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***Computational Methods
for Data Evaluation and
Assimilation***

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Preface

This book is addressed to graduate and post-graduate students and researchers in the interdisciplinary methods of data assimilation, which refers to the integration of experimental and computational information. Since experiments and corresponding computations are encountered in many fields of scientific and engineering endeavors, the concepts presented in this book are illustrated using paradigm examples that range from the geophysical sciences to nuclear physics. In an attempt to keep the book as self-contained as possible, the mathematical concepts mostly from probability theory and functional analysis—needed to follow the material presented in the books five chapters, are summarized in the books three appendices.

The authors wish to acknowledge the outstanding professional assistance of Dr. Erkan Arslan at the University of South Carolina, where this book was finalized. Moreover, this book would have not have appeared were it not for the patience, guidance, and understanding of Bob Stern (Executive Editor, Taylor and Francis Group), whom the authors appreciate immensely.

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