Below T_c the electrons form pairs (top).



- Movement is orderly.

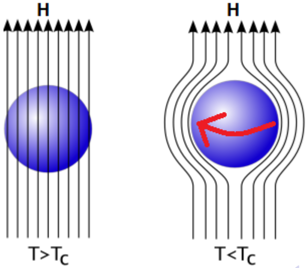
- No waste heat!



- Above T_c things break down (bottom).

The Meissner Effect

- Occurs when a superconductor (SC) is in a magnetic field.
- A resistance free current (super current) is induced.
- The current prevents penetration.
- This persists until the field reaches a critical strength H_c.
- Magnetic Field Penetration = NO Superconductivity.



Type I and Type II

- Type I SC are not penetrated at all (Meissner Effect) (top right).
- Type II SC are only penetrated by tubes of magnetic flux (Vortices) (bottom).
- Two critical H values, H_{c1} and H_{c2}.
- Vortex state: $H_{c1} < H < H_{c2}$.

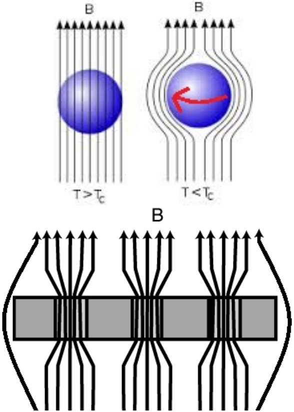


Figure : Normal and Type I (top). Type II (bottom)

Why You Should Care: Applications

Possible Superconducting Technology:

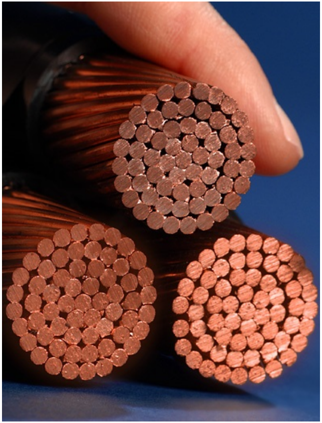
- Efficient Current Carriers
- Powerful Magnets (by magnetization)
- MRI
- Efficient Mag Lev

The Catch

- There is no free lunch.
- T_c is close to 0 K for most metals.
- Liquid helium is expensive.
- This rules out many applications such as power wires.
- Thankfully recent discoveries have overcome this.

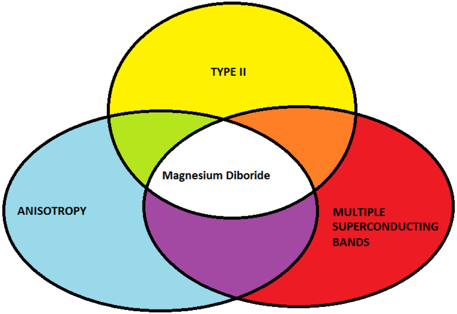
High Temperature Superconductors (HTS)

- New materials have revitalized superconductivity.
- Higher T_c values allow the use of liquid N or O coolants.
- Magnesium Diboride (MgB_2) is cheap and ductile ($T_c = 39$ K or -234° C).
- $HgBa_2Ca_2Cu_3O_8$ is used in MRIs ($T_c = 135$ K or -138° C).
- Hydrogen Sulfide under 150 G. Pascals of pressure ($T_c = 203$ K or -70° C).



High Temperature Superconductors (HTS)

- These materials come with new odd properties:
- Odd temperature dependencies in quantities.
- All of are Type II S.C.
- This complicates the modeling process.



Visualizing Vortices

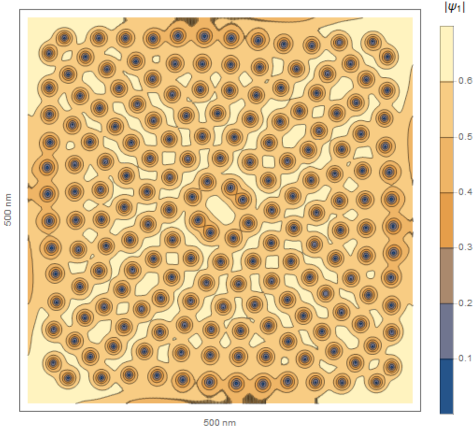


Figure : Simulation

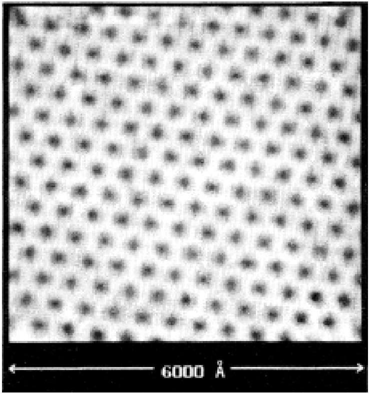


Figure : SEM image of vortices

Applied Currents

- So far we have T_c and H_c.
- What happens when we apply a current to a SC?
- Can it be carried without Resistance?
- Only below J_c!

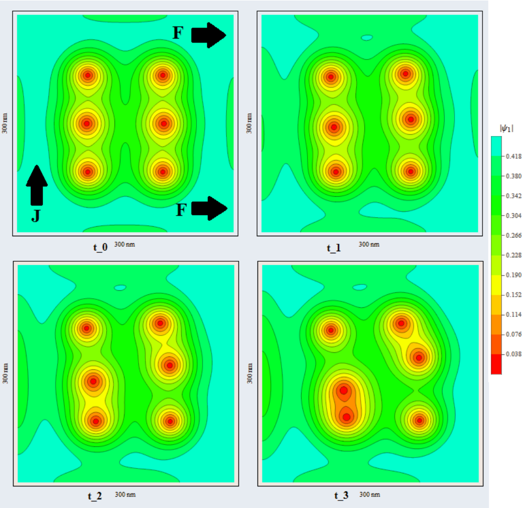
Why Vortex Dynamics are Important

- Vortices (B) and Current (J)= Flux Flow.
- Moving Vortices (flux flow) creates Resistance.

$$f\ \hat{\mathbf{x}} = J\ \hat{\mathbf{y}} \times B\ \hat{\mathbf{z}}$$

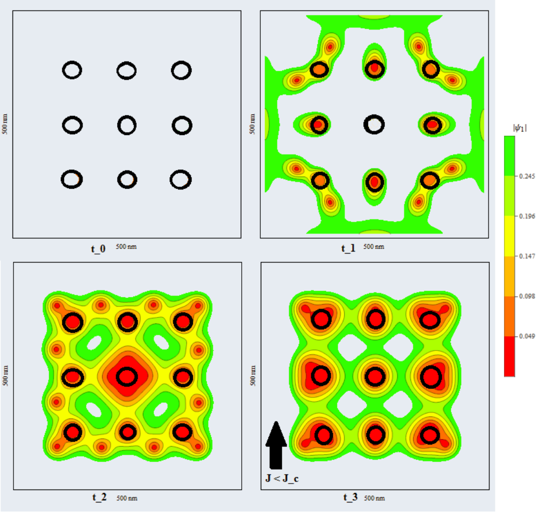
$$E\ \hat{\mathbf{y}} = B\ \hat{\mathbf{z}} \times u\ \hat{\mathbf{x}}$$

- Flux Flow induces Electric Field (E) and Voltage (V).
- Resistance now exists ($\frac{V}{I}$=R).



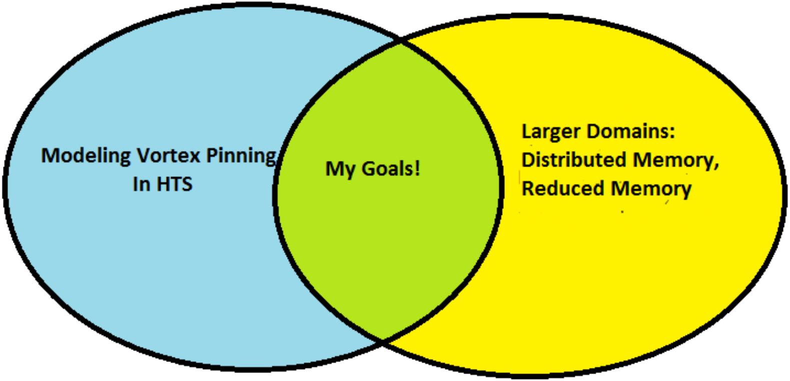
Vortex Pinning Comes to the Rescue

- Immobilizing the Vortices Is Crucial.
- Non Superconducting Metal= Normal Metal= Pinning Sites. (Outlined in Black)
- Vortices "Stick" To impurities.
- Limited increase In J_c.



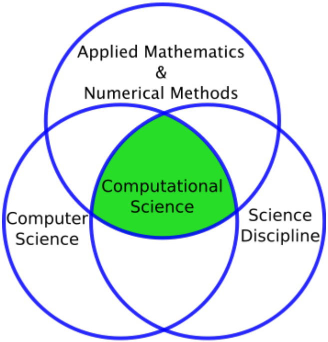
Simulations

- Simulations are critical to modeling new technology.
- No models for two-band SC and vortex pinning by impurities.
- Larger domains to avoid boundary effects.



The Framework

- A model: the Ginzburg-Landau model.
- Modify it for HTS and vortex pinning.
- Specify a material and model it.
- Modify for large scale simulations.



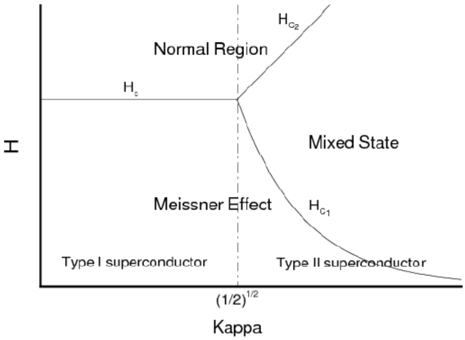
Ginzburg-Landau

Ginzburg-Landau (GL) Theory

- The G-L theory (or model) describes superconductivity as a phase transition for a valid temperature range.
- A free energy functional is formed.
- Its minimum is given by the G-L equations.
- This is done using calculus of variations.
- Gauge invariance.
- The model is non-dimensionalized using important material parameters.

Important Quantities

- Two variables:
  - ψ -The complex order parameter, describes the density of superconducting electrons.
  - **A** - The magnetic vector potential, $\nabla \times \mathbf{A} = \mathbf{B}$.
- Three material parameters:
  - λ -The penetration depth.
  - ξ - The coherence length.
  - κ - The G-L parameter $\kappa = \frac{\lambda}{\xi}$.
- Type I & II Revisited:



The Time Dependent G-L Model (TDGL)

- The solution $(\psi, \mathbf{A})$ minimizes the free energy.
- CGS units (no ϵ_0 or μ_0).

$$\Gamma(\frac{\partial\psi}{\partial t}) + i\kappa\Phi\psi + (|\psi|^2 - (1 - \frac{T}{T_c}))\psi + (-i\frac{\xi}{x_0}\nabla - \frac{x_0}{\lambda}\mathbf{A})^2\psi = 0 \quad (1)$$

$$\sigma(\frac{1}{\lambda^2}\frac{\partial\mathbf{A}}{\partial t} + \nabla\Phi) + \nabla(\nabla\cdot\mathbf{A}) + \nabla\times\nabla\times\mathbf{A} + \frac{i}{2\kappa}(\psi^*\nabla\psi - \psi\nabla\psi^*) + \frac{1}{\lambda^2}|\psi|^2\mathbf{A} = \nabla\times\mathbf{H} \quad (2)$$

+ B.C.s and I.C.s

- $\mathbf{H}$ is the applied magnetic field.
- Note $\mathbf{H} = \mathbf{B} - \mathbf{M}$; $\mathbf{M}$=magnetization.
- σ is the normal conductivity. T is temperature. Γ is relaxation constant.
- x_0 is scaling factor; Φ the potential is 0 by gauge choice.

Super and Normal Current

- Two components of the electrical current.

Normal Current Density

The resistive, normal current.

$$\mathbf{J}_n = \sigma \mathbf{E} = \sigma\left(\frac{1}{\lambda}\frac{\partial \mathbf{A}}{\partial t} + \nabla \Phi\right)$$

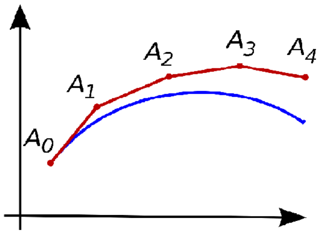
Super Current Density

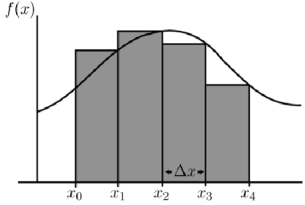
The resistance free super current.
This is the current that gives rise to the Meissner effect.

$$\mathbf{J}_s = -\frac{i}{2\kappa}(\psi \nabla \psi^* - \psi^* \nabla \psi) - \frac{1}{\lambda^2}|\psi|^2 \mathbf{A}$$

Solving The TDGL system

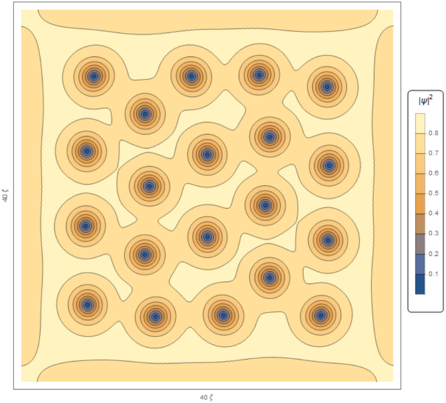
- Non-linear, time dependent, coupled system of PDEs.
- FEM for space. Quadratic triangular elements.
- Quadrature for integrals.
- Adaptive backward Euler for time.
- Newton for non-linearities.
- Direct or Krylov Solver? (SUPERLU_DIST at first)





TDGL Simulation

- $\psi \to 0$ where the material is normal (vortices or impurities).
- $\lambda = 60$ nm, $\xi = 5$ nm, $(1 - \frac{T}{T_c}) = 0.7$, $\frac{T}{T_c}$=0.3, $\mathbf{H} = 1.5 = 1.5H_c$, and $\kappa = 12$.



TDGL Simulation

G-L Variants

Anisotropy

Anisotropy can be modeled by assuming electrons have directional dependent masses $\rightarrow$ Effective mass model.
It also creates quantities for each direction: ξ^x, λ^x, κ^x, H^x_{c2} $+[.]^y$

Normal Inclusion

Impurities (Normal Inclusion model) can be modeled as well by solving a second set of equations. This is done by setting the reduced temperature $(1 - \frac{T}{T_c}) = -1$ and removing the $|\psi|^2\psi$ term.

Applied Current

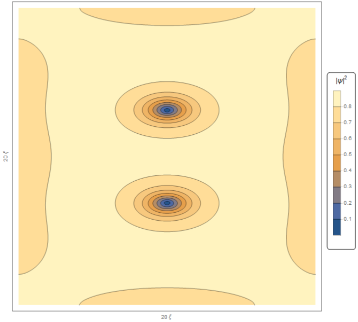
Applied currents can modeled by modifying the potential Φ.

$$-\sigma \nabla \Phi = \mathbf{J}$$

- Modeling Vortex Pinning= Applied Current + Normal Inclusions.

Anisotropy

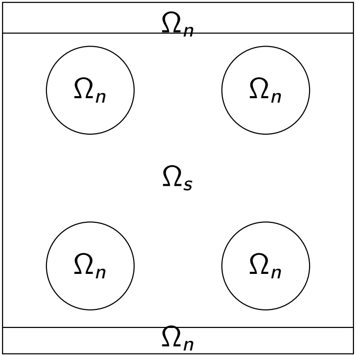
- Anisotropy distorts the shape of vortices.
- $\lambda^x = 60$ nm, $\xi^x = 5$ nm, $(1 - \frac{T}{T_c}) = 0.7$, $\frac{T}{T_c}$=0.3, $\mathbf{H} = 1.5 = 1.5\sqrt{2}\mathbf{H}_c^x$ and $m_y = \frac{1}{4}m_x$.



Anisotropy

G-L Variants: Normal Inclusion Model

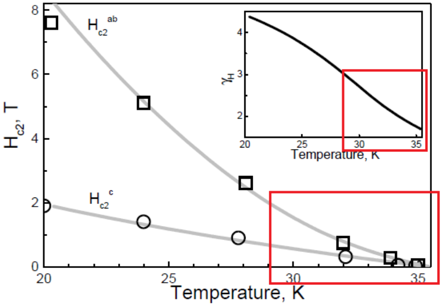
- Superconducting (Ω_s), Normal (Ω_n).



Two-Band Superconductivity

Two-Band Superconductivity

- Some HTS come with odd properties.
- Magnesium Diboride (MgB_2) ($T_c = 39$ K) is no exception.
- Anisotropic direction *ab*.
- Isotropic direction *c*.
- Upward curvature in T dependence of H_{c2}.



Two-Band Superconductivity

- Addition of second superconducting band explained behavior. Bands are "pathways".
- Two-band TDGL model (2B-TDGL)$\rightarrow \psi_1$ and ψ_2.
- λ_i, ξ_i, κ_i, $H_{i,c2}$, $T_{i,c}$
- Composite T_c, H_{c2} above each band's value from Coupling.
- Peculiarity: $T > T_{2,c}$, but $T < T_{1,c}$ and Superconductivity persists.
- Other HTS (Iron Pnictides) possess similar behavior.

Modified 2B-TDGL (M2B-TDGL) for HTS

- We would like to model HTS and all their odd properties.
- This composite model includes:
  - Two-band Behavior
  - Anisotropy
  - Applied Currents
  - Novel Strategy for Normal Inclusion
- How to ensure normal behavior?

Ensuring Normal Behavior

- For SC $(1 - \frac{T}{T_c}) > 0$
- One-band normals $(1 - \frac{T}{T_c}) \to -1$
- Two-band for MgB_2 $(1 - \frac{30K}{T_{2,c}}) \approx -1.5$
- No coupling in normal regions and $(1 - \frac{T}{T_{i,c}}) \to$.

$$\alpha_i(x, y)|_{\Omega_n} < \min\{(1 - \frac{T}{T_{1,c}}), (1 - \frac{T}{T_{2,c}})\} < 0$$

$\alpha(x, y) = -2 \in \Omega_n$.

ND M2B-TDGL

$$\left(\frac{\partial\psi_1}{\partial t} - i\frac{Jy}{\sigma}\kappa_1\psi_1\right) - \alpha_1(x,y)\psi_1 + b(x,y)|\psi_1|^2\psi_1 + \left(\hat{\mathbf{D}}_1 \cdot \Lambda_1(x,y) \cdot \hat{\mathbf{D}}_1\right)\psi_1 + \eta(x,y)\psi_2 = 0 \tag{3}$$

$$\Gamma\left(\frac{\partial\psi_2}{\partial t} - i\frac{Jy}{\sigma}\kappa_1\psi_2\right) - \alpha_2(x,y)\psi_2 + b(x,y)|\psi_2|^2\psi_2 + \left(\hat{\mathbf{D}}_2 \cdot \Lambda_2(x,y) \cdot \hat{\mathbf{D}}_2\right)\psi_2 + \eta(x,y)\nu^2\psi_1 = 0 \tag{4}$$

$$\nabla \times \mathbf{H} + \mathbf{J} = \sigma\frac{\partial \mathbf{A}}{\partial t} + \nabla \times \nabla \times \mathbf{A} + \Lambda_1(x,y) \cdot [\frac{i}{2\kappa_1}(\psi_1^*\nabla\psi_1 - \psi_1\nabla\psi_1^*) + \frac{x_0^2}{\lambda_{1,c}^2}\mathbf{A}|\psi_1|^2] + \Lambda_2(x,y) \cdot [\frac{i}{2\nu\kappa_2}(\psi_2^*\nabla\psi_2 - \psi_2\nabla\psi_2^*) + \frac{x_0^2}{\lambda_{2,c}^2}\mathbf{A}|\psi_2|^2] \tag{5}$$

ND TB-TDGL

$$\hat{\mathbf{D}}_1 = -i\frac{\xi_{1,c}}{x_0}\nabla - \frac{x_0}{\lambda_{1,c}}\mathbf{A}$$

$$\hat{\mathbf{D}}_2 = -i\frac{\xi_{2,c}}{x_0}\nabla - \nu\frac{x_0}{\lambda_{2,c}}\mathbf{A}$$

$$\nu = \frac{\lambda_{2,c}\xi_{2,c}}{\lambda_{1,c}\xi_{1,c}}$$

Modeling MgB_2.

Modeling MgB_2

- MgB_2 is an ideal candidate for our model.
- MgB_2 is cheap and ductile, being ideal for wires.
- Superconducting wires $\rightarrow$ Transport currents.
- Validating our model and it's simulation?
- Flux flow, vortex pinning, and transport currents.

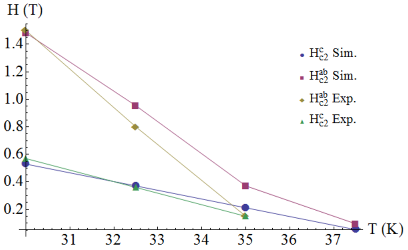
Material Parameters

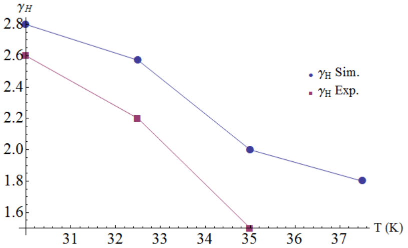
- The material parameters for MgB_2.
- Notice one band is Type II, the other is Type I.
- Γ_i and ϵ (or η) derived.

$\xi_{1,c}$=13.0 nm	$\lambda_{1,c}$=47.8 nm	κ_1=3.62
$\xi_{2,c}$=51.0 nm	$\lambda_{2,c}$=33.6 nm	κ_2=0.66
Γ_1=0.0288$\hbar$	Γ_2=0.001875$\hbar$	ϵ(0K)= -2.7016×10^{-17} J
T_{c1}=35.6 K	T_{c2}=11.8 K	T_c=39.0 K
$H_{1,c}$(0K)=0.3745 T	$H_{2,c}$(0K)=0.1358 T	ρ_n=0.7 $\mu\Omega$/cm
$\gamma_1(0K) = 4.55$	γ_2(0K)=1.0	

Validation: Curvature in H_{c2}

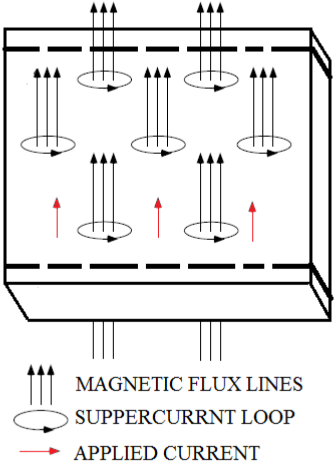
- One of the well known properties is the curvature in H_{c2}.
- Can we reproduce this in simulations?
- Our coupling is simplified.
- Qualitative behavior.





Vortex Pinning and Transport Currents

- Now we model vortex pinning.
- Our numerical domain.
  (Current $+y$, Field $+z$)
- Normal bands (dashed lines) are metal leads.
- We can see how much current is transported resistance free.



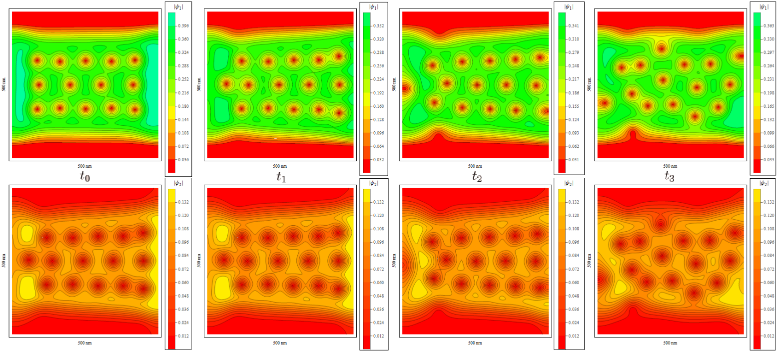
Simulation

- 1. Flux flow with field.
- 2. Vortex pinning.
- 3. Are the normal inclusions pinning?

Simulation 1: Flux Flow in Field

- Now $\mathbf{H} = 0.2648$ T,$\mathbf{J}$=33.717 MA cm^{-2}, $T = 30$K

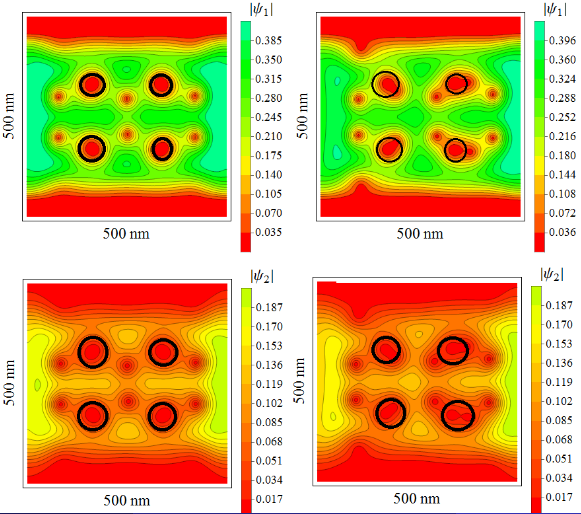
- Movie time frame: 1203.84 ps (1.2 ns)



Simulation 1: Flux Flow in Field

Simulation 2: Flux Flow and Normal inclusions

- $\mathbf{H} = 0.2648$ T $\mathbf{J}$=4.214 MA cm^{-2}, $T = 30$K.
- 4 Normal Inclusion, outlined in black.

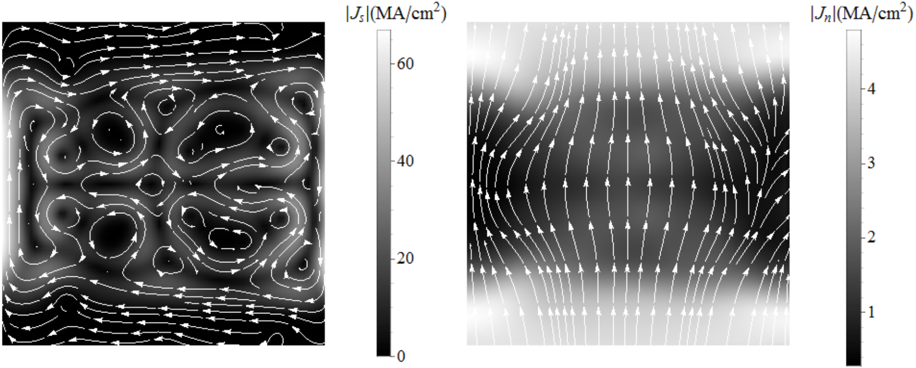


Simulation 2: $\mathbf{J}$=4.214 MA cm^{-2}

Simulation 2: Resistance Free Current?

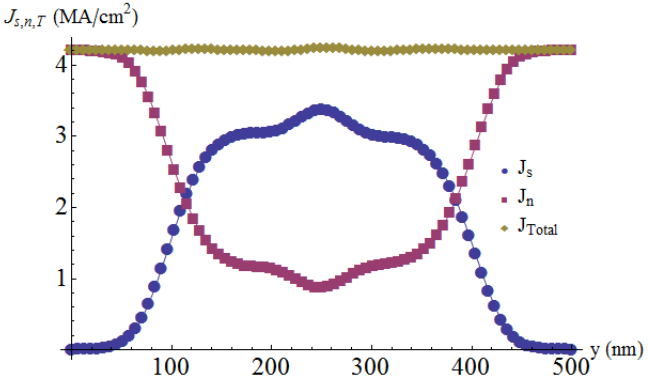
- Super and normal current stream density plots.

- Hard to determine!



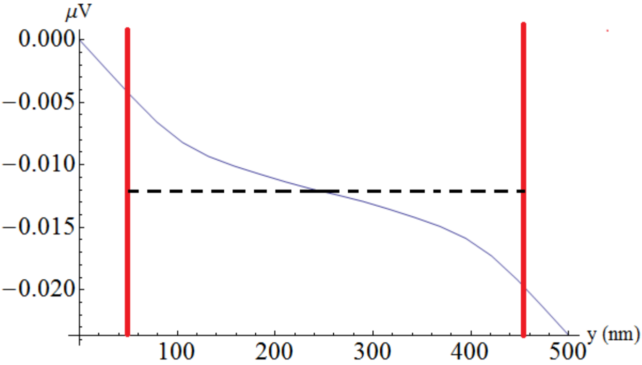
Average Currents

- We can also look at values in the *y*-direction averaged over *x*.
- There is a large minimum in J_n.



Resistance By Voltage

- Can the voltage be used as a proxy to resistance?
- No Resistance = No Voltage in S.C.! $V(y) = V(0) - \int_0^y E_{y,avg}(y')y'$

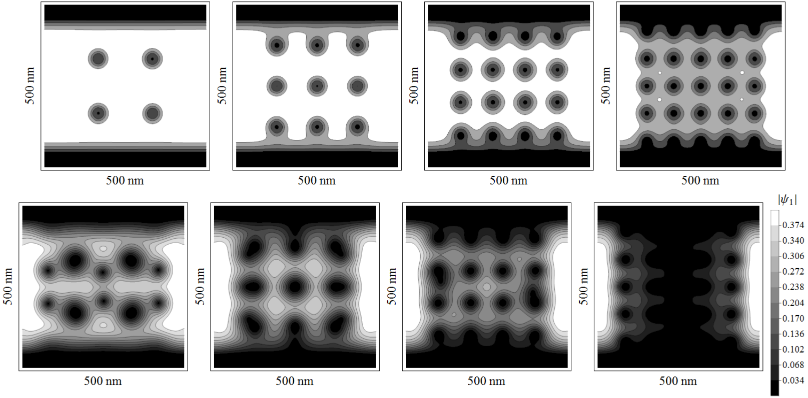


Resistance By Voltage

- J_c is hard to find. Envelope of simulations.
- How can you tell if normal inclusions are working?
- If the vortices are pinned, the voltage change should be small (Metric).
- Implies less flux flow and resistance.

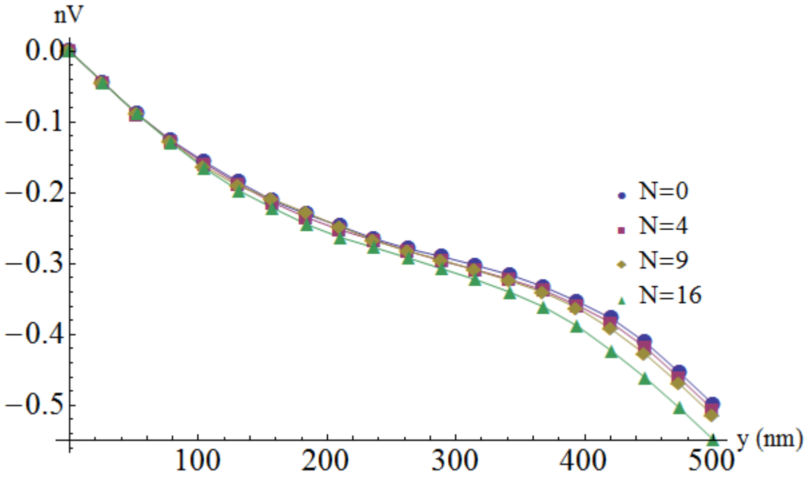
Vortex Pinning

- The normal inclusion arrangement (top) for $N = 4, 9, 16, 25$ normal inclusions.
- Their respective steady state solutions with $\mathbf{H} = 0.2648$ T and $T = 30$K. (bottom)



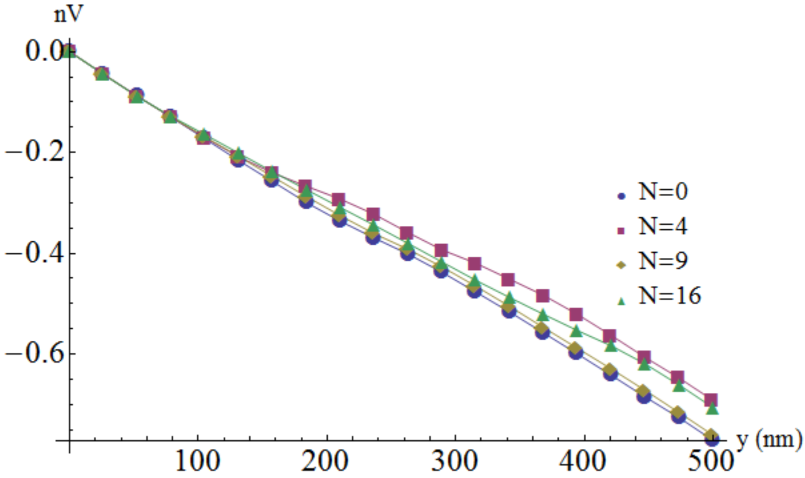
J= 0.0843 MA cm^{-2} at $t = 0$

- N=0 has the smallest voltage change at first. Notice the trend with *N*?



J= 0.0843 MA cm^{-2} at 100 TS.

- Now $N = 4$ and $N = 16$ have the smallest change due to pinning.



Various Applied Currents

- While the exact pattern is not known, the normal inclusions pin vortices for mild current densities.
- Δ V in nV

N	**J**=33.717 MA cm^{-2}	**J**=0.08429 MA cm^{-2}	**J**= 0.8429 KA cm^{-2}
0	-326.656 nV	-0.793243 nV	-5.16257×10^{-3} nV
4	-326.954 nV	-0.714098 nV	-6.37429×10^{-3} nV
9	-337.007 nV	-0.782900 nV	-4.33417×10^{-3} nV
16	-335.168 nV	-0.728167 nV	-6.97961×10^{-3} nV

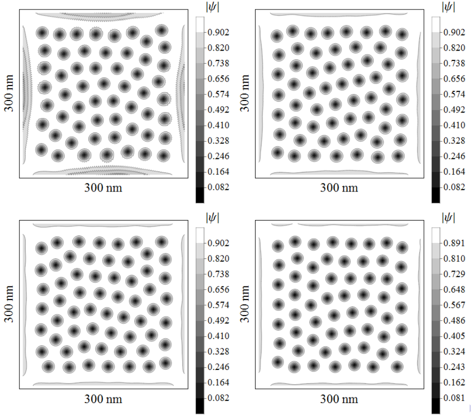
Computational Issues

Computational Issues

- HTS modeling is only part of the endeavor.
- To simulate superconducting technology, large domains are needed.
- Limited computational resources call for superior methods.
- The needed resolution also grows non-linearly with domain size.
- In this second endeavor, we to hope find ways to improve storage and shorten solve times.
- One-band: $\lambda = 50$ nm, $\xi = 5$ nm, $(1 - \frac{T}{T_c}) = 1.0$, $\frac{T}{T_c}$=0.0, $\mathbf{H} = .15\kappa H_c$, and $\kappa = 10$.

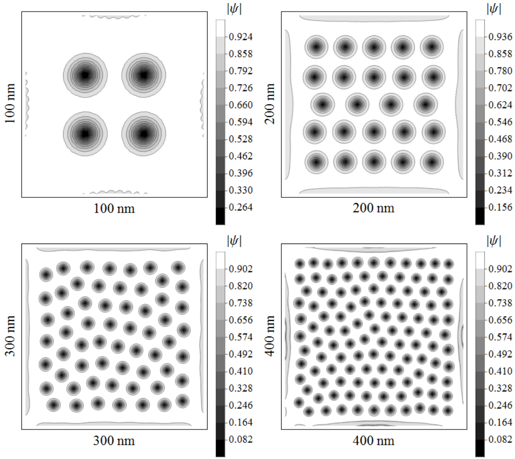
Resolution Issues

- A 300 nm by 300 nm domain. The number of vortices changes with resolution.
- 103040 DOFs and 56 vortices (top left), 231360 DOFs and 59 vortices (top right), 410880 DOFs and 60 vortices (bottom left), and 641600 DOFs and 60 vortices (bottom right)



Non-Linear Growth in Resolution

- Domain sizes of $(100\text{ nm})^2$, $(200\text{ nm})^2$, $(300\text{ nm})^2$, $(400\text{ nm})^2$
- $h = 4.761$ nm for $(100\text{ nm})^2$, $h = 3.278$ nm for $(200\text{ nm})^2$, $h = 2.127$ nm for $(300\text{ nm})^2$, and $h = 1.990$ nm for $(400\text{ nm})^2$.



Chronology

- Banded solver and symmetric Cholesky solver.
- CSR and SuperLU gave storage improvements.
- Parallelization through Trilinos and distributed matrices.
- Geometry is still a problem.
- Trilinos provides vast suite of solvers (and preconditioners).

Decoupling

- Decoupling methods can reduce memory.
- Decoupling large systems $\rightarrow$ Reducing in DOFs.
- Decoupling ψ and **A** equations?
- This give one non-linear system $(\mathcal{R}\{\psi\}, \mathcal{I}\{\psi\})$ and one linear systems ($\mathbf{A}_x$,$\mathbf{A}_y$).

Decoupling of Type I Vs Full Equations

- The storage advantage is clear!
- Going to steady state may take longer with decoupling.
- Global transient behavior.
- Is it more appropriate for dynamic studies?

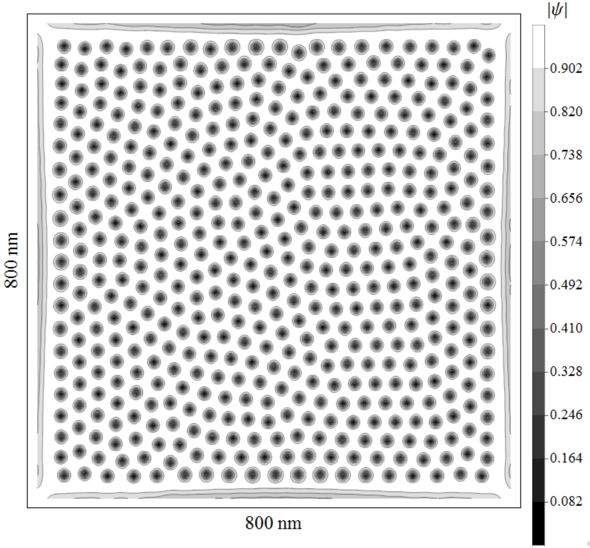
Domain Expansion

- Now we can see what the improvements have done for us.
- The serial Banded solver limited us to 24 vortices.
- Now we have $\approx$ 450 when the geometry routine maxed out the memory.

Implementation	Method Max	Domain (nm^2)	Domain (ξ^2)	DOFs*	Vortices
		50^2	$(10\xi)^2$	1,680	0
		100^2	$(20\xi)^2$	6,560	4
		150^2	$(30\xi)^2$	58,080	12
Serial	Banded	200^2	$(40\xi)^2$	58,080	24
		300^2	$(60\xi)^2$	314,720	60
Serial	CSR Full Eq.	400^2	$(80\xi)^2$	641,600	116
Serial	CSR Decoup. Type 1	500^2	$(100\xi)^2$	1,640,960	172
Parallel	Serial Geom max	800^2	$(160\xi)^2$	23,049,600	$\approx$450

A Small Hurdle

120 processors for 96 hours (11520 CPU hours)



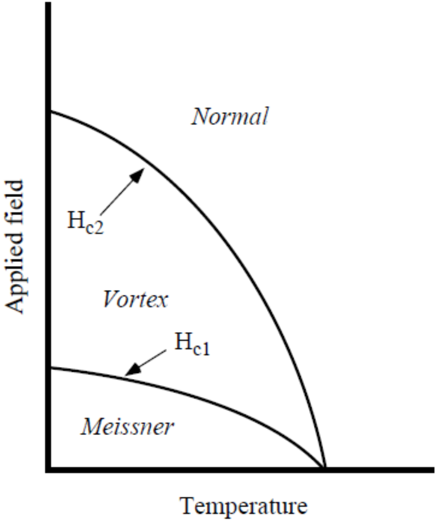
Future Work

- Improve some aspects of the M2B-TDGL.
- More realistic modeling: Normal inclusion metals.
- Apply computational methods to two-band model.

Back Up Slides

Type II

- Type II SC have 2 critical values for **H**.
- H_{c1}: Transition form Meissner to Vortex state.
- H_{c2}: Transition form Vortex to Normal state.
- H_c is still used for ND in Type II calculations.



Preconditioning

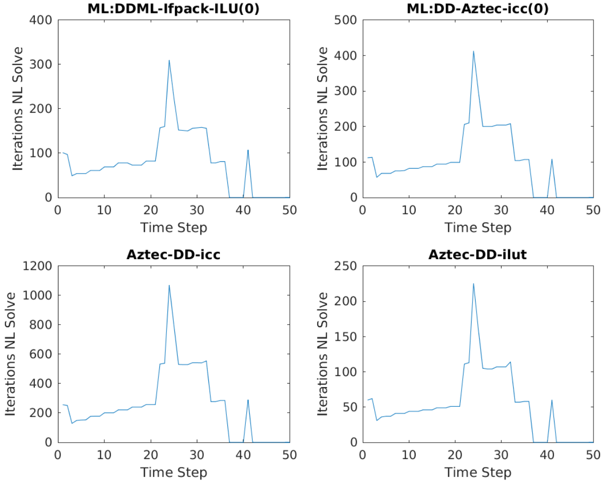
- Several we tested on 100 time steps, with 25921 degrees of freedom on a 20 nm by 20 nm domain.

- $\lambda = 50$ nm, $\xi = 5$ nm, $\kappa = 10$, $(1 - \frac{T}{T_c}) = 1.0$, $\mathbf{H} = 0.15\kappa\sqrt{2}H_c\ \hat{z}$

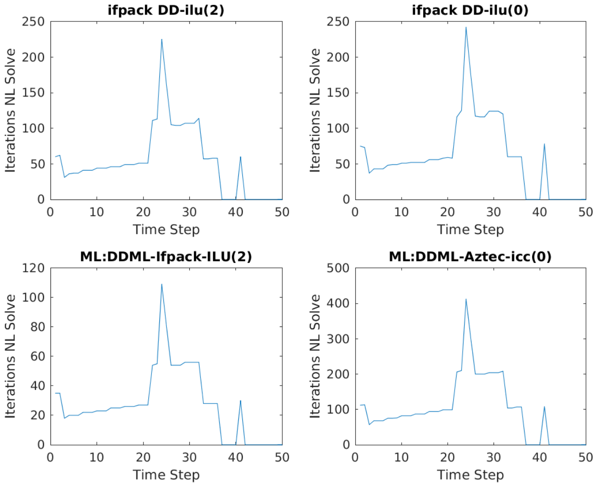
Preconditioner	NL solver Timing (sec)
Aztec-DD-icc(0)	1193.74
Aztec-DD-ilut(0)	1319.49
ifpack DD-ilu(0)	1332.03
ifpack DD-ilu(2)	1411.68
ML:DD-Aztec-icc(0)	3118.95
ML:DDML-Aztec-icc(0)	3325.07
ML:DDML-Ifpack-ILU(2)	4108.58
ML:DDML-Ifpack-ILU(0)	4319.17

Preconditioning

- GMRES iterations for each non-linear solve.
- Aztec-DD-icc has the shortest run time but largest iteration count.
- Iteration count before non-linear tolerance tightened.



Preconditioning



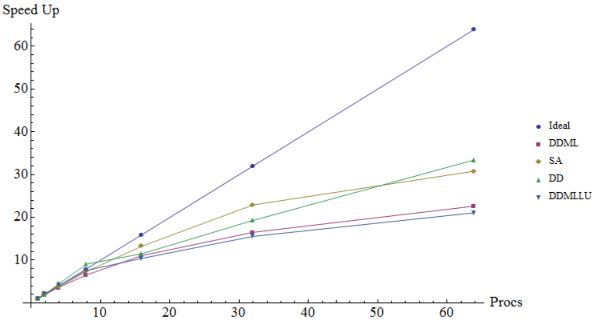
Preconditioning

- Iteration counts and solve times?
- How do they perform with more DOFs?

Preconditioner	DOFs	wall time	avg iters per GMRES solve
Aztec-DD-icc(0)			
	103040	1193.74	540.9
	1640960	123479.0	859.0
ifpack DD-ilu(0)			
	103040	1411.68	204.6
	1640960	220355.2	103.3
ML:DD-Aztec-icc(0)			
	103040	3118.95	237.14
	1640960	106508.0	261.6
ML:DDML-Aztec-icc(0)			
	103040	3325.07	237.14
	1640960	47905.8	91.457

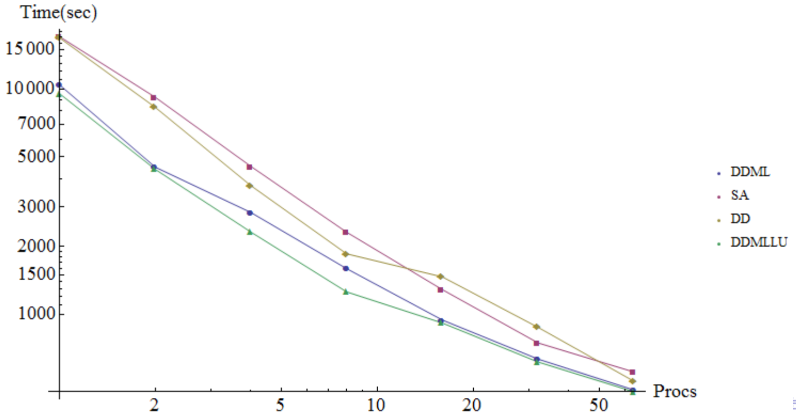
Preconditioning

- Clearly the ML preconditioners perform well for large domains.
- How do they scale for a small set of processors? (103041 DOFS)



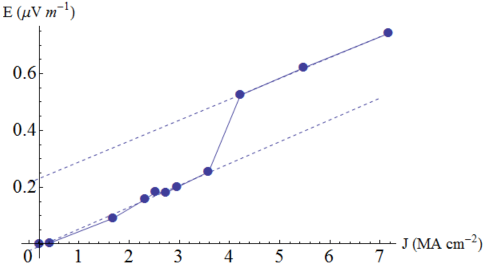
Preconditioning

- The Domain Decomposition (DD) and Smooth Aggregation (SA) scale well.
- But have the worst timings.



$E - J$ Curve

- The $E - J$ curve helps characterize J_c in Materials.
- Finding J_c numerically can be a chore!
- However this observable is still important.
- Flux Flow!



initialization of time;
set initial time step;
Newton iterates, $\psi(0) = \psi_{0,0}$ and $\mathbf{A}(0) = \mathbf{A}_{0,0}$,;
set tol$=10^{-8}$;
for *each time step i* **do**
  Non Linear Solve;
  if *steady state* **then**
    STOP;
  else
    update solution and continue;
  end
end

Algorithm 1: General TDGL Algorithm

Let: G(ψ,**A**) be the G-L equation
Let: M(ψ,**A**) be the Maxwell equation
for $k = 1, \cdots, k_{max}$ *iterate between equations* **do**

- **Solve**: $\text{Jac}[\text{G}(\psi_{k,i},\mathbf{A}_{k,i})]\delta\psi_{k+1,i}{=}{-}\text{Resid}[\text{G}(\psi_{k,i},\mathbf{A}_{k,i})]$;
- $\psi_{k+1,i} = \psi_{k,i} + \delta\psi_{k+1,i}$;
- **Solve**: $\text{M}(\psi_{k+1,i},\mathbf{A}_{k,i})\mathbf{A}_{k+1,i}{=}\nabla \times \mathbf{H}$;
- **Calculate**: $R_1{=}\text{Resid}[\text{G}(\psi_{k,i+1},\mathbf{A}_{k,i+1})]$;
- **Calculate**: $R_2{=}\text{Resid}[\text{M}(\psi_{k+1,i},\mathbf{A}_{k,i+1})]$;
- set Max R=max$\{R_1, R_2\}$;
- **if** *Max R* $<$ *tol* **then**
  - go to next time step;
- **else**
  - continue iterating;
- **end**

end

Algorithm 2: The decoupling of type 1 algorithm for the TDGL system

Decoupling of Type I Vs Full Equations

- Performance test to steady state.
- Full Equations DOFs: 25920, Decoupling of Type 1 DOFs: 13122.
- Since our matrix size has been reduced by 1/2, our storage is cut by 1/4.
- At the cost of more time steps and non-linear iterations.

Method	Storage	Time steps	Horizon	Wall Time
Full	1165896	145	5002.05	526.226 s
Decoup. type 1	296964	328	5477.00	1463.32 s

Method	NL time (sec)	NL time Avg. (sec)	Avg NL steps
Full	522.459	3.60	1.55
Decoup. type 1	1463.32	4.461	3.71

Decoupling of Type I Vs Full Equations

- The adaptive time step does not give a fair comparison.
- For a fixed non-dimensionalized time step of 0.5.
- Full Equations DOFs: 410,880, Decoupling of type 1 DOFs: 206,082.
- The Decoupling Method gives a shorter solve time! (2 small vs 1 big)

Method	Time steps	NL. Time (sec)	NL time Avg. (sec)	Avg NL steps
Full Eq.	1000	63733.7	63.733	1.455
Decoup. type 1	1000	44974.0	44.974	2.128

Decoupling

- The next clear step is to decouple the TDGL system into 4 four systems.
- This would cut the matrix storage by $1/16$ when compared to the full equations.
- Possibly more time steps to steady state.
- What if we just want the steady state?

Decoupling of Type 2

- What if we decouple and linearize using previous time steps (ψ_n , $\mathbf{A}_n$).
- Now we have 4 decoupled linear equations.
- Best case scenario?

$$\frac{\partial \psi}{\partial t} - \Delta \psi^{n+1} = -(|\psi^n|^2 - (1 - \frac{T}{T_c})\psi^n - \frac{i}{\kappa}\psi^n \nabla \cdot \mathbf{A}^n - \frac{i}{\kappa}\mathbf{A}^n \cdot \nabla \psi^n - \frac{1}{\lambda^2}\psi^n |\mathbf{A}^n|^2 \text{ in } \Omega \times (0, T) , \quad (6)$$

$$\sigma(\frac{1}{\lambda^2}\frac{\partial \mathbf{A}}{\partial t}) + \nabla \times \nabla \times \mathbf{A}^{n+1} - \nabla(\nabla \cdot \mathbf{A}^{n+1}) = \sigma(\frac{1}{\lambda^2}\frac{\partial \mathbf{A}}{\partial t}) - \Delta \mathbf{A}^{n+1}$$
$$= -\frac{i}{2\kappa}(\psi^n \nabla \psi^{*n} - \psi^{*n} \nabla \psi) - \frac{1}{\lambda^2}|\psi^n|^2 \mathbf{A}^n + \nabla \times \mathbf{H} \text{ in } \Omega \times (0, T) . \quad (7)$$

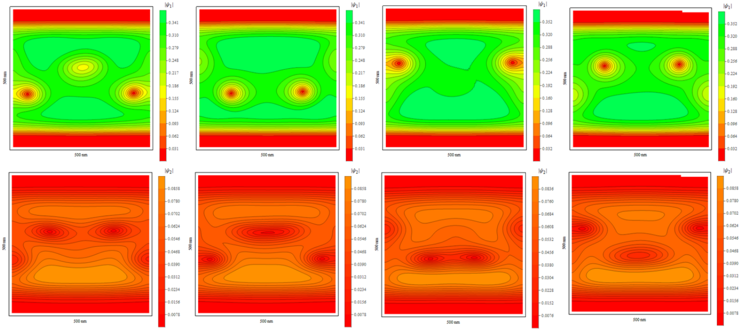
Decoupling of Type 2

- Time step size restriction for “backward Euler”.
- The restriction has some relations to resolution.
- Could another time method help (Exponential Integrators?)

DOFs	Domain Size (nm^2)	Acceptable Time Step size (ND units)
6561	10^2	0.5
25921	20^2	0.3
103041	20^2	0.3
103041	30^2	0.0.0625
410881	50^2	$<$0.0.0625

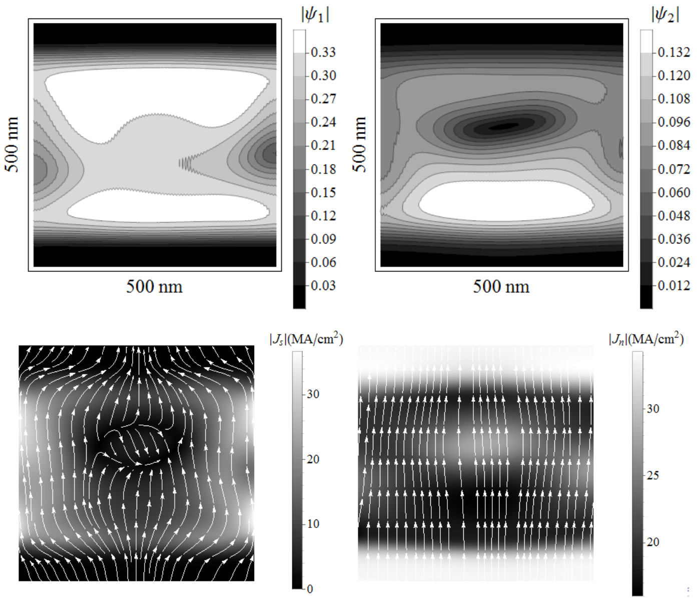
Simulation 1: Flux Flow

- $\mathbf{H} = 0$, $J = 33.717$ MA cm^{-2}, $T = 30$ K, ψ_1 (top), ψ_2 (bottom).
- $t = 0.6912$ ns, $t = 0.6933$ ns, $t = 0.6974$ ns, $t = 0.6992$ ns.



$J = 33.717$ MA cm^{-2}

Resistance Free Current?



Resistance Free Current?

- $\mathbf{J}_s$ and $\mathbf{J}_n$ in y-direction averaged over x.
- 1/2 of the current is Normal! ($J = 33.717$ MA cm^{-2})
- Flux flow is a problem!