Comparative Numerical Analysis Using Reduced-Order Modeling Strategies for Nonlinear Large-Scale Systems

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Abstract

We perform a comparative analysis using three reduced-order strategies – Missing Point Estimation (MPE) method, Gappy POD method, and Discrete Empirical Interpolation Method (DEIM) – applied to a biological model describing the spatio-temporal dynamics of a predator-prey community. The comparative study is focused on the efficiency of the reduced-order approximations and the complexity reduction of the nonlinear terms. Different variants are discussed related to the projection-based model reduction framework combined with selective spatial sampling to efficiently perform the online computations. Numerical results are presented.

1 Introduction

In the case of spatial-temporal systems, numerical simulation is typically achieved by spatial discretization of the governing PDEs using, for example, finite volume or finite element methods. The spatial discretization procedure leads to large-scale systems of ordinary differential equations (ODEs), typically of order $10^3 - 10^8$, depending on the complexity of the governing equations and the desired level of accuracy [10, 35, 36]. The underlying governing equations are generally nonlinear and the model parameters are often functions of state variables (hence time-varying), which adds considerably to the degree of complexity. Thus, for problems of practical interest, the computational effort required to simulate these systems is substantial.

The paper is organized as follows. The methods of reduced-order modeling are described in Section 2. Subsection 2.1 is devoted to Proper Orthogonal Decomposition (POD) method. Subsection 2.2 is focused on reduced order strategies for nonlinear term approximation by presenting Gappy POD method and Discrete Empirical Interpolation Method (DEIM). Subsection 2.3 presents Missing Point Estimation (MPE) method, an approach which is based upon Gappy POD. The numerical comparative study focused on the efficiency of the reduced-order approximations and the complexity reduction of the nonlinear terms is performed in Section 3. There are described different variants of the projection-based model reduction framework, combined with selective spatial sampling to efficiently perform the online computations. Results of extensive numerical experiments are presented, while conclusions are drawn in Section 4.