Convex Hulls and the Calliper Algorithm

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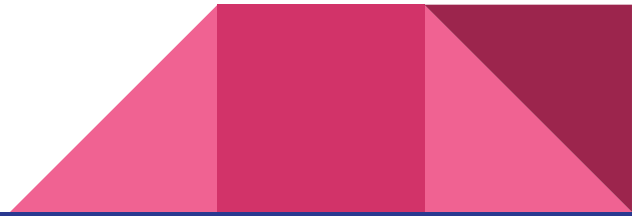
Motivation

If you had a set of points on a 2D plane, what is the most efficient way to draw a rectangular box around them?

Maybe there’s stains on your carpet, and you want to find out the smallest rectangular piece of carpet you can cut out and replace.

Maybe there’s bullet holes no your wall, and you want to cut out the smallest piece of dry-wall to replace it.



The naive approach

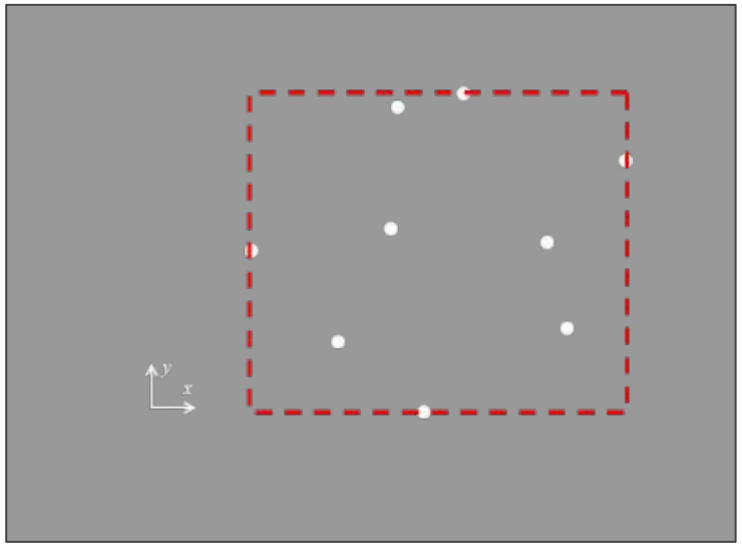
- Find the min and max x-coordinates.
- Find the min and max y-coordinates.

Advantages:

- Fast
- Easy to implement
- Axis aligned bounding box

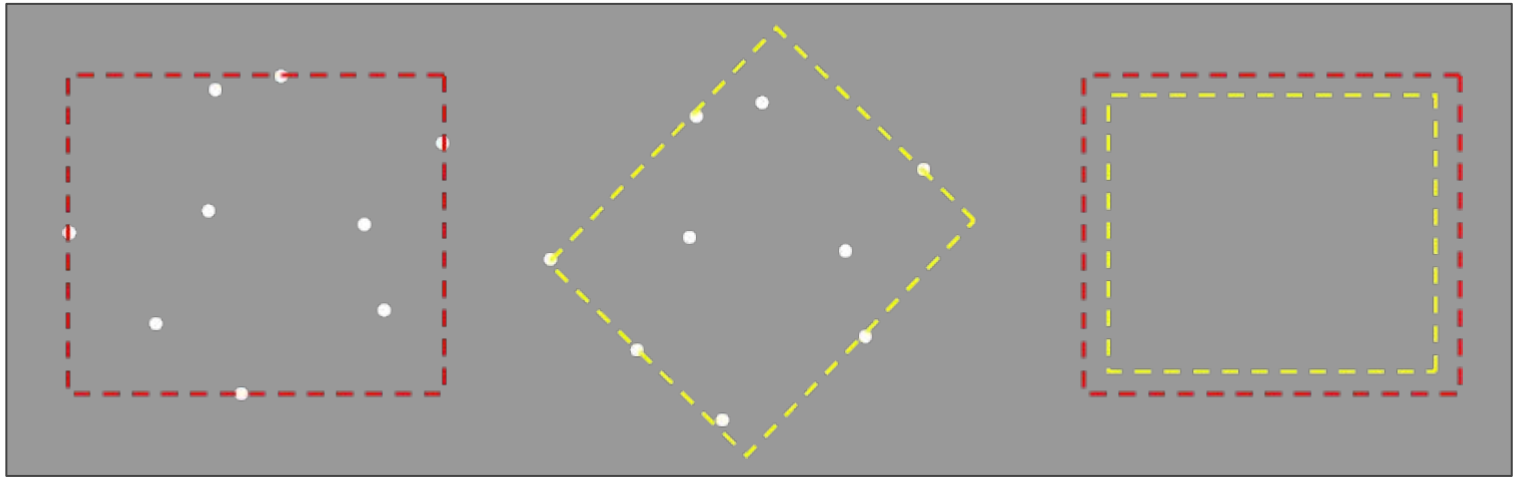
Disadvantages:

- Not the optimal bounding box



Slightly less naive

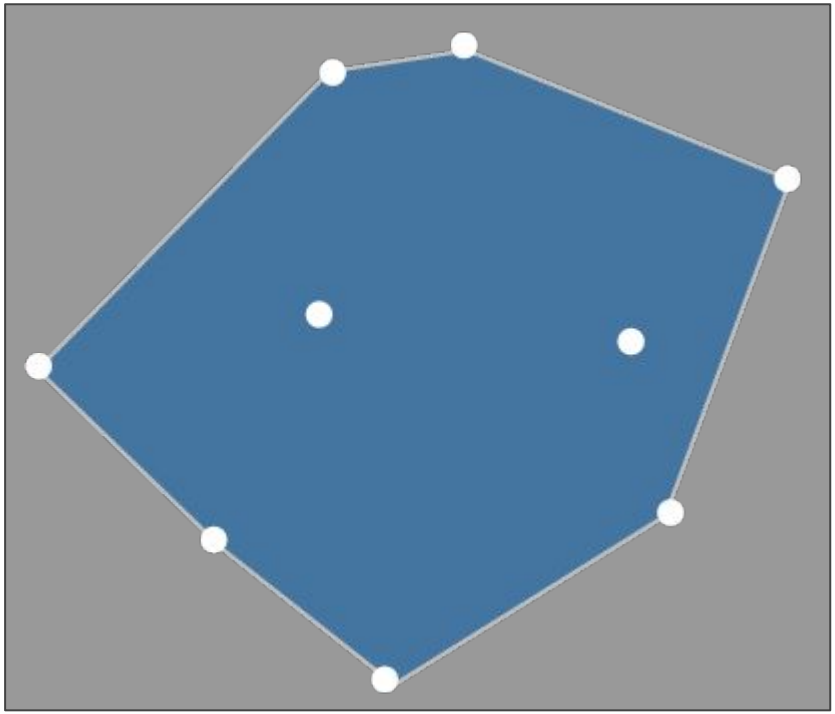
What if we can rotate the bounding box?



Convex Hull

A convex hull is the smallest polygon that encloses all the points, and has internal angles that are less than 180 degrees.

We can compute it by starting with a point on the edge, and then sweeping around. As you touch each point, you add an edge to it until you connect back to the original point.

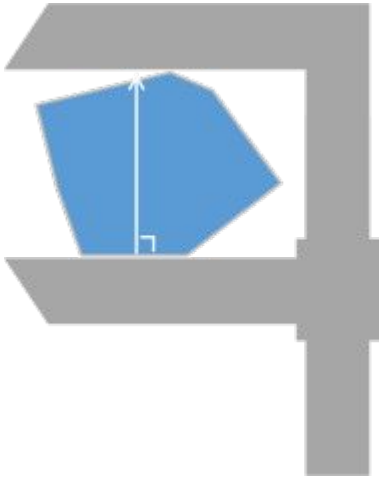


The Calliper Algorithm

In 1975, Freeman and Shapira proved that the minimum area rectangle of a set of points is collinear with one of the edges of the convex hull that encloses the set.

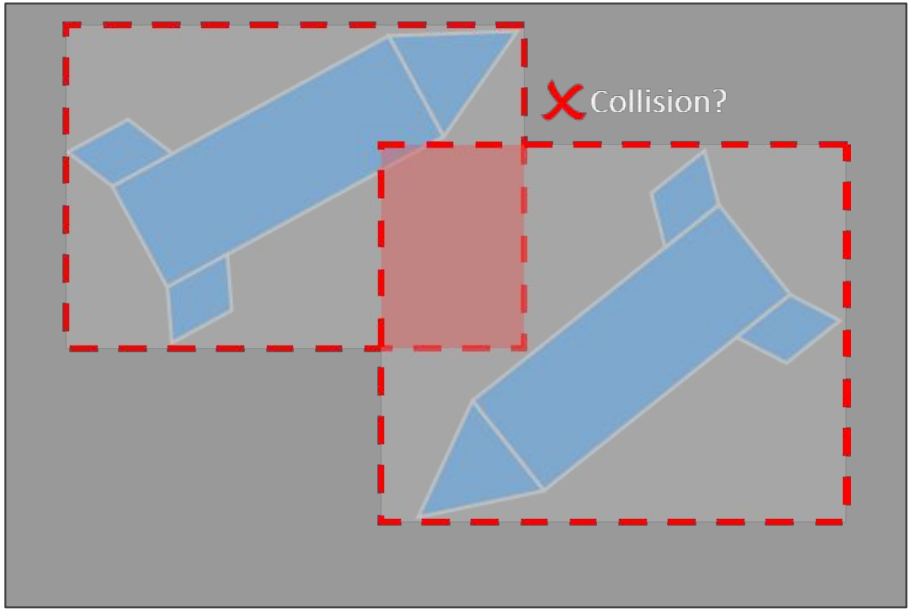
So, we can rotate our convex hull onto each of its edges and measure the perpendicular distance to the opposite side.

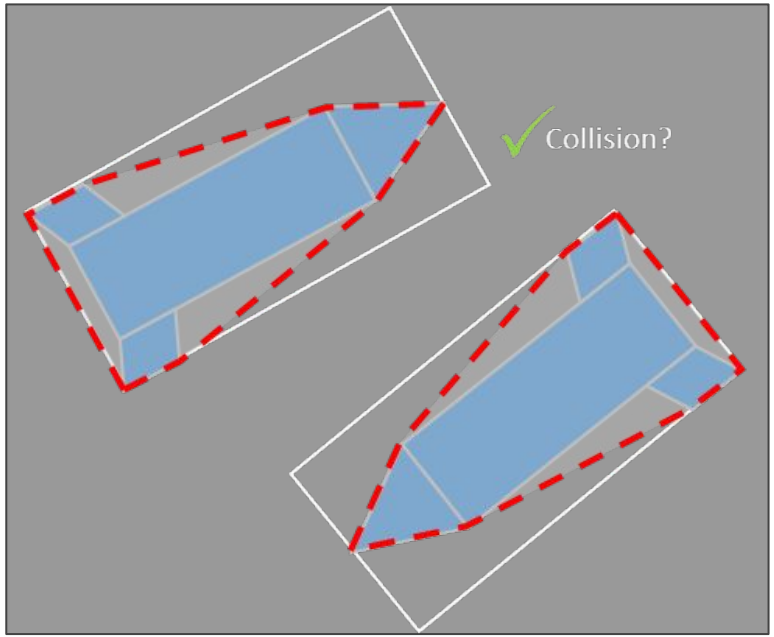
Assuming the convex hull has n sides, once we've checked the n bounding boxes we'll know which bounding box has the smallest area.



One important application

Video Games!





References

Nick Berry - “Bounding Boxes”: http://datagenetics.com/blog/march12014/index.html