## Python \#10 Lots of Plots

Location: https://people.sc.fsu.edu/~jburkardt/classes/python_2022/python10/python10.pdf
Freely adapted from the Python lessons at https://software-carpentry.org/


## Graphics

- The eye can spot patterns in a plot that are invisible in raw data;
- The matplotlib.pyplot library enables a wide range of graphics options;
- The plot() function creates a basic image of data;
- A plot can be improved and annotated using matplotlib.pyplot features;
- Several plots can be grouped into a single figure;
- Plots can be saved as graphics files, in formats such as jpg, pdf, png.

How can we illustrate our data and mathematical results?
Plotting allows us to visualize machine learning data. Before we do any analysis, this allows us to explore the data; after an analysis, we use various kinds of plots to analyze and publish our results. MATLAB has a rich set of plotting commands that we will explore, including

- line plot, a sequence of points $\left(x_{i}, y_{i}\right)$ connected by lines;
- line plots, several sets of points $\left(x_{i}, y_{i}\right)$, each connected by lines;
- between plot, the area between two curves is filled with color;
- scatter plot, a set of points $\left(x_{i}, y_{i}\right)$ indicated by markers;
- bar plot, a set of data $y_{i}$ displayed as bar heights;
- horizontal bar plot, data $y_{i}$ displayed as bar lengths (good for labels);
- histogram, data to be grouped into bins before bar plotting;
- contour lines, contour lines of a function $f(x, y)$;
- contour colors, contour colors of a function $f(x, y)$;
- surface plot, a 3D plot of the surface $z=f(x, y)$;
- vector plot, a 2D plot using arrows for a gradient or flow field;
- filled polygons, a plot of colored polygons;
- tree plot, representing a branching process;
- graph plot, a set of labeled points $p_{i}$, and a list of two-way connections $\left(p_{i} \leftrightarrow p_{j}\right)$;
- digraph plot, a set of labeled points $p_{i}$, and a list of one-way connections $\left(p_{i} \rightarrow p_{j}\right)$.


## 1 The matplotlib.plot library

In order to create graphics with the matplotlib. pyplot library, we need to request access using the import statement. Everyone abbreviates the library name to plt:

```
import matplotlib.pyplot as plt
```

Once the library is imported, its functions may be used. Suppose we want to plot the function $y=\sin (x)$ over the interval $0 \leq x \leq 2 \pi$. We might start by creating data vectors of a reasonable size, such as 101 entries;

```
x = np.linspace ( 0, 2.0 * np.pi, 101)
y = np.sin ( x )
```

To make our plot, we will call certain functions from mathplotlib.pyplot. When we use a function name, we will prefix it with plt. Here, for instance, is a bit of plotting code in which we are calling the functions clf(), plot(), and grid().

```
plt.clf ( ) # Clear the current graphics figure
plt.plot ( x, y ) # Plot data vectors x and y
plt.grid ( True ) # Add grid lines to the plot
```


## 2 Getting the data files

Many of the following plotting exercises start by reading data from a file. For instance, the first exercise reads from the file bulgaria_data.txt. Thus, to carry out the exercise on your computer, you need to download a copy of this file.

As we have seen in previous exercises, if you do not have a copy of this file, it can be downloaded from the class website. Another way to get it is to go to

```
https://people.sc.fsu.edu/~jburkardt/classes/python_2022/python_2022.html
```

then choose datasets, which will give you a menu that includes the file we are interested in.
For this set of exercises, you will want to download the files

- album_data.txt
- bulgaria_data.txt
- geyser_data.txt
- price_data.csv
- schoolyear_data.csv
- snowfall_data.txt
- volcano_data.txt


## 3 graphviz and pydotplus

For some of the later exercises, we will need to make some plots that matplotlib can't handle. Instead, we will rely on graphviz and pydotplus.

The graphviz home page is https://graphviz.org/.

```
Windows: download and run graphviz-3.0.0 installer
Macintosh with MacPorts: sudo port install graphviz
Linux: sudo apt install graphviz
```

The pydotplus home page is https://pypi.org/project/pydotplus/ Depending on your operating system, you may be able to install it with the command

```
pip install pydotplus
```


## 4 A Line Plot: the BULGARIA Data

Load the text data file bulgaria_data.txt, which records a sequence $\left(y_{i}, p_{i}\right)$, of years and population. The population counts are given at somewhat irregular intervals. Instead of connecting the data points in a curve, we want to show just the data points.

```
import matplotlib.pyplot as plt
import numpy as np
filename = 'bulgaria_data.txt,
data = np.loadtxt ( filename )
# Split data into separate year and population vectors.
year = data [:,0]
population = data [:,1]
#
# Create a plot of the data points.
# plt.plot ( year, population, 'bo' ) # Mark data with blue circles.
#
# Show the plot.
plt.show ( )
```

\#

## 5 "Decorating" a plot; saving a plot

The result of the Bulgaria example is not particularly pretty, and could be more professional-looking. We can try again, adding a title, axis labels, and a grid. The revised plotting code would be:

```
plt.plot ( year, population, 'r.' ) # Just to be different, use red dots this time
plt.grid ( True )
plt.xlabel ( '<-- Year -->' )
plt.ylabel ( '<-- Population -->' )
plt.title ('The population of Bulgaria')
plt.show ( )
```

Depending on your operating system, there may be ways to take a snapshot of the plot so that you can save it for later. But Python includes the savefig() command to do that. If we want to save this plot, we might simply call plt.savefig('bulgaria.png'); This call should come before the call to plt.show().

You can use pretty much any name you want for the graphics file that you create. Python guesses the actual file format from the filename extension. Aside from 'png', other formats include jpg and pdf.

## 6 Lines, points, or both

Since the Bulgarian population data represents actual census measurements, it's worth emphasizing that these measurements came at specific, somewhat irregular times. We do that by displaying the data as a sequence of dots.

If, on the other hand, we want to suggest the trend evident from the data, then the eye will be much happier looking at a connected curve that passes through the points. The easiest way to do this is to change the third input in the plot() call, which is the "format" of the drawing. The "dash" symbol indicates that line segments are to be used.
plt.plot ( year, population, 'g-' ) \# Green line segments connect the data

In some cases, it is worth showing both the original data and the curve that passes through them. This can be done by specifying a format involving both a dash for the line segments, and your choice of a marker for the data (the choices include '.', '*', 'o'):
plt.plot ( year, population, 'm-*' ) \# Magenta stars and line segments

There are more ways to draw the line or mark the data, but these require you to look for help on the plot () parameters linestyle and marker.

## 7 Line Plots: the PRICE Data

The file price_data.csv is a table of average monthly prices for 11 consumer products, between February 2008 and February 2018. There are 241 records, and each record contains 13 items: the month, the year, and the prices of 11 goods. Columns 3,10 and 13 contain the prices of bananas, gas, and milk, respectively. We want a single plot that shows line plots for all three items together.

Because the data is stored in comma separated value (CSV) format, with an initial line containing the names of each data column, we use a special function that reads the names and values together, creating an object we can name data. We can then create a list of the numeric values in any given column by using the data column name as an index. Here, we extract from data the three price lists we want, each of length $n=241$. We also create a corresponding array called month which simply runs from 1 to $n$.

```
data = np.genfromtxt ( 'price_data.csv', dtype = None, delimiter=',', names = True )
bananas = data['Bananas']
gas = data['Gas']
milk = data['Milk']
n = len ( bananas )
month = np.linspace ( 1, n, n )
```

Now we can simply issue a plot command for each pair of (month, price) values. All three curves will appear in the same figure. To make the curves more visible, we request linewidth=3, rather than the default value of 1 .

```
plt.plot ( month, bananas, linewidth = 3 )
plt.plot ( month, gas, linewidth = 3 )
plt.plot ( month, milk, linewidth = 3)
```

Now that we have our three curves plotted, we can "decorate" the figure with a grid, labels and a legend. A legend helps to interpret a plot containing multiple curves, by displaying a small chart pairing a line color with a line label.

```
plt.grid ( True )
plt.legend ( [ 'Bananas', 'Gas', 'Milk' ] )
plt.xlabel ( '<-- Month index }->', fontsize = 16 ),
plt.ylabel ( '<-- Price ($) -->', fontsize = 16)
plt.title ('Prices, Feb 2008-Feb 2018', fontsize = 16 )
```

It is important to understand that in matplotlib, a new plot() statement doesn't clear the current figure, but simply adds more information to it. Compare this to MATLAB, where each plot statement wipes out any previous information, unless the hold ( 'on' ) statement is issued.

## 8 A "Between" Plot: the INTEGRAL Data

The integral of a function $f(x)$ over the limits $a \leq x \leq b$ can be pictured as the area "under the curve", that is, the area between the $x$ axis and the curve $y=f(x)$. If the curve goes below the $x$ axis, this portion counts as negative area.

As an example, we'd like to plot $y=\sin (x)$ and illustrate the integral from $\frac{\pi}{3} \leq x \leq \pi$, displaying positive area in blue and negative area in red. In order to do this, we will have to call plt.subplots() to define a figure and axis, and then use fill_between() to fill in the colors.

```
\(\mathrm{x} 1=\mathrm{np}\). linspace \((-0.25,6.5,101)\) \# plot range
\(\mathrm{y} 1=\mathrm{np} \cdot \sin (\mathrm{x} 1)\)
\(\mathrm{x} 2=\mathrm{np}\). linspace ( \(\mathrm{np} . \mathrm{pi} / 3.0\), np.pi, 101) \# positive integral range
\(y 2 \_h i=n p . \sin (x 2)\)
\(y 2 \_1 o=n p . z e r o s(101)\)
\(\mathrm{x} 3=\mathrm{np}\). linspace ( \(\mathrm{np} \cdot \mathrm{pi}, 4.0 * \mathrm{np} . \mathrm{pi} / 3.0,101\) ) \# negative integral range
y 3 _hi \(=\mathrm{np} \cdot \sin (\mathrm{x} 3)\)
y 3 _lo \(=\mathrm{np} . \mathrm{zeros}(101)\)
fig, \(a x=\) plt.subplots ( )
ax.plot ( \(\mathrm{x} 1, \mathrm{y} 1, \quad \mathrm{k}-'\), linewidth \(=2\) ) \# Curve
ax.fill_between ( x 2 , y 2 _lo, y 2 _hi, linewidth \(=0\), color \(={ }^{\prime} \mathrm{b}\), ) \# Positive part
ax.fill_between ( x 3 , y3_lo, y3_hi, linewidth \(=0\), color \(\left.={ }^{\prime} \mathrm{r}{ }^{\prime}\right) \quad\) \# Negative part
\(\operatorname{ax} \cdot \operatorname{plot}([-0.25,6.5],[0.0,0.0], ' \mathrm{k} —\),\() \# Draw x-axis\)
plt. show ()
```

Previously, we simply used the plt.plot() function to make our plots. The plt.subplots() approach is a more sophisticated method which allows control over the figure (the entire picture) and one or more axes (individual curves or graphic elements). The format for "decorating" the plot is somewhat different from what we are used to:

```
ax.grid (True )
ax.set_title ('Integral = Blue area - red area')
ax.set_xlabel ( '<-- x ->' )
ax.set_ylabel ( '<-- y = sin(x) -->' )
filename = 'integral_between.png'
fig.savefig ( filename )
print (',Graphics saved as ", + filename + '", )
plt.show()
plt.close ( )
```


## 9 A Scatter Plot: the GEYSER Data

The file geyser_data.txt contains 272 observations of the Old Faithful geyser. Each record lists the eruption duration in minutes, and the pause in minutes until the next eruption. A line plot of the durations, or of the pauses, doesn't show a clear pattern. However, it might seem reasonable that the duration and successive pause variables are related. A long eruption might mean a long subsequent wait, for instance. Such patterns can be investigated using a scatter plot, in which we simply put a mark at each point $(x, y)$ for which we have data.

Load the text data file geyser_data.txt, and split the data:

```
data = np.loadtxt ( filename )
duration = data[:,0]
pause = data[:, 1]
```

We do not want to use the plot() command. Instead we use scatter:

```
plt.scatter ( duration, pause )
```

If we wish to use an 'o' for the marker, of size 5 , and color red, we could give the fancier command:

```
plt.scatter ( duration, pause, s = 5, c = 'r', marker = 'o' )
```

More options are available by looking for help on matplotlib scatter.
This example shows how a scatter plot can reveal interesting patterns in data. What relationship between eruption duration and pause duration is suggested here?

## 10 A Bar Plot: the ALBUM Data

Line plots connect pairs of data values in which $x$ is steadily increasing and $y$ is varying smoothly. A bar plot also considers data pairs, but the variable playing the $x$ role might be an increasing numeric value, or it might simply be a set of named categories. The main purpose of a bar plot is to emphasize the variation in height of the bars, inviting the user to draw a conclusion as to the pattern.

Our first example considers the sales of music albums over a period of years. We could instead display the data using a line plot, but, by using bars, the change in behavior over the years is much more dramatically displayed.

The file album_data.txt lists the year, and total music album sales, for each year from 2007 to 2017. We begin by reading the dat from the file, and copying out the year and sales information separately:

```
filename = 'album_data.txt'
data = np.loadtxt ( filename )
year = data[:,0]
sales = data[:,1]
```

To create a bar plot, we can call the matplotlib function bar():
plt.bar ( year, sales )
One of the useful optional parameters is width=value, which defaults to 0.8 . This draws bars that are only $80 \%$ of the maximum width, thus leaving gaps to make the bars more clearly separate. You can easily specify this value, and adjust it as you like. Similarly, the parameter color has the default value 'b', but you can change the color of the bars to green by specifying color='g' instead.

Especially for bar plots, specifying a background grid makes it easier for the viewer to compare the heights of different bars.

## 11 A Horizontal Bar Plot: the SCHOOLYEAR Data

Usually, a bar plot exhibits the bars vertically. The length of the bar represents a count, but what does the position mean? Frequently, the bars represent counts of data over bins of equal size, and evenly spaced. But other times, the data is associated with different people, or different countries, or other items, so that each bar needs a readable label.

For such data, a horizontal bar plot might be best. The labels might appear on the left, with a horizontal bar to the right, as suggested by this simple typewritten example:

```
Votes in favor: xxxxxxxxxx
Votes against: xxxxxxxxxxxxxxxxx
Votes withheld: xxxx
```

For this exercise, our data will involve the length of the school year in days, for various countries. We need to read a CSV data file, schoolyear_data.csv, in which the first item is the name of a country, and the second is the number of data in a school year. The first four lines of this file are:

```
"Country", "Days"
"Belgium (French)", 175
"Belgium (Flemish)", 160
"British Columbia", 185
```

so you can see that line $\# 1$ gives names for the two pieces of data, and subsequent lines are a country name in quotes, a comma, and a number of days. Data like this is very common, but a little tricky to read.

As before, we will use genfromtxt () to read the data. But now, because the data is not simply numeric, we need to include some information describing the type of each item. MATLAB has some trouble reading data files that contain a mixture of text and numeric data, so instead, we will get our data from a MATLAB function. We will describe the type of the first item as ' $\mathrm{U} 20^{\prime}$, that is, a Unicode text string of up to 20 characters. The second item is listed as 'i4', that is, a 32 bit signed integer. So here's how we read from the file, split out our two data items, and make an index variable country:

```
types \(=\) ['U20', 'i4' ]
data \(=\) np.genfromtxt ( 'schoolyear_data.csv', dtype \(=\) types, delimiter \(=\) ', ', names \(=\) True \()\)
days \(=\) data ['Days']
name \(=\) data ['Country']
\(\mathrm{n}=\operatorname{len}\) (days)
country = np.linspace (1, n, n )
```

The point here is that the bars will be meaningless unless we can label them, preferably with the country name. That would be very difficult with vertical bars, but we may be able to manage it with horizontal bars, since the labels will read across. So we call barh() to make the bars, and then specify the country names as "y tick marks"

```
plt.barh ( country, days )
plt.grid ( True )
plt.title ( 'Schoolyear lengths', fontsize = 16 )
plt.xlabel ( '<-- Days ->', fontsize= = 16 )
plt.yticks ( country, name, fontsize = 6 )
```

We specify the fontsize in order to keep the country names small enough to fit. Again, a background grid might be helpful for comparing the lengths of various bars.

The data items were given in alphabetical order, but in the plot, the first data item appears at the bottom, and the last at the top. To rectify this, you could use np.flip to reverse the order of the two data arrays:

```
plt.barh ( country, np.flip ( days ) )
plt.grid ( True )
plt.title ('Schoolyear lengths', fontsize = 16 )
plt.xlabel ( '<-- Days -->', fontsize = 16 )
plt.yticks ( country, np.flip ( name ), fontsize = 6 )
```

This may seem a small matter, but it's enough to drive some people crazy!

## 12 A Histogram: the SNOWFALL Data

Suppose we want to display a large set of observations of some numerical quantity, $\left(x_{1}, x_{2}, \ldots, x_{n}\right)$. If the data has some slowly varying trend, a line plot might be suitable. If the data is not too numerous, a bar plot might work. But if the data is numerous and somewhat chaotic, we are better off with a histogram, which simplifies the data by grouping it into a small number of bins. A histogram then shows how many data items fall into each range.

When preparing a histogram, it's important to pick a reasonable number of bins for the plot. A good number of bins will allow the data to suggest a smoothly varying behavior, for instance. Specifying too few or too many bins will result in an ugly and uninformative plot.

The students at Michigan Technical University have been collecting snowfall data for years. The file snowfall_data.txt contains data from 1890 to 2022 . Column 9 of the data lists the total snowfall for that year. That is the data we want to display.

```
data = np.loadtxt ( filename )
inches = data [:,9]
plt.hist ( inches, rwidth = 0.95 )
```

The rwidth=0.95 argument specifies that the bars in the plot should be just $95 \%$ of their maximum width.
The argument bins=n can be used to specify the number of bins to be $n$. If this is not specified, a suitable value is chosen automatically. Experiment with choosing this value yourself. Even small changes to $n$ can sometimes make a noticeable difference in the plot. As stated before, using $n$ too large or too small will result in an awfully ugly plot.

What features of the data does the histogram allow you to see right away?
To understand the difference between a histogram and a bar plot, try calling plt.bar () directly for this same data. Does the bar plot do a good job of presenting the information?

## 13 A Contour Line Plot: the VOLCANO Data

We may have a collection of measurements of temperature, or elevation, or pollution levels, sampled at regular points on an $(x, y)$ grid. This means that we are really interested in a 3 D plot, of $z(x, y)$. One way to display such data on a 2 D screen or piece of paper is to use some kind of contour plot. On a map of a mountainous area, for instance, contour lines are used to indicate points that have the same height: 100 meters, 200 meters, and so on. By choosing the right number of such contour lines for your data, you can suggest the pattern formed by your 3D data.

A map of a volcano has been created by creating an $87 \times 67$ grid of $(x, y)$ points, and recording a table of the vertical height $z(x, y)$ at each point. This data has been stored in the text file volcano_data.txt. The first 5 rows and columns of this data look like:
$1.00 \mathrm{e}+021.00 \mathrm{e}+021.01 \mathrm{e}+02 \quad 1.01 \mathrm{e}+02 \quad 1.01 \mathrm{e}+02 \ldots$
$1.01 \mathrm{e}+021.01 \mathrm{e}+021.02 \mathrm{e}+021.02 \mathrm{e}+021.02 \mathrm{e}+02 \ldots$
$1.02 \mathrm{e}+021.02 \mathrm{e}+021.03 \mathrm{e}+02 \quad 1.03 \mathrm{e}+021.03 \mathrm{e}+02 \ldots$
$1.03 \mathrm{e}+021.03 \mathrm{e}+021.04 \mathrm{e}+02 \quad 1.04 \mathrm{e}+021.04 \mathrm{e}+02 \ldots$
$1.04 \mathrm{e}+021.04 \mathrm{e}+021.05 \mathrm{e}+021.05 \mathrm{e}+021.05 \mathrm{e}+02 \ldots$
...
We can read the $z$ data from the file using np.loadtxt().

```
zmat = np.loadtxt ( 'volcano_data.txt', dtype = 'f', delimiter = ' ' )
```

Here, dtype='f' is saying the data involves real numbers, while delimiter $=$, ' indicates that values are separated by blank spaces, rather than, say, commas, as we saw in CSV files.

In order to use the contour plotting function, we need to supply values for $(x, y)$ as well as $z$. For this plot, the exact locations don't matter, and we will just assume that the data is equally spaced. In that case, we can use np.linspace() to make lists of $x$ and $y$ coordinates. We actually need to create a two-dimensional table of $(x, y)$ data. This is done by calling np.meshgrid().

```
m, n = zmat.shape
xvec = np.linspace ( 0, n-1, n ) # xvec =(0, 1, ..., n-1)
yvec = np.linspace ( 0, m-1, m )
xmat, ymat = np.meshgrid ( xvec, yvec )
```

Now we are ready to call plt. contour (). We include a value for the number of contour levels. We can also specify that the contour lines have a color that varies with value. To do so, we specify a particular colormap. In order to use this option, we must also load the cm library from matplotlib:

```
from matplotlib import cm
levels = 15
plt.contour ( xmat, ymat, zmat, levels, cmap = cm.coolwarm )
```


## 14 A Contour Color Plot: the VOLCANO Data

You may have noticed that using color for the contour lines of the volcano data made it easier to "read" the image. An even more vivid picture can often be made by dropping the contour lines and simply using color by itself.

We have already seen in the previous discussion how to create the arrays xmat, ymat, zmat. To create a color contour plot, we call plt.contourf() instead of plt.contour ().

Since the colors you choose will dominate the image, it can be important to find a colormap that suits your taste. There is a large set of such maps available in matplotlib. You might like to redraw your plot using one of the following:

- viridis
- viridis
- Greys
- binary
- PiYG
- twilight
- Pastel1
- flag


## 15 A Surface Plot: the SOMBRERO Function

A contour plot can also be useful to examine the behavior of a function $z=f(x, y)$; even better is a 3D surface plot. In this case, we have to generate the $(x, y)$ grid values, and set up a way to evaluate the function.

Because we want to do 3D plotting, we have to import the Axes3D library from matplotlib. We will also try using a colormap for the surface, so let's import cm as well.

```
from mpl_toolkits.mplot3d import Axes3D
from matplotlib import cm
```

For this example, we are interested in the function $z=\sin (r) / r$ where $r=\sqrt{x^{2}+y^{2}}$. Since our plot domain will include the point $(0,0)$, we want to avoid the value $r=0$, so we will instead compute $r=\sqrt{x^{2}+y^{2}+\epsilon}$, where $\epsilon$ is a small, positive real number.

```
xvec = np.linspace( - 8.0, 8.0, 33)
yvec = np.linspace( - 8.0, 8.0, 33)
xmat, ymat = np.meshgrid ( xvec, yvec )
rmat = np.sqrt ( xmat**2 + ymat**2 + np.finfo(float).eps )
zmat = np.sin ( rmat ) / rmat
```

At this point, having created the arrays xmat, ymat, zmat, we could easily create a contour line or color contour plot of our data. You may want to try this, so that you can compare these contour results with the surface plot we are about to make. While surface plots can be much more dramatic, it is sometimes the case that a simple contour plot is more readable.

Because we want to make a 3D plot, we have to use a more elaborate way to define our figure and axis. Then we call ax.plot_surface() to make the surface plot. Notice that we again can choose a colormap. By default, the surface will be covered by grid lines, which can be quite distracting. For our plot, we can suppress these gridlines by specifying edgecolor='none'.

```
fig = plt.figure ( )
ax = fig.add_subplot ( 111, projection='3d' )
ax.plot_surface ( xmat, ymat, zmat, \
    cmap = cm.ocean, edgecolor = 'none' )
```


## 16 A Vector Plot: the FLOW Function

Suppose we want to visualize the direction and magnitude of the wind over some 2D region. While a contour plot deals with a scalar function $z(x, y)$, now we are considering a vector function $(u, v)(x, y)$. The standard way to represent a velocity field like this is to draw arrows at each point $(x, y)$, of the appropriate direction and magnitude.

Another case in which a vector plot is needed is when we are trying to minimize a function $z(x, y)$, in which case we want to plot the gradient vectors $(d z d x, d z d y)(x, y)$.

The matplotlib function quiver () can produce the plots we want. All we need to supply is the arrays of $x, y, u, v$ values.

In the following example, we consider a flow field over the unit square, for which we have an explicit formula:

$$
u=-\left(x^{4}-2 x^{3}+x^{2}\right)\left(2 y^{3}-3 y^{2}+y\right) v=\left(2 x^{3}-3 x^{2}+x\right)\left(y^{4}-2 y^{3}+y^{2}\right)
$$

For convenience, then, we can write a function flow_field( $n$ ) for which the input $n$ is the number of nodes in the $x$ and $y$ directions, over the unit square. In the usual way, we use np.linspace() to define vectors
xvec and yvec, then np.meshgrid() to create tables xmat and ymat. Using the flow formula above, we can then create tables umat and vmat. The function then returns xmax, ymat, umat, vmat.

Now it's simple to set up our plot:

```
n = 16
xmat, ymat, umat, vmat = flow_field ( n )
plt.quiver ( xmat, ymat, umat, vmat, color = 'c' )
plt.axis ( 'Equal' )
```

Here, the extra argument color='c' has the arrows drawn in cyan color.
Additional arguments to quiver () can alter the arrow length or thickness. Vector plots are a fundamental tool when working in fields such as computational fluid dynamics, electromagnetism, and climate modeling.

## 17 Filled Polygons: the ELL data

Many geometric images can be constructed by combining a number of colorful polygons. A checkerboard, for instance, can be drawn using 32 red and 32 black squares. A $6 \times 10$ rectangle can be tiled by the 12 pentominoes (distinct shapes of 5 squares), each of which can be shown in a different color. A polygon can be triangulated, with each triangle a different color.

Such images are known as filled polygon plots. The matplotlib library offers the function plt.fill() to display such figures. The format of the call can be

```
plt.fill ( x, y ) # one polygon, default color
plt.fill ( x, y, c ) # one polygon, color 'c'
plt.fill ( x1, y1, x2, y2, ..., xn, yn ) # n polygons
plt.fill ( x1, y1, c1, x2, y2, c2 ) # 2 polygons, colored 'c1', 'c2'
```

Our target for this exercise is to create an L-shaped figure composed of three polygonal shapes colored blue, red, and cyan. Here is a sketch of what we want:

```
4 +-+
    |B|
3 + +
    |B|
2 + +-+---+
    |B B|R R|
1 +---+-+ +
    |C C C|R|
0 +-----+-+
    01234
```

You should see that ' $B$ ' polygon has $x 1=[0,1,1,1,1,0], y 1=[1,1,2,2,4,4]$ and $c 1=' b$ '. Determine $\mathrm{x} 2, \mathrm{y} 2, \mathrm{c} 2$ and $\mathrm{x} 3, \mathrm{y} 3, \mathrm{c} 3$ for the ' $\mathrm{C}^{\prime}$ and ' R ' polygons. Call plt.fill() to create the filled polygon version of this plot.

You may notice that your plot is slightly distorted. Squares are rectangular instead of squares. This is because matplotlib, like MATLAB, chooses by default a rectangular picture frame, and stretches that data to fit it, if necessary. To avoid this unpleasant habit, we can simply say

```
plt.axis ( 'Equal' )
```


## 18 A Tree Plot: the GENEALOGY data

A tree plot is way to illustrate a process which involves branching or dependence. A tree plot can illustrate all the paths that start at the initial point and branch one way or another at decision points.

The locations on a tree are called nodes, and the node representing the initial state is called the root node. If we assign each node an identifying index, then the structure of the tree can be described if each node identifies its "parent node", which connects it back to the root node.

In order to make our plot, we are going to need to install graphviz and pydotplus, and then import the directed graph library from graphviz.
from graphviz import Digraph
For our data, we will assume we have a simple sort of genealogy, in which one person is the ancestor (and hence will be represented by the root node). This ancestor had several immediate descendants, and so an arrow will be directed from the ancestor node to the nodes of each of these descendants. If any of these descendants had children, more nodes and arrows will be drawn, in a natural way.

The graphviz library allows us to specify labels for each node, and the arrows we wish to have drawn from one node to another.

We begin by creating an object we name dot, adding a comment, and indicating that the final figure should be saved as a png file.

```
dot = Digraph ( comment = 'A genealogy tree', format = 'png' )
```

Now we want to describe the nodes. We begin by requesting that the nodes be drawn with an egg shape. Then we list each node, giving a short name (which might as well be just a guoted numerical index), and a label, for each.

```
dot.attr ( 'node', shape = 'egg' )
dot.node ( '1', 'Alan' )
dot.node ( '2,', 'Bert' )
dot.node ( '3', 'Chad' )
dot.node ( '4,', 'Dian' )
dot.node ( '5', 'Enid' )
dot.node ( '6', 'Fran' )
dot.node ( '7', 'Gert' )
dot.node ( '8,', 'Hank' )
dot.node ( '9', 'Iona' )
dot.node ( '10', ''Jean'')
dot.node ( '11', 'Kate' )
dot.node ( '12', 'Lynn' )
```

Now we specify the connections between nodes:

```
dot.edge ( '1, , '2', )
dot.edge ( '1', '3')
dot.edge ( '2', '4' )
dot.edge ('2', '6',)
dot.edge ( '3', '5' )
dot.edge ( '3', '10' )
dot.edge ( '4', '7' )
dot.edge ( '4,','8, )
dot.edge ( '4', '9' )
dot.edge ( '5', '11' )
dot.edge ( '5,', '12, )
```

Finally, we can ask for a printed description of the plot, as verification, and then we render the image to a file, to be called genealogy_tree.dot.png', and optionally display the plot to the screen.

```
print ( dot.source )
dot.render ('genealogy_tree.dot', view = True )
```


## 19 A Graph Plot: the NETWORK data

Mathematically, a graph is a set of nodes, some of which are connected by edges. (A tree is a special kind of graph which is connected, and has no circuits.) In a graph, the edges usually represent two-way connections. We will see one-way connections in a moment, in the directed graph example.

We will again use graphviz for this example. The primary difference from the tree plot is that now we import the Graph library.
from graphviz import Graph
We begin by calling Graph to create an object we will name dot.

```
dot = Graph ( comment = 'A network', format = 'png' )
```

Now, just like we did for the tree example, we must list the nodes, with a short identifier and a label, followed by a list of the edges. This procedure will seem identical to what we did for the tree, and so we leave most of the work to you. Here is just a bit of the commands you will need:

```
dot.node ( '1', 'A' )
dot.node ( '11', 'K' )
dot.edge ( '1', '2' )
dot.edge ( '10', '11',)
```

To fill in the details, look at the information in the following plot:


Once you have specified the nodes and edges, make your plot:

```
dot.render ( 'network_graph.dot', view = True )
```


## 20 A Directed Graph Plot: the WEB data

A directed graph displays nodes connected by one-way edges. There may be a value associated with each edge, representing a weight, length or probability of access. A good display should include that information.

Let us consider the following network, and assume that nodes 1 through 6 are labeled Alpha, Beta, Gamma, Delta, Epsilon and Zeta, respectively.


To create a corresponding image, we will again use the Digraph function from the graphviz library.

$$
\text { dot }=\text { Digraph ( comment }=\text { 'A network', format }=\text { 'png' })
$$

We will specify nodes as before, but this time, we will add a third argument to the edge specifications, a label to be printed, which indicates some property of that edge. Working from the plot above, you should be able to fill in the missing details:

```
dot.node ( '1', 'Alpha' )
dot.node ( '6', 'Zeta' )
dot.edge ( '1', '5', '0.1' )
dot.edge ( '1,', '6', ,0.9, )
dot.edge ( '6', '4', '1.0' )
```

And of course, once we are done with the specifications, we can view our plot and save it as a file:

```
dot.render ('web_digraph.dot', view = True )
```


## 21 Conclusion

A variety of plots can be made for illustrating computational work. Once the basic plot has appeared, it's important to consider ways to improve or "decorate" it. Line plots may look fine on a computer screen, but usually should be drawn with a larger linewidth for clarity, especially when being displayed in print or on a slide presentation. A background grid improves the observer's ability to estimate or compare data in the plot. Labels, titles, and legends change a plot from a picture to a "document", something that tells a story even if you aren't there to state it. Occasionally, it's necessary to force a plot to use the same units in $x$ and $y$, especially when a geometric object is being displayed.

When we began plotting, we simply used the plot() command from matplotlib. However, for 3D plotting and certain other applications, it may be useful or necessary to work with functions like

$$
\begin{aligned}
& \text { fig }=\text { plt.figure } \\
& \text { ax }=\text { fig.add_subplot }\left(111, \text { projection }={ }^{\prime} 3 d, \quad\right)
\end{aligned}
$$

or

$$
\text { fig, } \quad \text { ax }=\text { plt. subplots ( ) }
$$

You should try to find more examples of these functions and get to know them better.
The matplotlib library currently doesn't have good abilities for drawing mathematical trees, graphs and networks, and so we have looked at graphviz. These kinds of plots are constantly coming up in computational, mathematical and scientific work. It's important to pick up the skills you need in order to get plots that express your ideas. The graphviz package has online help and a useful document to help you.

