Homework # 2

Due: Wednesday, 9/18

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1. (20 pts) In Section 2.1 we derived a second order *explicit* Taylor series method.

a. Modify the derivation in Section 2.1 to derive the second order *implicit* Taylor series method

$$Y_0 = y_0 \qquad Y_i = Y_{i-1} + \Delta t f(t_i, Y_i) - \frac{\Delta t^2}{2}\left[f_t(t_i, Y_i) + f(t_i, Y_i) f_y(t_i, Y_i)\right], i = 1, 2, \ldots N$$

for the IVP (1.2)

b. (G) Derive a third order accurate explicit Taylor series method for the IVP (1.2).

c. Do a hand calculation to perform one step of the implicit method from (a) for the IVP

$$y'(t) = ty(t) \quad 0 < t \leq 1 \qquad y(0) = 2$$

using $\Delta t = 0.1$. Compute to the actual error. Give your answers to six digits of accuracy.

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2. (15pts) The Heun method is given by

$$Y_{i+1} = Y_i + \frac{1}{4}\Delta t f(t_i, Y_i) + \frac{3}{4}\Delta t f\big(t_i + \frac{2}{3}\Delta t, Y_i + \frac{2}{3}\Delta t f(t_i, Y_i)\big).$$

a. (UG) Verify that the local truncation error is at least order $(\Delta t)^3$; this means that you have to verify that terms through $(\Delta t)^2$ disappear but you don't have to verify that the terms of order $(\Delta t)^3$ don't disappear (although they don't so the local truncation error is exactly third order).

b. (G) Verify that the local truncation error is exactly $(\Delta t)^3$.

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3. (15 pts) Consider the two-step method

$$Y_{i+1} = Y_{i-1} + 2\Delta t f(t_i, Y_i).$$

This differs from the Adams-Bashforth methods that we derived because it uses the approximation Y_{i-1} and not just Y_i. These types of multistep methods can still be derived by using an interpolating polynomial for f but in this case we use a polynomial over the interval $[t_{i-1}, t_{i+1}]$ and thus we must integrate the differential equation $y'(t) = f(t, y)$ over this interval.

a. Use an appropriate constant interpolating polynomial for $f(t, y)$ to derive this method.

b. Determine the exact local truncation error for the method; i.e., don't just say for example order Δt but rather give the constant in front of Δt.
