

ISC 5935 - Computational Tools for Finite Elements

Homework #9 Solutions

Assigned 05 November 2014, Due 12 November 2014

http://people.sc.fsu.edu/~jburkardt/classes/fem_2014/homework9.pdf

For these problems, assumed that $\sigma(x)$ is the flux, $k(x)$ is the thermal conductivity, $f(x)$ is the source term, and $u(x)$ is the temperature. Assume we are working in the interval $0 \leq x \leq 10$. Assume that, in general (but not necessarily in question 4):

$$\text{Jump}(\sigma(x)) = 0$$

$$\sigma(b) - \sigma(a) = \int_a^b f(x) dx$$

$$\sigma(x) = -k(x) \frac{du}{dx}$$

1. Suppose that the graph of $\sigma(x)$ as a function of x is a straight line that is 1 at $x = 0$ and 3 at $x = 10$.

- what is an expression for $f(x)$?

$$f(x) = \frac{1}{5}$$

- if $k(x) = \frac{1}{10}$, what is a formula for $u(x)$?

$$u(x) = -x^2 - 10x + u(0)$$

2. Suppose that the plot of $f(x)$ as a function of x is 1 for $0 < x < 5$ and 4 for $5 < x < 10$.

- what is a formula for $\sigma(x)$?

$$\sigma(x) = \begin{cases} \sigma(0) + x & \text{for } 0 \leq x \leq 5 \\ \sigma(5) + 4(x - 5) & \text{for } 5 \leq x \leq 10 \end{cases}$$

- if $k(x) = 3$, what is a formula for $u(x)$?

$$u(x) = \begin{cases} u(0) - \frac{1}{3}(\sigma(0)x + x^2/2) & \text{for } 0 \leq x \leq 5 \\ u(5) - \frac{1}{3}(\sigma(5)(x-5) + 2(x-5)^2) & \text{for } 5 \leq x \leq 10 \end{cases}$$

3. Suppose that the plot of $k(x)$ as a function of x is 5 for $0 < x < 5$ and 2 for $5 < x < 10$. Suppose that $f(x)$ is 1 (*correction!*), and that $u(0)$ is 0.

- what is a formula for $\sigma(x)$?

$$\sigma(x) = \sigma(0) + x$$

- what is a formula for $u(x)$?

$$u(x) = \begin{cases} -\frac{\sigma(0)x + x^2/2}{5} & \text{for } 0 \leq x \leq 5 \\ u(5) - \frac{\sigma(5)(x-5) + (x-5)^2/2}{2} & \text{for } 5 \leq x \leq 10 \end{cases}$$

4. Suppose that $f(x) = 2 + 10\delta(x - 7)$.

- what is a formula for $\sigma(x)$?

$$\sigma(x) = \begin{cases} \sigma(0) + 2x & \text{for } 0 \leq x \leq 7 \\ \sigma(0) + 2x + 10 & \text{for } 7 \leq x \leq 10 \end{cases}$$

- if $k(x) = 3$, what is a formula for $u(x)$?

$$u(x) = \begin{cases} u(0) - \frac{1}{3}(\sigma(0)x + x^2) & \text{for } 0 \leq x \leq 7 \\ u(7) - \frac{1}{3}(\sigma(0)(x - 7) + (x - 7)^2 + 10(x - 7)) & \text{for } 7 \leq x \leq 10 \end{cases}$$
