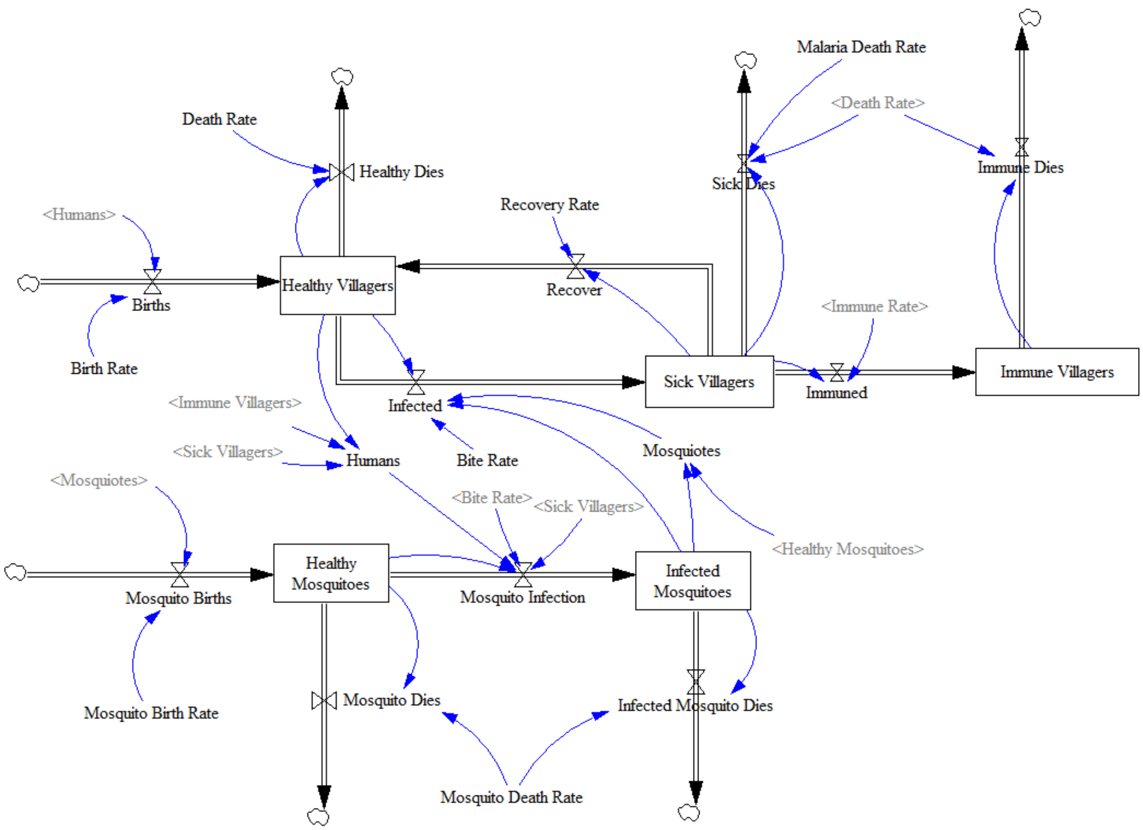
Mathematical Modeling for Malaria Transmission Dynamics

Malaria is one of the most devastating diseases and a leading cause of death in tropical regions of the world. A mathematical model will help public health professionals to have a better understanding of the disease transmission and identify effective measures for the prevention and elimination of the disease.

Malaria is a vector borne disease. The diagram of malaria transmission dynamics model below shows a relatively complete model of the transmission of the malaria parasite in host-Human and vector-Mosquitoes.



The malaria parasite is transferred to humans through the bites of infected female mosquitoes that carry the parasites. Assuming that there is no latent period for infection, and part of the sick villagers get immunity to the disease, humans can be partitioned into three categories: healthy villagers, sick villagers and immune villagers. For any given time, the change in the number of healthy villagers depends on the numbers of births, deaths, infected, and recovered villagers. Change in the number of sick villagers depends on the number of infected villagers, recovered villagers, villagers who gained immunity, and deaths of sick villagers. Change in immune villagers depends on the number of villagers who gained immunity and the number of deaths in immune villagers.

Mosquitos get infected through a blood meal from an infected human. The life cycle of the malaria parasite in mosquitos is a little simpler since infected mosquitos end with death, with no recovery or immunity. For any time period, changes in the number of healthy mosquitoes depends on the numbers of births, deaths, and infected mosquitoes, and change in the number of infected mosquitoes depends on infected and death of infected mosquitoes during that period of time.

The following equations give detailed relationships among variables.

Humans= Healthy Villagers+Sick Villagers+Immune Villagers

*Births= Birth Rate*Humans*

*Healthy Dies= Healthy Villagers*Death Rate*

*Infected= Bite Rate*Healthy Villagers*(Infected Mosquitoes/Mosquitoes)*

*Sick Dies= (Malaria Death Rate+Death Rate)*Sick Villagers*

*Immune Dies=Death Rate*Immune Villagers*

*Recover= Sick Villagers*Recovery Rate*

*Immune=Sick Villagers*Immune Rate*

Sick Villagers= Infected-Recover-Immune-Sick Dies

Immune Villagers=Immune-Immune Dies

Healthy Villagers= Births+Recover-Healthy Dies-Infected

Mosquitoes= Healthy Mosquitoes+Infected Mosquitoes

*Mosquito Births= Mosquitoes*Mosquito Birth Rate*

*Mosquito Dies=Healthy Mosquitoes*Mosquito Death Rate*

*Mosquito Infection= (Sick Villagers/(Humans+1))*Healthy Mosquitoes*Bite Rate*

*Infected Mosquito Dies= Infected Mosquitoes*Mosquito Death Rate*

Infected Mosquitoes= Mosquito Infection-Infected Mosquito Dies

Healthy Mosquitoes= Mosquito Births-Mosquito Dies-Mosquito Infection

For the first part of this project, you are asked to build a malaria transmission dynamics model and plot the time series of the variables in the model. A partial SAGE script is given to help you get started.

The code is set up to represent the changes in each of the populations as a difference equation where the new values are calculated as a function of the rates of change for each population.

For example, for the healthy population;

$H_{t+1} = H_t + H_t$ *BirthRate - H_t *DeathRate + S_t * RecoveryRate - H_t * InfectionRate

Where

H_t = Healthy population at time t
S_t = Sick population at time t

Similarly, you can construct an equation for each part of the human population using the relationships given above. For the mosquito population, a simpler equation taking into account births, deaths, and infected can be added. Your final model should include a plot the populations of each sector of the human population and plot of the mosquito population.

For your model, here are some starting parameter values:

Starting number of healthy mosquitoes = 1000
Starting number of infected mosquitoes = 0
Starting number of healthy humans = 499
Starting number of sick humans = 1
Starting number of immune humans = 0
Human birth rate = 0.000092/day based on an annual rate of 0.03358
Human death rate = 0.00001945/day based on an annual rate of 0.0071
Human malaria induced death rate = 0.00016123/day based on an annual rate of 0.0588486
Human recovery rate = 1- malaria induced death rate
Human immunity rate = 0.01

The parameters for mosquitos:
Mosquito birth rate = 0.01/day
Mosquito death rate = 0.011/day
Bite rate from mosquitos = 0.24

Create the basic model and run it for 200 days. Examine the resulting breakdown of the human populations and the infected and uninfected mosquito population. Then, run the model for 1500 days and compare what happens to the trends in the disease. Analyze the sensitivity of the model results to changes in the immunity rate and in the bite rate for mosquitos.

Prepare a final submission package to be given to the contest judges for scoring. The package should include the following:

1. All relevant versions of the code you used to test the range of circumstances you analyzed. Each version should be internally documented to indicate which of the circumstances was tested and what parameters were changed. For sensitivity runs, you need not present multiple versions of

the code with each parameter change but should indicate in the internal documentation the range of runs that were made and the range of values used in those test.

2. Prepare a summary presentation of the project that contains the following:
   a. A title, list of team members, and date of the contest
   b. A summary of the project results including the following sections:
      i. Description of the final main code along with a code listing
      ii. A summary of the model assumptions and how the model simplifies the systems that it represents
      iii. A summary of the model runs made to derive the values of the parameters that met the objective of the modeling exercise. The summary should embed graphs and/or table to illustrate the model results.
      iv. A summary of the sensitivity testing that was completed along with their results using appropriate graphs and/or tables.
3. If you undertake any of the extra projects below, add slides that provide the relevant summary, code changes, graphs, and tables as appropriate as a separated section for each of the extra projects.

Extra Model Additions

1. The progression of the disease is not nearly as simple as depicted in this basic model. Consider a different form of malaria in which sick villagers can go into remission stage and have relapses. The probability of a relapse for some forms of malaria is 17%. Refine the model to calculate the impacts of relapses on the sick population.
2. Suppose there is a seasonal increase in the number of mosquitoes, such as a rainy season in tropical areas. The mosquito birth rate mbr and death rate mdr are then defined by:

   mbr=(cos((time_in_days*(2*PI))/(365)+1))*max_mosquito_birth_rate/2,

   mdr=(cos((time_in_days*(2*PI))/(365)+1))max_mosquito_death_rate/2.

   Where the max_mosquito_birth_rate = 0.02
   max_mosquito_death_rate = 0.022

   Refine the malaria model to reflect these seasonal variations and analyze the results as compared with the original model.
3. According to a study about the effectiveness of mosquito nets in reducing the infection rate, insecticide treated nets (ITN) reduced the incidence of uncomplicated malarial episodes by 50% compared to no nets, and 39% compared to untreated nets. If the cost of treated nets is $3.50 and untreated nets is $2.50, how much will it cost to provide nets to your population and how will this impact the incidence of the disease? Assume that each net can cover two people. Assume your village is in Ghana so that you can look insert the average family size into your calculations. Adjust the model to test this policy and describe both the cost of treatment and the benefit in terms of reduced cases of malaria. (See http://en.wikipedia.org/wiki/Mosquito_net)

References:

http://www.cdc.gov/malaria/about/biology/

http://www.malariajournal.com/content/10/1/202

http://apps.who.int/gho/data/node.main.A1362

(http://www.capital.edu/uploadedFiles/Capital/Academics/Schools_and_Departments/Natural_Sciences,_Nursing_and_Health/Computational_Studies/Educational_Materials/Content/Modeling%20Malaria.pdf)