MATH 4974 - Modeling Seminar:

Bifurcation, Catastrophe, Singularity, and All That

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The Buckling Spring

Plot the 4-dimensional Manifold in 2-dimensional space. We want to consider points of equilibrium, $\sum F = 0$.

Therefore, for an array of L_i and Θ_j values, we compute λ and μ such that $\sum F = 0$.

This amounts to solving the linear system:

$$F = \begin{pmatrix} -2(1-L_i) + 2\lambda_{i,j}\cos(\Theta_j) + \mu_{i,j}\sin(\Theta_j) \\ \frac{1}{2}\Theta_j - 2\lambda_{i,j}L_i\sin(\Theta_j) + \mu_{i,j}L_i\cos(\Theta_j) \end{pmatrix} = \overrightarrow{0}$$

$$\Rightarrow \begin{pmatrix} 2\cos(\Theta_j) & \sin(\Theta_j) \\ -2L_i\sin(\Theta_j) & L_i\cos(\Theta_j) \end{pmatrix} \begin{pmatrix} \lambda_{i,j} \\ \mu_{i,j} \end{pmatrix} = \begin{pmatrix} 2(1-L_i) \\ -\frac{1}{2}\Theta_j \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \lambda_{i,j} \\ \mu_{i,j} \end{pmatrix} = \begin{pmatrix} 2\cos(\Theta_j) & \sin(\Theta_j) \\ -2L_i\sin(\Theta_j) & L_i\cos(\Theta_j) \end{pmatrix}^{-1} \begin{pmatrix} 2(1-L_i) \\ -\frac{1}{2}\Theta_j \end{pmatrix}$$

Provided that the matrix inverse exists.

We solve this system to create a collection of $i \times j$ $(\lambda_{i,j}, \mu_{i,j})$ coordinate pairs, which are plotted below.

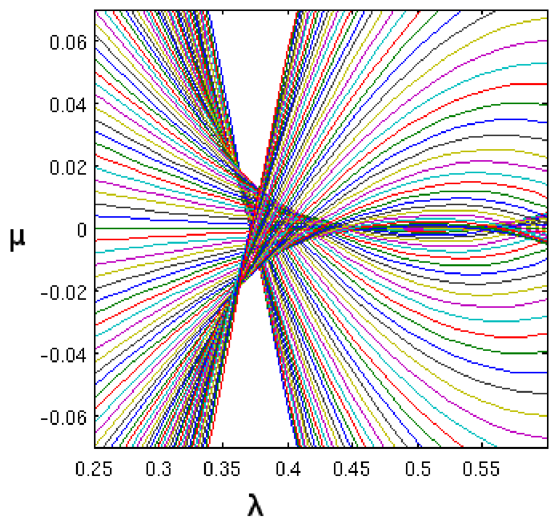


Figure 1: Problem 1: μ vs. λ.

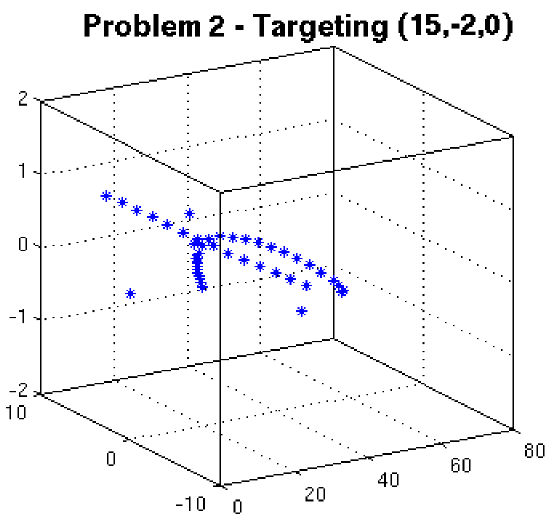
The Freudenstein Roth Function

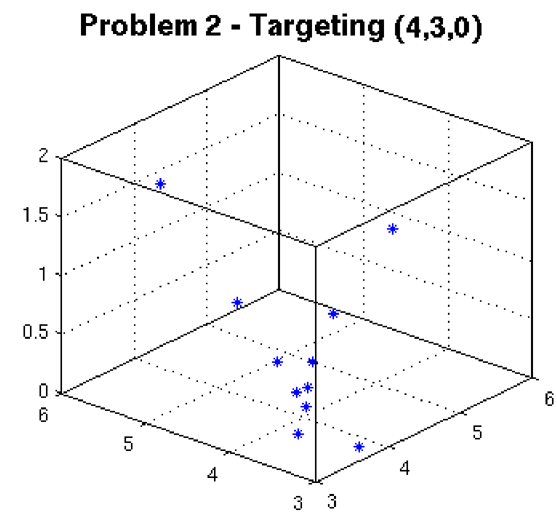
Plot the iterations of Newton's Method to solve:

$$g(x_1, x_2) = \begin{pmatrix} x_1 - x_2^3 + 5x_2^2 - 2x_2 - 13 = 0 \\ x_1 + x_2^3 + x_2^2 - 14x_2 - 29 = 0 \end{pmatrix}$$

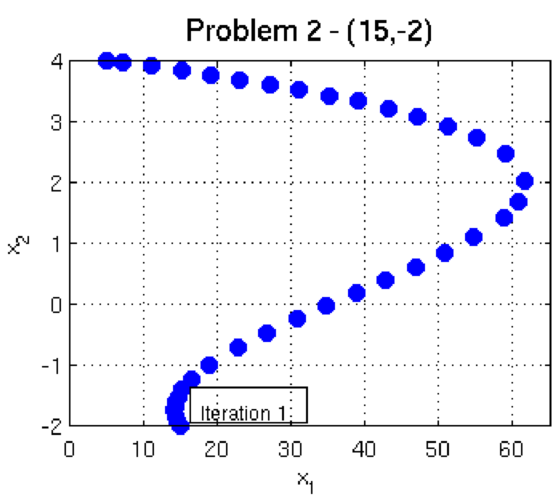
Defining the auxiliary $f(x_1, x_2, x_3) = g(x_1, x_2) + (x_3 - 1)g(x_1^*, x_2^*)$ we can help to ensure convergence of Newton's Method, *the continuation method*. This helps us to ensure that if our starting guess is not close to the solution of $g(x_1, x_2)$ it will be close to the solution of $f(x_1, x_2, x_3)$, and from here we can solve Newton's Method a number of times while pushing x_3 from $0 \to 1$. This method provided the following solution:

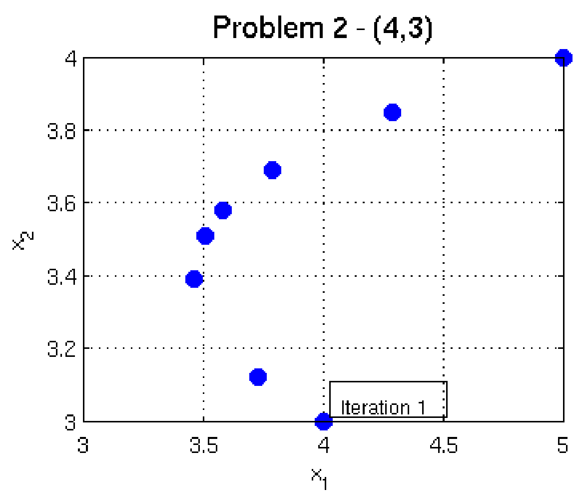
$$x = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$





Both starting locations $(15, -2)$ and $(4, 3)$ give the same solution of $(5, 4)$ even though the initial points are quite different. However by starting farther away (as in **part a**), we see that it takes many more iterations.





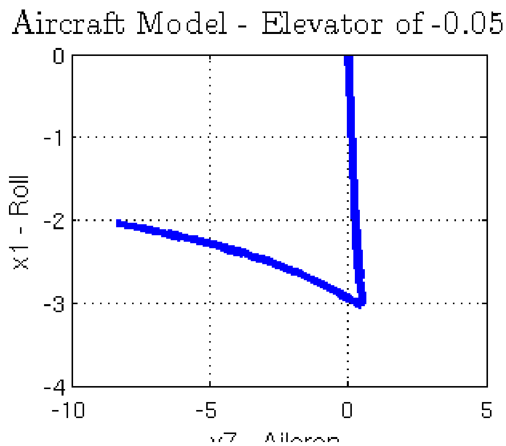
The Aircraft Model

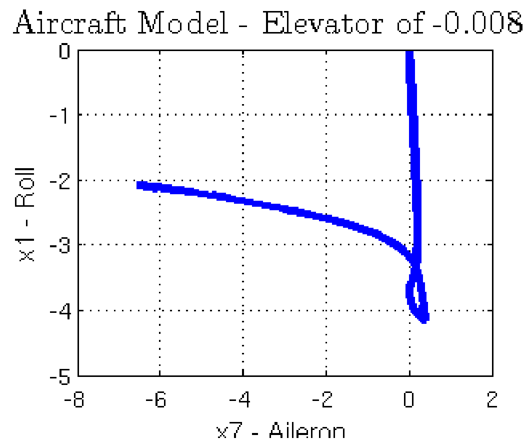
Find limit points in the roll subject to changes in the aileron at set values for the elevator.

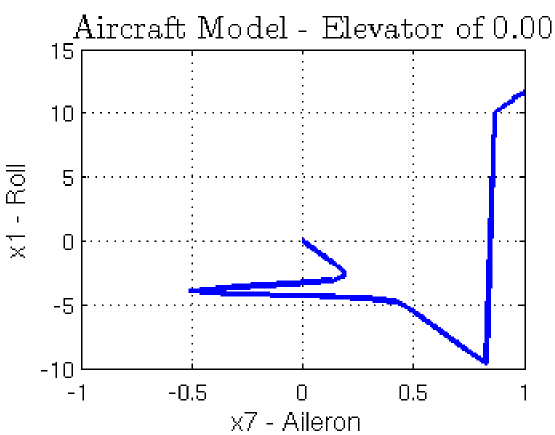
We will see that making slight changes in the aileron, which would be natural for a pilot to make in an attempt to correct the flight path, may cause catastrophic changes in the roll. These limits in the roll may cause severe damage to the plane.

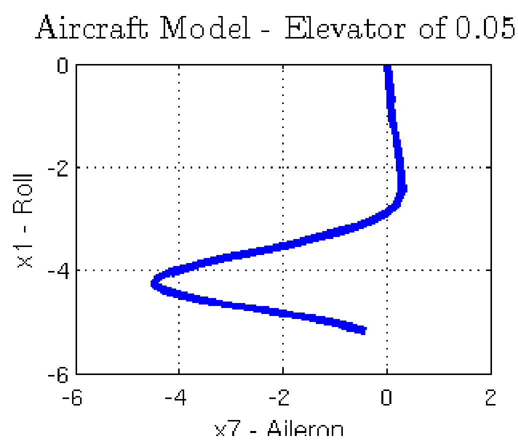
By analyzing the graphs, Elevator = 0.00 for example, we see that if we push the aileron in the positive direction, from 0.5 to 1, the roll will jump from -10 to 10 *very* quickly. This will likely cause serious damage to the plane, and should be avoided.

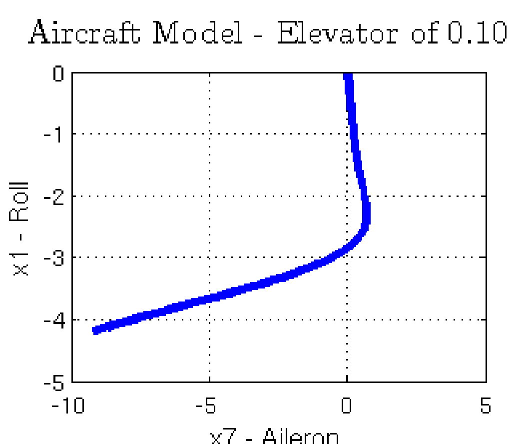
The Aircraft Model continued on next page. . .


Aircraft Model - Elevator of -0.05
x1 - Roll


Aircraft Model - Elevator of -0.008
x1 - Roll
x7 - Aileron


Aircraft Model - Elevator of 0.00
x1 - Roll
x7 - Aileron


Aircraft Model - Elevator of 0.05
x1 - Roll
x7 - Aileron


Aircraft Model - Elevator of 0.10
x1 - Roll
x7 - Aileron