Volume 2, No. 3, March 2014
Journal of Global Research in Mathematical Archives
ISSN 2320 – 5822
MATHEMATICAL SECTION
Available online at http://www.jgrma.info

SOFT EXPERT *pg SET

T.A.Albinaa* and I.Arockiarani*
* Nirmala College For Women, Coimbatore, Tamil Nadu , India
albinakavi@yahoo.in

Abstract: In the present paper, we explore a class of sets namely soft *pre generalized closed sets in expert soft topological spaces and obtain their characterizations.

1. INTRODUCTION

In many complicated problems arising in the fields of engineering, social science, economics, medical science,etc involving uncertainties, classical methods are found to be inadequate in recent times. Molodtsov [13] pointed out that the important existing theories viz. Probability Theory, Fuzzy set Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. which can be considered as a mathematical tools for dealing with uncertainties, have their own difficulties. He further pointed out that the reason for these difficulties is, possibly, the inadequacy of the parameterization tool of the theory. In 1999 he initiated the novel concept of soft set as a new mathematical tool for dealing with uncertainties. Soft set theory, initiated by Molodtsov [13], is free of the difficulties present in these theories. Soft systems provide a very general framework with the involvement of parameters. Therefore, researches work on soft set theory and its applications in various fields are progressing rapidly.Maji et al [10] presented an application of soft sets in decision making problems and studied basic notions of soft set theory.

Many researchers have studied this theory and they created models to solve problems in decision making. But most of these models deal with only one opinion (or) with only one expert. This causes a problem with the user when questioners are used for the data collection. Alkhazaleh and Salleh [15] defined soft expert set and created a model in which the user can know the opinion of the experts in the model without any operations.

The present paper is mainly concerned with soft expert generalized preclosed sets and their basic properties in soft expert topological spaces which is introduced with the help of soft expert open sets.

II. PRELIMINARIES

Let U be an initial universe, E be a set of parameters and X be the set of opinions, $Z = E \times X \times O$ and $A \subseteq Z$. Let P(U) denote the power set of U and A be a non-empty subset of Z. A pair (F,A) is called a soft expert set over U,where F is a mapping given by $F : A \to P(U)$.

Definition 2.1: [7]

(i) Two soft sets (F,A) and (G,B) over a common universe U, we say that (F,A) is a soft subset of (G,B) if $A \subseteq B$ and for all $e \in B, G(e) \subseteq F(e)$.

We write (F,A) $\tilde{\subseteq}$ (G,B). (F,A) is said to be a soft superset of (G,B), if (G,B) is a soft subset of (F,A). We denote it by (F,A) $\tilde{\supseteq}$ (G,B).

(ii) Two soft sets (F,A) and (G,B) over a common universe U are said to be soft equal if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A)

Definition 2.2: [7]

The union of two soft sets of (F,A) and (G,B) over the common universe U is the soft set (H,C),where $C = A \bigcup B$ and for all $e \in C$,H(e) = F(e) if $e \in A - B$, G(e) if $e \in B - A$ and $F(e) \bigcup G(e)$ if $e \in A \cap B$. We write $(F,A) \bigcup (G,B) = (H,C)$.

Definition 2.3: [7]

The intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe U, denoted $(F,A) \cap (G,B)$, is defined as $C = A \cap B$, $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition 2.4: [7]

Let $x \in U$, (x,A) denotes the soft expert set over U for which x(e) ={x}, for all $e \in A$. Let (F,A) be a soft expert set over U and V be a non-empty subset of U. Then the sub soft expert set of (F,A) over V denoted by (V_F,A),is defined as follows: $V_F (e) = V \cap F(e)$, for all $e \in A$. In other words $(V_F,E) = \widetilde{V} \cap (F,A)$.

Definition 2.5: [7]

Let τ be the collection of soft expert sets over U, then τ is said to be a soft expert topology on U if

(1) $\phi, \widetilde{U}$ belong to τ

(2) the union of any number of soft expert sets in τ belongs to τ

(3) the intersection of any two soft expert sets in τ belongs to τ

Definition 2.6: [7]

The triplet (U, τ ,A) is called a soft expert topological space over U. Let (U, τ ,A) be a soft expert space over U, then the members of τ are said to be soft expert open sets in U.

Definition 2.7: [7]

Let (U, τ ,A) be a soft expert space over U. A soft expert set (F,A) over U is said to be a soft expert closed set in U, if its complement $(F,A)'$ belongs to τ .

Definition 2.8: [7]

Let $\tau = \{ \phi, \widetilde{U} \}$. Then τ is called the soft expert indiscrete topology on U and (U, τ ,A) is said to be a soft indiscrete space over U. Let τ be the collection of all soft expert sets which can be defined over U. Then τ is called the soft expert discrete topology on U and (U, τ ,A) is said to be a soft expert discrete space over U.

Definition 2.9: [7]

Let (U, τ ,A) be a soft expert topological space over U and (F,A) be a soft expert set over U. Then, the soft expert closure of (F,A) denoted by cl(F,A) is the intersection of all soft expert closed supersets of (F,A). Clearly cl(F,A) is the smallest soft expert closed set over U which contains (F,A).

Definition 2.10: [7]

The soft expert interior of (F,A), denoted by int(F,A) is the union of all soft expert open subsets of (F,A). Clearly int(F,A) is the largest soft expert open set over U which is contained in (F,A).

Definition 2.11: [7]

A soft set (F,A) in a soft expert topological space (U, τ ,A) over U is called soft expert generalized closed (soft expert g-closed) if cl(F,A) $\tilde{\subseteq}$(G,A) whenever (F,A)$\tilde{\subseteq}$(G,A) and (G,A) is soft expert open in U.

Definition 2.12: [7]

A soft expert set (F,A) is called soft expert generalized open (soft expert g-open) if and only if (F,A)′ is soft expert g-closed.

III. SOFT EXPERT *pg CLOSED SET

Definition 3.1:

A soft set (F,A) in a soft expert topological space (U, τ ,A) over U is called

soft expert *pre generalized closed **(soft expert *pg-closed)** if pcl (F,A) $\tilde{\subseteq}$(G,A) whenever (F,A)$\tilde{\subseteq}$(G,A) and (F,A) is soft expert preopen in U.

Example 3.2:

Let X = {p, q, r}, U = {u_1, u_2, u_3, u_4}, E = {e_1, e_2, e_3}.

Let τ = { $\tilde{\phi}$,$\tilde{\bigcup}$, (F,A), (G,A)} where

(K,A) = {((e_1, p, 1), {u_1, u_2, u_4}),((e_1, q, 1), {u_1, u_4}), ((e_2, p, 1), {u_4}), ((e_2, q, 1), {u_1, u_3}),((e_2, r, 1), {u_1, u_2, u_4}), ((e_3, p, 1), {u_3, u_4}), ((e_3, q, 1), {u_1, u_2}), ((e_3, r, 1),{u_4}),((e_1, p, 0), {u_3}), ((e_1, q, 0), {u_2, u_3}), ((e_1, r, 0), {u_1, u_2}), ((e_2, p, 0), {u_1, u_2, u_3}),((e_2, q, 0), {u_2, u_4}), ((e_2, r, 0),{u_3}),((e_3, p, 0), {u_1, u_2}), ((e_3, q, 0), {u_3, u_4}),((e_3, r, 0), {u_1, u_2, u_3})} is soft expert *pg-closed but not soft expert closed.

Theorem 3.3:

If (F,A) is soft expert *pg-closed in U and (F,A) $\tilde{\subseteq}$ (G,A) $\tilde{\subseteq}$ pcl(F,A), then (G,A) is soft expert *pg-closed.

Proof:

Suppose (F,A) is soft expert *pg-closed in U and (F,A) $\tilde{\subseteq}$ (G,A) $\tilde{\subseteq}$ pcl (F,A). Let (G,A) $\tilde{\subseteq}$ (H,A) and (H,A) is soft expert preopen in U. Since (F,A) $\tilde{\subseteq}$(G,A) and (G,A) $\tilde{\subseteq}$ (H,A),we have (F,A) $\tilde{\subseteq}$ (H,A). Hence pcl(F,A) $\tilde{\subseteq}$ (H,A) {Since (F,A) is soft expert *pg-closed}.

This implies pcl(G,A) $\tilde{\subseteq}$ pcl(F,A) $\tilde{\subseteq}$ (H,A).Therefore, (G,A) is soft expert *pg-closed.

Next it is observed that soft expert *pg-closed sets are closed under finite union. This is shown in the following**.**

Theorem 3.4:

If (F,A) and (G,A) are soft expert *pg-closed sets then (F,A) $\tilde{\bigcup}$ (G,A) is also *pg-closed.

Proof:

Suppose that (F,A) and (G,A) are soft expert *pg-closed sets. Let (H,A) be soft expert preopen in U and (F,A) $\tilde{\cup}$ (G,A) $\tilde{\subseteq}$ (H,A). Since (F,A) and (G,A) are soft expert *pg-closed sets, we have pcl(F,A) $\tilde{\subseteq}$ (H,A) and pcl(G,A) $\tilde{\subseteq}$ (H,A). Therefore, soft pcl {(F,A) $\tilde{\cup}$ (G,A) } $\tilde{\subseteq}\{pcl(F,A)\}\tilde{\cup}\{pcl(G,A)\}\tilde{\subseteq}(H,A)$

Theorem 3.5:

If a set (F,A) is soft expert *pg-closed in U, then pcl(F,A) \(F,A) contains only null soft expert preclosed set.

Proof.

Suppose that (F,A) is soft expert *pg-closed in U. Let (H,A) be soft expert preclosed and (H,A) $\tilde{\subseteq}$ pcl(F,A) \ (F,A). Since (H,A) is soft expert preclosed, we have (H,A)′ is soft expert preopen. Since (H,A) $\tilde{\subseteq}$ pcl(F,A) \ (F,A), we have (H,A) $\tilde{\subseteq}$ pcl(F,A) and (H,A) $\tilde{\subseteq}$ pcl(F,A)′.

Hence (F,A) $\tilde{\subseteq}$ (H,A)′. Consequently pcl(F,A) $\tilde{\subseteq}$ (H,A)′ {Since (F,A) is soft expert *pg-closed in U}. Therefore,(H,A) $\tilde{\subseteq}$ {pcl(F,A)}′. Hence (H,A) $\tilde{\subseteq}$ {pcl(F,A)}′ $\tilde{\cap}$ pcl(F,A).

Hence pcl(F,A)\(F,A) contains only null soft expert preclosed set.

Theorem 3.6:

If (F,A) is soft expert *pg-closed and (F,A) $\tilde{\subseteq}$ (G,A) $\tilde{\subseteq}$ pcl(F,A) then pcl(G,A) \ (G,A) contains only null soft expert preclosed set.

Proof.

Let (F,A) be soft expert *pg-closed and (F,A) $\tilde{\subseteq}$ (G,A) $\tilde{\subseteq}$ pcl(F,A).Then (G,A) is soft expert *pg-closed .Therefore, pcl(G,A) \ (G,A) contains only null soft expert preclosed set.

Definition 3.7:

A soft expert set (F,A) is called soft expert *pre generalized open **(soft expert *pg-open)** if (F,A) $\tilde{\subseteq}$ pint(G,A) whenever (F,A) $\tilde{\subseteq}$ (G,A) and (F,A) is soft expert preclosed in U.

Theorem 3.8:

A set (F,A) is soft expert *pg-open if and only if (H,A) $\tilde{\subseteq}$ pint(F,A) whenever (H,A) is soft expert preclosed and (H,A) $\tilde{\subseteq}$ (F,A).

Proof:

Suppose that (F,A) is soft expert *pg-open. Then, (F,A)′ is soft expert *pg-closed.

Suppose that (H,A) is soft expert preclosed and (H,A) $\tilde{\subseteq}$ (F,A).Then, (H,A)′ is soft expert preopen and (F,A)′ $\tilde{\subseteq}$ (H,A)′.Therefore, pcl(F,A)′ $\tilde{\subseteq}$ (H,A)′ {Since (F,A)′ is soft expert *pg-closed}. Since pcl(F,A)′ = [pint(F,A)]′,we have [pint(F,A)]′ $\tilde{\subseteq}$ (H,A)′.

Hence (H,A) $\tilde{\subseteq}$ pint(F,A).

Conversely,

suppose (H,A) $\tilde{\subseteq}$ pint(F,A) whenever (H,A) is soft expert preclosed and (H,A) $\tilde{\subseteq}$ (F,A).

Then, (F,A)′ $\tilde{\subseteq}$ (H,A)′ and (H,A)′ is soft expert preopen. Take (M,A) = (H,A)′.

Since (H,A) $\tilde{\subseteq}$ pint(F,A), [pint(F,A)]′ $\tilde{\subseteq}$ (H,A)′ = (M,A). Since pcl(F,A)′ = [pint(F,A)]′, pcl(F,A)′ $\tilde{\subseteq}$ (M,A). Therefore, (F,A)′ is soft expert *pg-closed. This completes the proof.

Example 3.9:

Let X = {p, q, r}, U = {u_1, u_2, u_3, u_4}, E = {e_1, e_2, e_3}.

Let τ = {$\tilde{\phi}$,$\tilde{\bigcup}$, (F,A), (G,A)} where

$(H,A)^C$ = {((e_1, p, 1),{u_1,u_2,u_4}), ((e_1, q, 1), {u_1, u_4}), ((e_1, r, 1), {u_3, u_4}),
((e_2, p, 1),{u_4), ((e_2, q, 1),{u_1, u_3}), ((e_2, r, 1), {u_1,u_2,u_4}), ((e_3, p, 1), {u_3, u_4}),
((e_3, q, 1), {u_1, u_2}), ((e_3, r, 1),{u_4}), ((e_1, p, 0), {u_3}), ((e_1, q, 0), {u_2, u_3}), ((e_1, r, 0),
{u_1, u_2}),((e_2, p, 0), {u_1,u_2,u_3}), ((e_2, q, 0), {u_2, u_4}), ((e_2, r, 0), {u_3}), ((e_3, p, 0), {u_1, u_2}),
((e_3, q, 0), {u_3, u_4}), ((e_3, r, 0), {u_1,u_2,u_3})} is a soft expert *pg-open but not soft expert open set .

Theorem 3.10:

If (F,A) and (G,A) are soft expert *pg-open sets then so is (F,A) $\tilde{\bigcap}$ (G,A).

Proof:

Suppose that (F,A) and (G,A) are soft expert *pg-open sets.

Let (H,A) be soft expert preclosed and (H,A) $\tilde{\subseteq}$ (F,A) $\tilde{\bigcap}$ (G,A). This implies,(H,A) $\tilde{\subseteq}$ (F,A) and (H,A) $\tilde{\subseteq}$ (G,A). By assumption (H,A) $\tilde{\subseteq}$ pint(F,A) and (H,A) $\tilde{\subseteq}$ pint(G,A) {Since(F,A) and (G,A) are soft expert *pg-open}. Therefore,(H,A) $\tilde{\subseteq}$ pint(F,A) $\tilde{\bigcap}$ pint(G,A). (H,A)

$\tilde{\subseteq}$ pint[(F,A) $\tilde{\bigcap}$ (G,A)]. Hence (F,A) $\tilde{\bigcap}$ (G,A) is soft expert *pg-open.

Theorem 3.11:

If (F,A) is soft expert *pg-open in U and pint(F,A) $\tilde{\subseteq}$ (G,A) $\tilde{\subseteq}$(F,A), then (G,A) is soft expert *pg-open.

Proof:

Suppose (F,A) is soft expert *pg-open in U and pint(F,A) $\tilde{\subseteq}$ (G,A) $\tilde{\subseteq}$ (F,A).

Let (H,A) be soft expert preclosed and (H,A) $\tilde{\subseteq}$ (G,A). Since (H,A) $\tilde{\subseteq}$ (G,A), (G,A) $\tilde{\subseteq}$ (F,A), we have (H,A) $\tilde{\subseteq}$ (F,A). Since (F,A) is soft expert *pg-open, we have (H,A)$\tilde{\subseteq}$ pint(F,A) . Since pint(F,A) $\tilde{\subseteq}$ (G,A) we have (H,A) $\tilde{\subseteq}$ pint(F,A) $\tilde{\subseteq}$ pint(G,A). Hence (G,A) is soft expert *pg-open in U.

Theorem 3.12:

If a set (F,A) is soft expert *pg-closed in U, pcl(F,A)\ (F,A) is soft expert*pg-open.

Proof:

Suppose that (F,A) is soft expert *pg-closed in U. Let (H,A) be soft expert preclosed and (H,A) $\tilde{\subseteq}$ pcl(F,A) \ (F,A). Since (F,A) is soft expert *pg-closed in U, we have pcl(F,A) \ (F,A) contains only null soft expert preclosed set. Since (H,A) $\tilde{\subseteq}$ pcl(F,A) \ (F,A), we have (H,A) = ϕ $\tilde{\subseteq}$ pint [pcl(F,A) \ (F,A)]. Therefore, pcl(F,A) \ (F,A) is soft expert *pg-open.

Theorem 3.13:

If a soft expert set (F,A) is soft expert *pg-open in a soft expert topological space (U, τ ,A), then (G,A) = $\tilde{U}$ whenever (G,A) is soft expert preopen and pint(F,A) $\tilde{\bigcup}$ (F,A)′ $\tilde{\subseteq}$ (G,A).

Proof:

Suppose that (F,A) is soft expert *pg-open in a soft expert topological space and (G,A) is soft expert preopen and pint(F,A) $\tilde{\bigcup}$ (F,A)′ $\tilde{\subseteq}$ (G,A).

Then, (G,A)′ $\tilde{\subseteq}$ {pint(F,A) $\tilde{\bigcup}$ (F,A)′}′ = [(F,A)′] \ (F,A)′. Therefore,[(F,A)′] \ (F,A)′ contains only null soft expert preclosed set in X . Consequently (G,A)′ = ϕ . Hence (G,A) = $\tilde{U}$

Theorem 3.14:

The intersection of a soft expert *pg-open set and soft expert preopen set is always soft expert *pg-open.

Proof:

Suppose that (F,A) is soft expert *pg-open and (G,A) is soft expert preopen. Since (G,A) is soft expert preopen, we have (G,A)′ is soft expert preclosed. Then (G,A)′ is soft expert *pg-closed. Hence, (G,A) is soft expert *pg-open. Hence (F,A) $\tilde{\bigcap}$ (G,A) is soft expert *pg-open .

REFERENCES

[1] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1986),87–96.

[2] K. Atanassov, Operators over interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems, 64 (1994), 159–174.

[3] D. Chen, The parametrization reduction of soft sets and its applications, Computers and Math. with Appl., 49, (2005), 757–763.

[4] W. Dunham, $T_{1/2}$ -spaces, Kyungpook Mathematical Journal, 17 (2),(1977),161–169.

[5] M.B. Gorzalzany, A method of inference in approximate reasoning based on interval- valued fuzzy sets, Fuzzy Sets and Systems, 21, (1987), 1–17.

[6] S. Hussain, On soft expert topological spaces, SpringerPlus(2012),:42.doi:10.1186/2193-1801,1-42

[7] K. Kannan, Soft Expert generalized closed sets in soft topological spaces, International Journal of Pure and Applied Mathematics,Volume 93,No 2,(2014),233-242

[8] N. Levine, Generalized closed sets topology, Rendiconti del Cirwlo Matematico di Palermo, Serie II, 19 (1970), 89–96.

[9] N. Levine, Semi-open sets and semi-continuity in topological spaces, AmerMath. Monthly, 70 (1963), 36–41.

[10] P.K. Maji, R. Biswas, R. Roy, An application of soft sets in a decision making problem,Comput. Math. Appl., 44, (2002), 1077–1083.

[11] P. K. Maji, R. Biswas, R. Roy, Soft set theory, Comput. Math. Appl., 45, (2003), 555– 562.

[12] J. Mahanta, P. K. Das, On soft topological space via semiopen and semi-closed soft sets, arXiv math.GN 1203.4133v1.

[13] D. Molodtsov, Soft set theory first results, Comput. Math. Appl.,37,(1999), 19–31.

[14] Muhammad Shabir, Munazza Naz, On soft topological spaces, Comput.Math. Appl., 61,(2011), 1786–179 9

[15] S. Alkhazaleh and A.R. Salleh, Soft Expert Sets, Advances in Decision Sciences,Volume 2011, Article ID 757868, 12 pages doi:10.1155/2011/757868.

