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Data assimilation of three-dimensional free surface flows using an adaptive adjoint model

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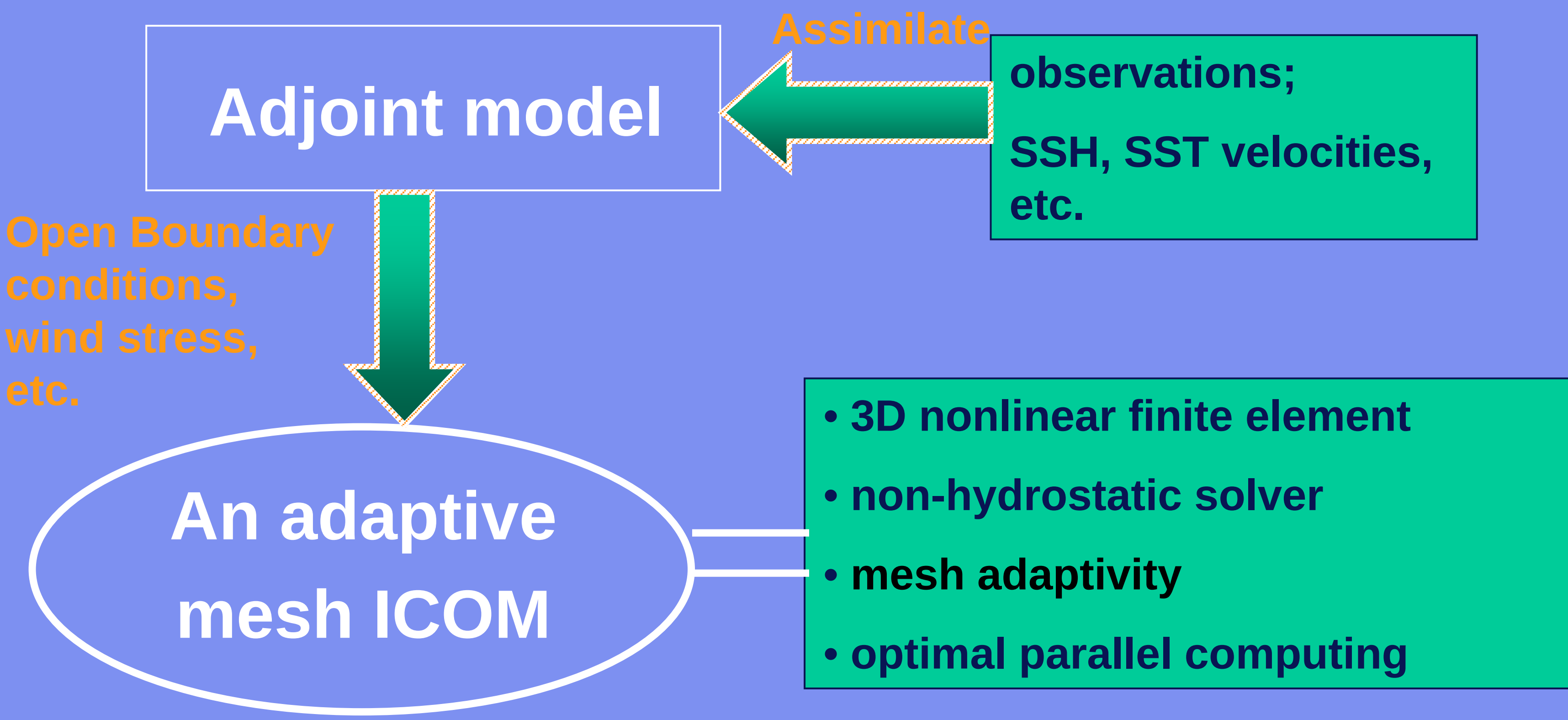
Outline

- **Objective**
- **Features of the forward and adjoint models**
- **Applications**
- **Discussion and future work**

OBJECTIVE

To develop an adjoint model to assimilate observations into a 3D unstructured prognostic model with free surface



Features of the forward and adjoint models

- Different options for non-linear discretisation and adaptive meshes for the forward and adjoint models;

- Dynamically adapt the mesh to optimise the accuracy of the inversion problem and forward solution;

- Incorporation of changing computational domain (free surface) into the 3-D adjoint model and sensitivity analysis;

- Inclusion of penalty terms to remove ill-posedness of the inversion problems and regularise control variables spatially and temporarily;

- Potential to accelerate the inversion with a hierarchy of increasingly fine mesh inversions.

Application to 2D/3D tidal flows

Special attention is given to

- **the accuracy of the gradient computed by the adjoint model;**
- **the feasibility of using adaptive meshes;**
- **the robustness of the adjoint model;**
- **the advantage of the 3-D flow model;**
- **the evaluation of the quality of the inversion.**

Test cases: Inversion of free surface height along the open boundary for 2D/3D tidal flow

The cost function (when considering assimilation of the sea level)

$$\Im(\zeta,\zeta_b) = \frac{1}{2}\int_t\int_\Omega \sum_{k=1}^{Nos} (\zeta-\zeta_o)^T W_{o,k} (\zeta-\zeta_o) d\Omega dt + \frac{1}{2}\lambda \int_t \int_{\partial\Omega} \zeta_b^T \zeta_b d(\partial\Omega) dt$$

The observations are obtained using an identical twin experiment

The water depth: $H_0 = 65m$

The exact inlet tidal height: $\eta_{exact} = 1.0\sin(t/T); T = 12\times 3600s$

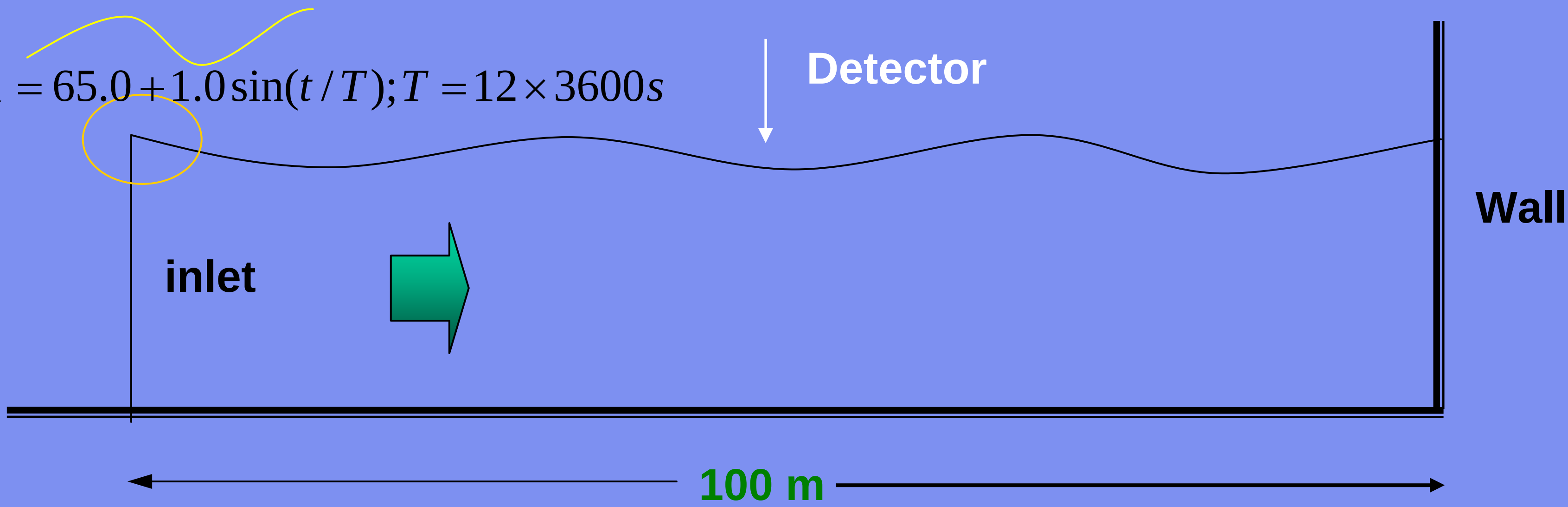
The corresponding inlet velocity: $u_b = \sqrt{g/(\eta + H_0)}\cdot\eta$

Slip boundary conditions are applied at coast and at bottom; Stress free condition on the free surface

Test case1: 2D tidal flow

To invert for free surface height at an inlet by assimilating observational data

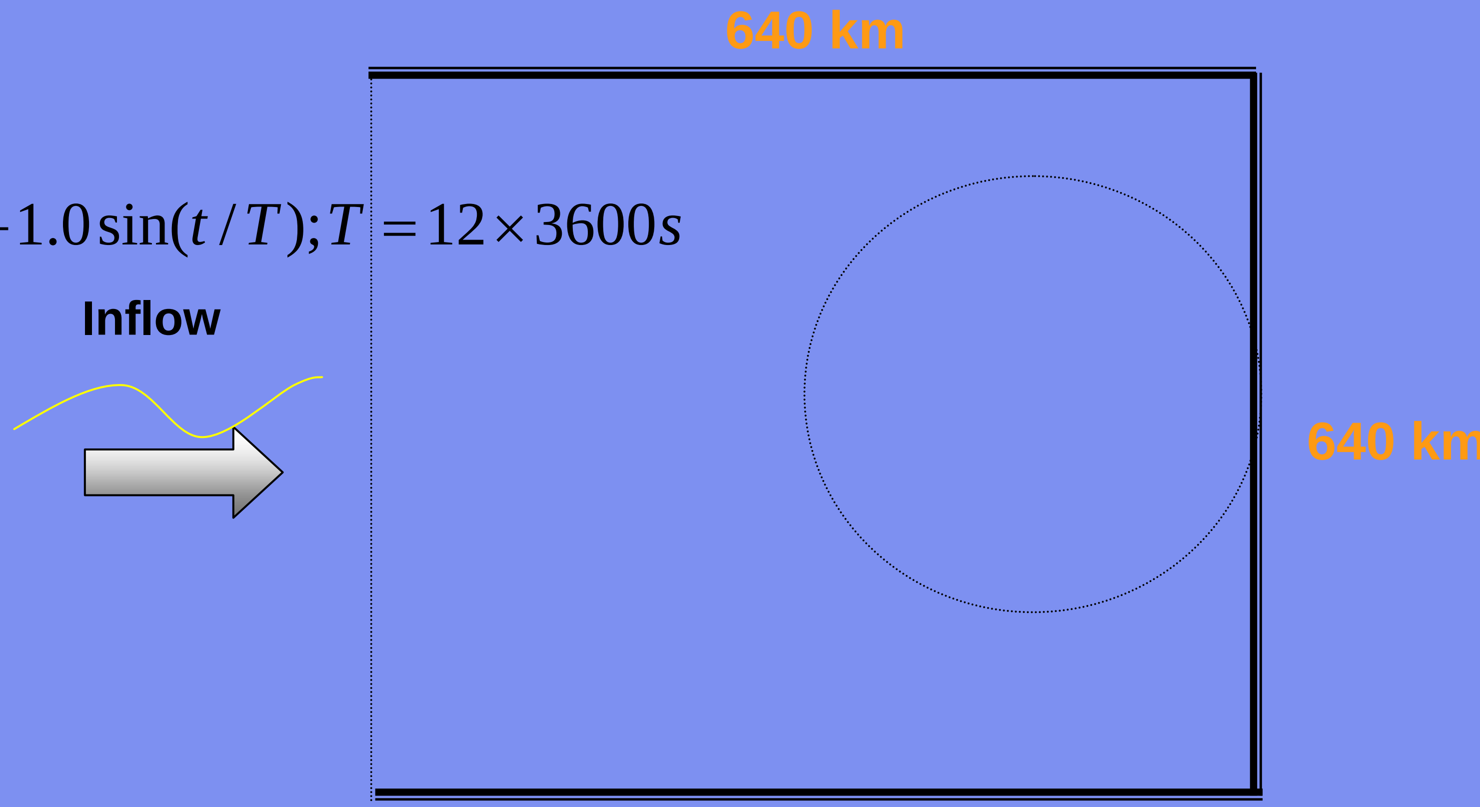
$$\eta_{exact} = 65.0 + 1.0\sin(t/T); T = 12 \times 3600s$$



Initial guess of free surface height

$$\eta_{ini} = 65.0 + 0.5\sin(t/T); T = 12 \times 3600s$$

Test case 2: Inversion of 3D free surface flow

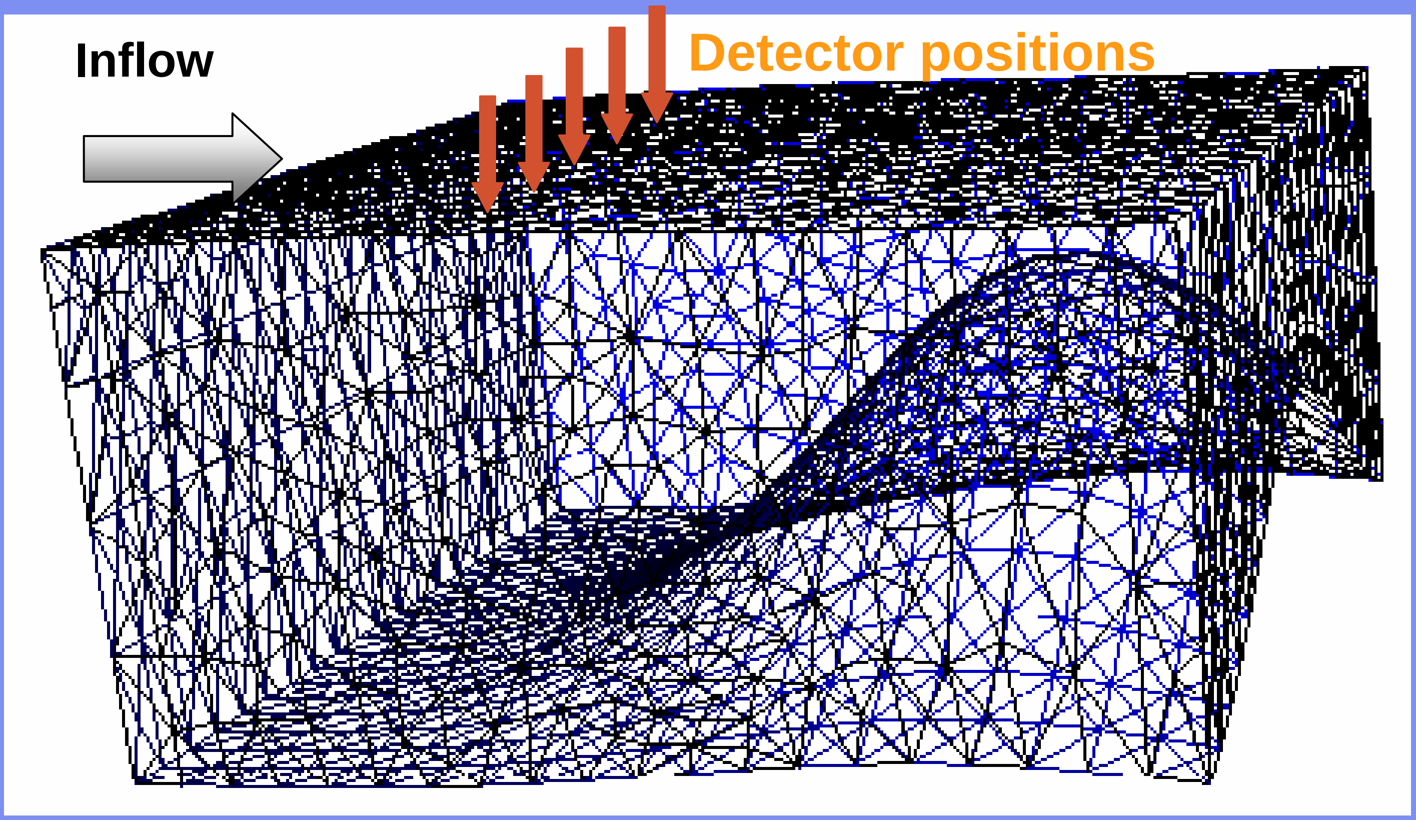


$$\eta_{exact} = 65.0 + 1.0\sin(t/T); T = 12 \times 3600s$$

Initial guess of free surface height: $$\eta_{ini} = 4(t/T - 0.5)^2 + 1; T = 12 \times 3600s$$

Seamount: Gaussian function: $$h_{seamount} = 50.0e^{[(x-500000)^2 + (y-320000)^2]/2*150000^2}$$

Test case 2: Inversion of 3D free surface



Rectangular gulf: 640 k*m* long and 640 k*m* wide

Seamount: Gaussian function:

$$h_{seamount} = 50.0e^{[(x-500000)^2+(y-320000)^2]/2*150000^2}$$

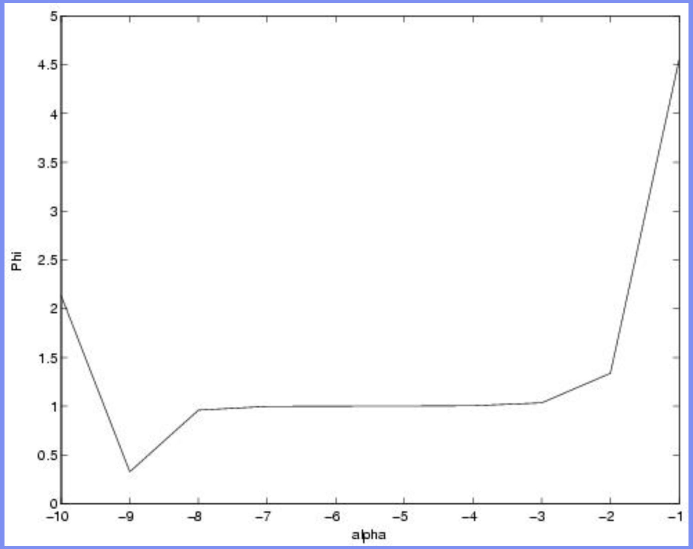
Accuracy of the adjoint model

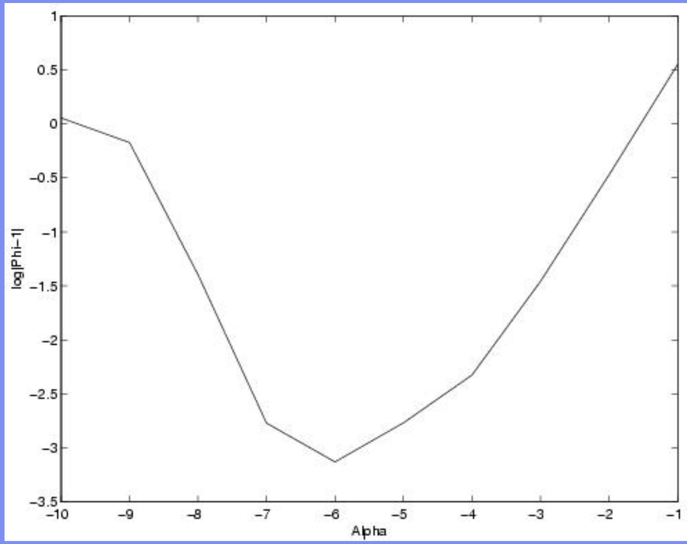
Test the consistency of the gradient (Navon, 1992)

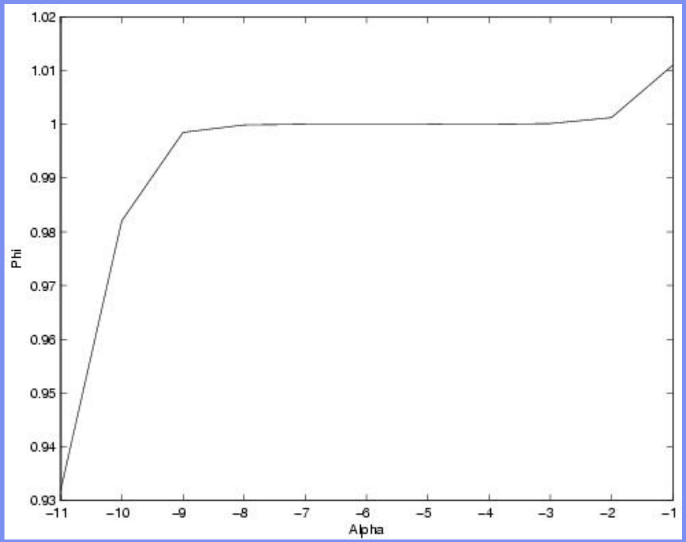
$$\Phi(\alpha) = \frac{\Im(m+\alpha h) - \Im(m)}{\alpha h^T \nabla \Im)m)} = 1 + O(\alpha)$$

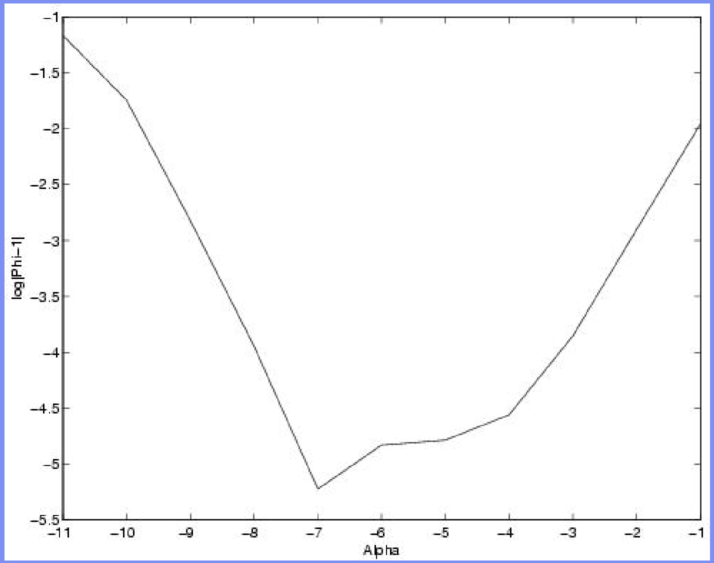
$\alpha \Rightarrow$ **0** **then** $\Phi(\alpha) \Rightarrow$ **0**

Accuracy of the adjoint model

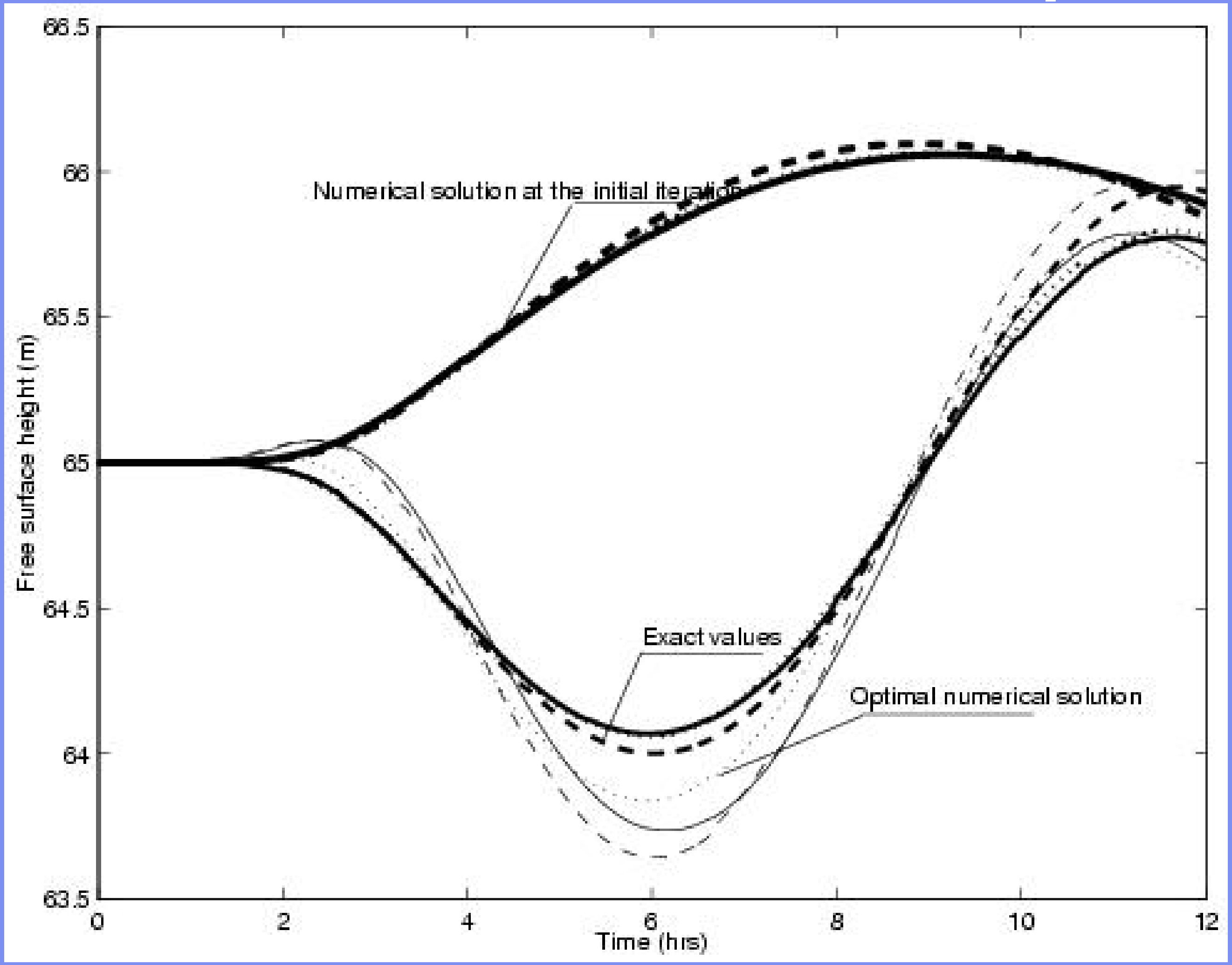








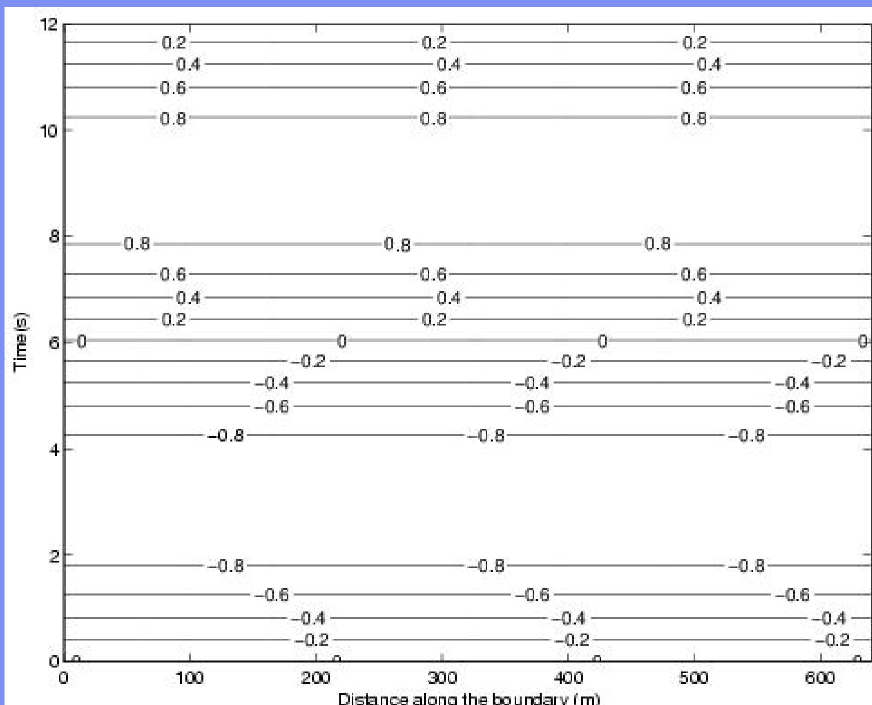
Comparison between the numerical solution and observed data at the detector positions

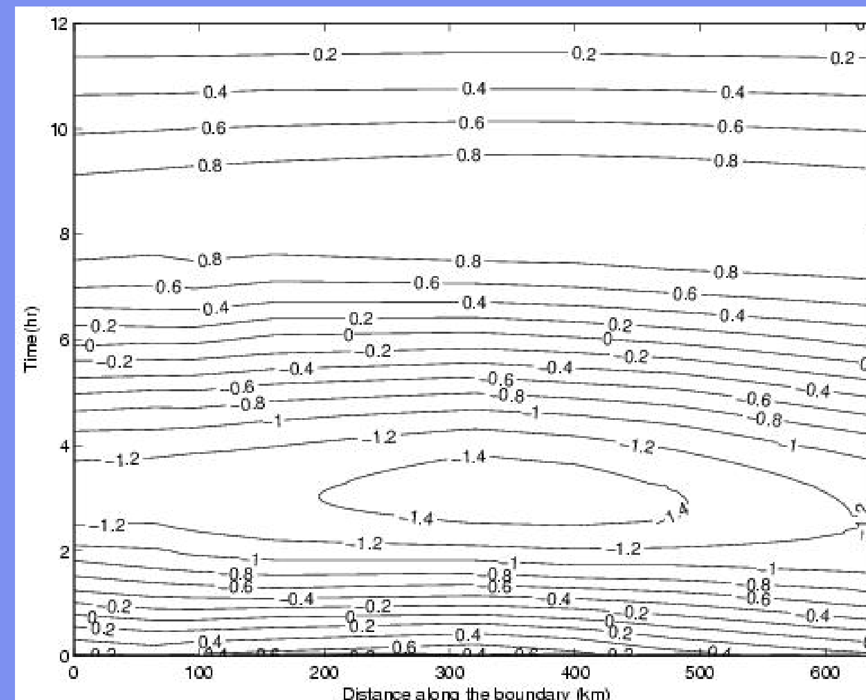


Case 3 3D tidal flow with seamount

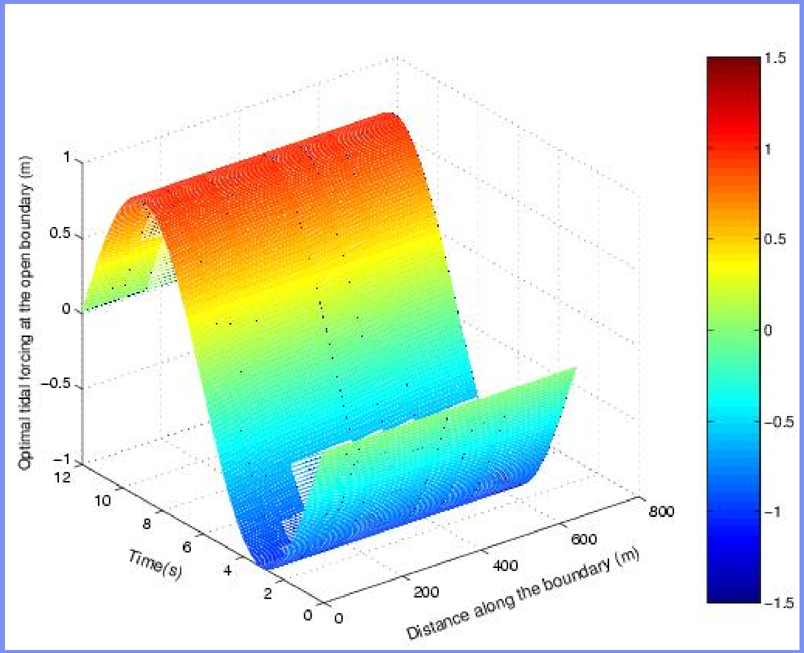
(a) x= 80km, y=250 km (dot line); (b)x=320,y=250km (dash line); © x=560, y=250 km

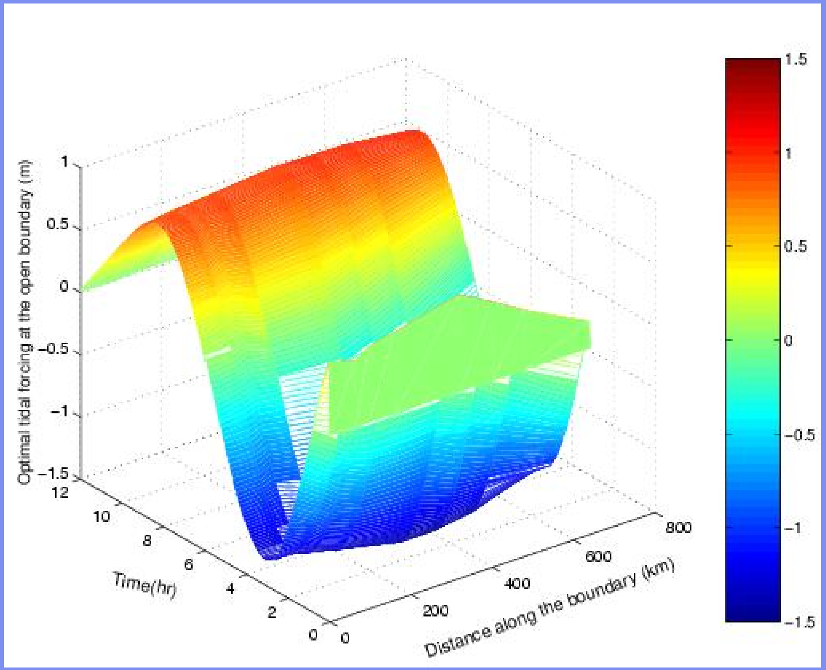
Optimised inlet tidal height.
Case 2: 3D tidal flow with a seamount



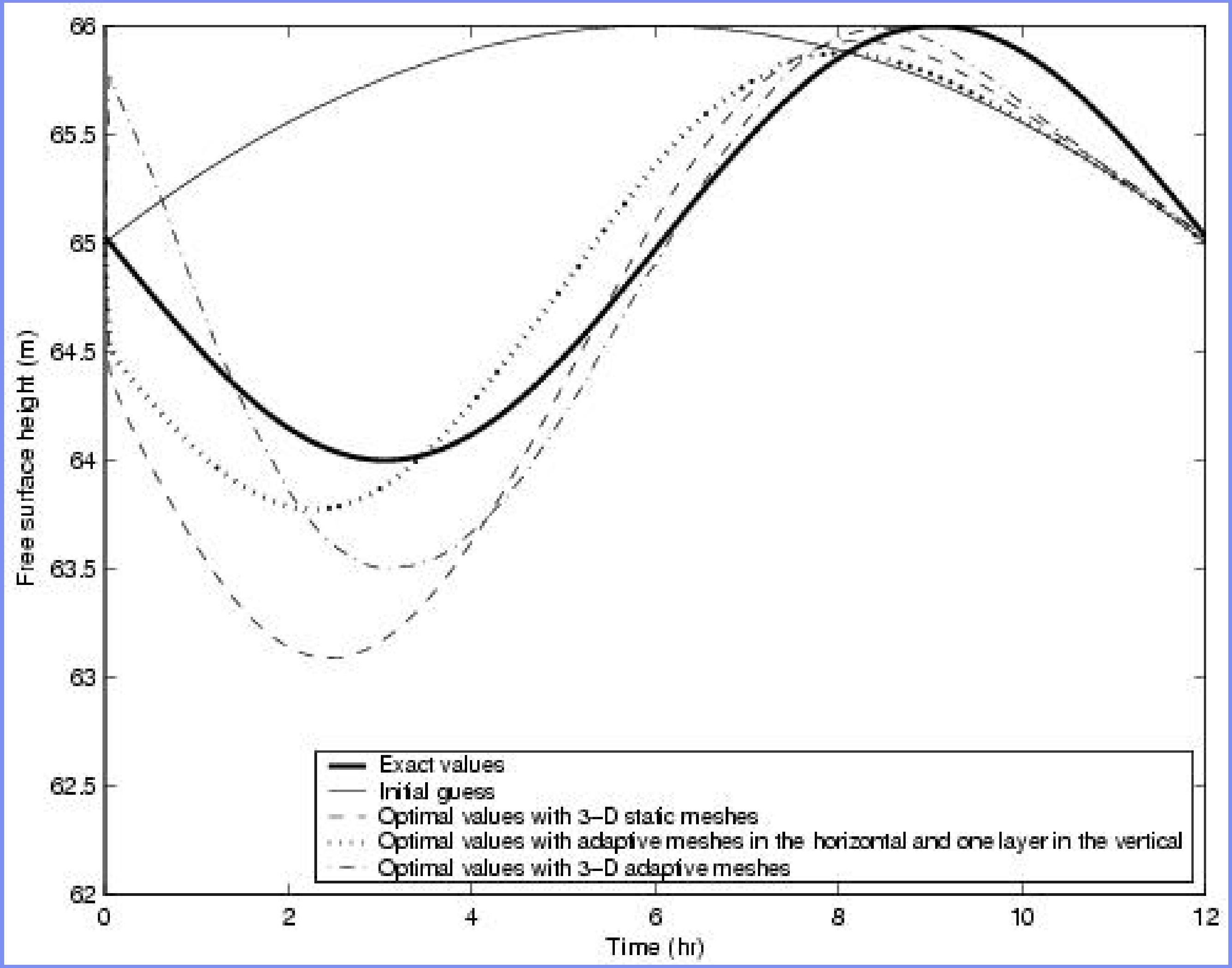


Optimised inlet tidal height.
Case 2: 3D tidal flow with a seamount

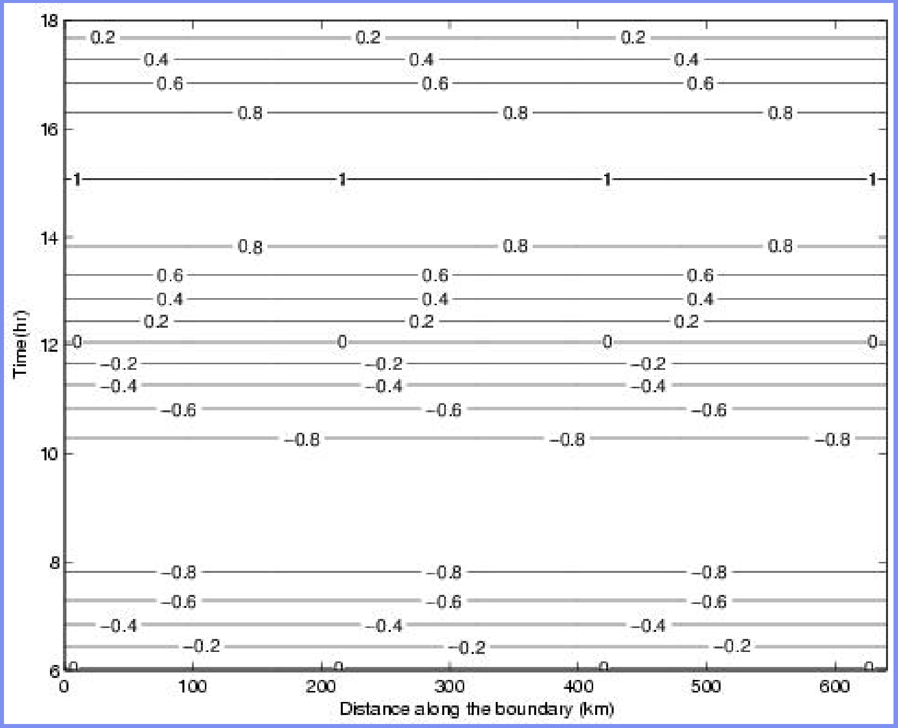


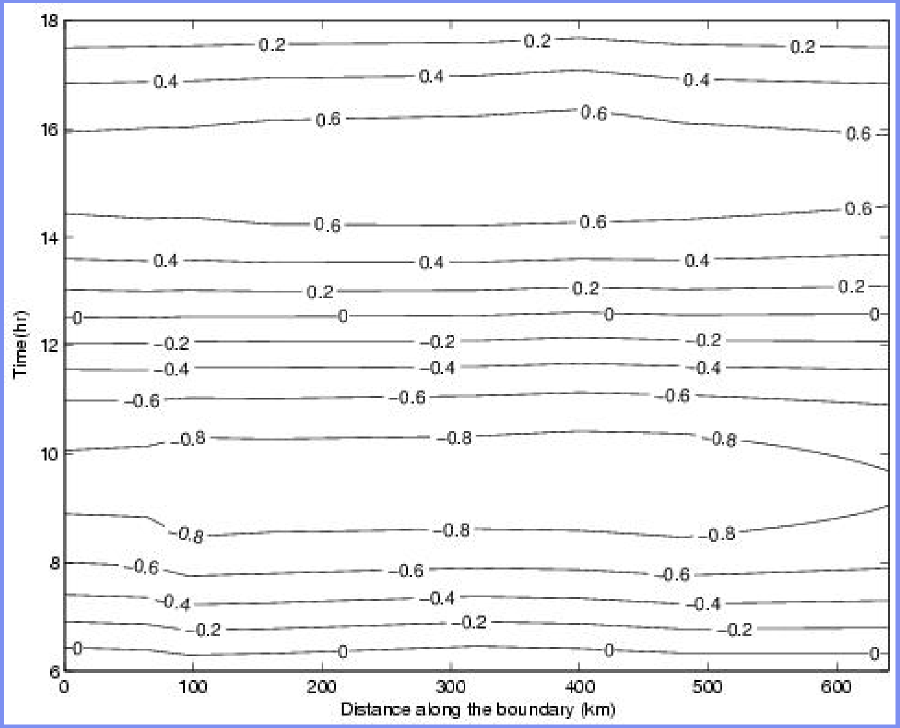


Optimised inlet tidal height at position (x= 320 km)
Case 2: 3D tidal flow with a seamount

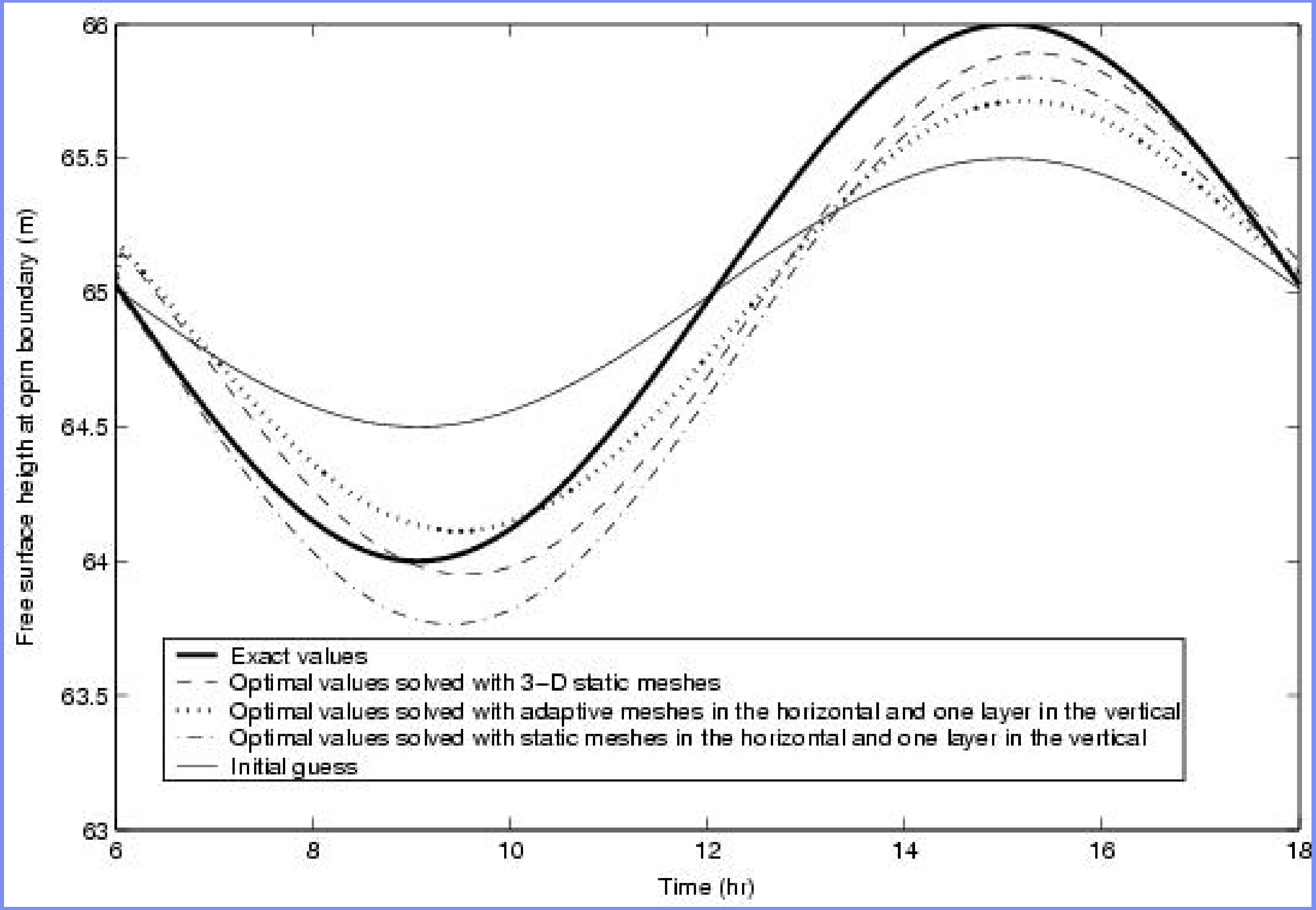


Optimised inlet tidal height.
Case 2: 3D tidal flow without a seamount

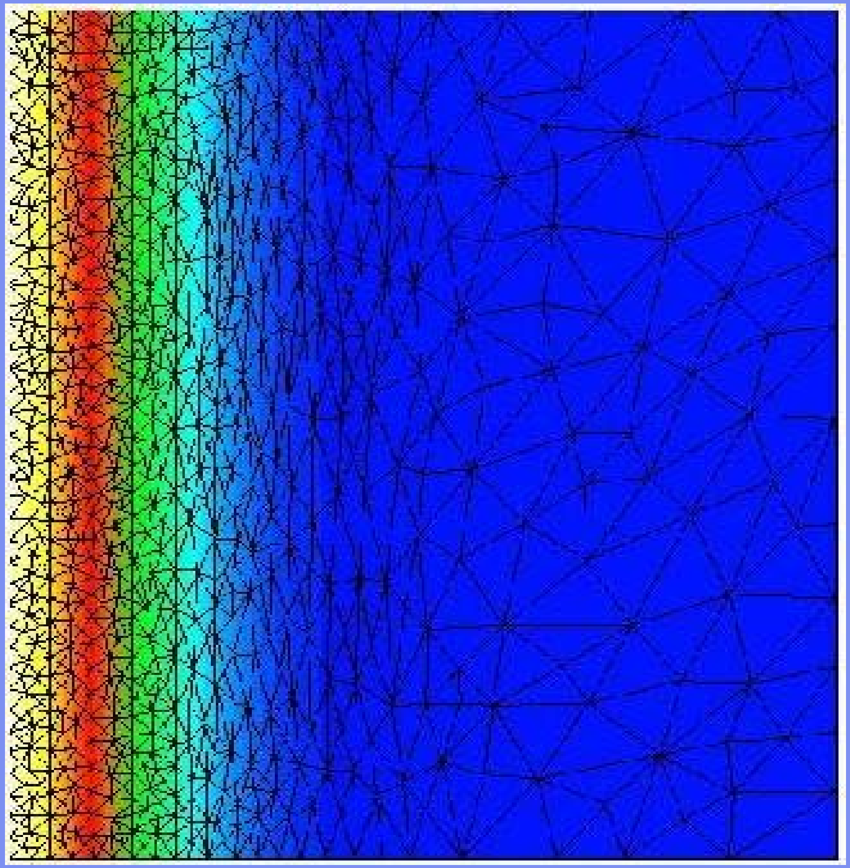


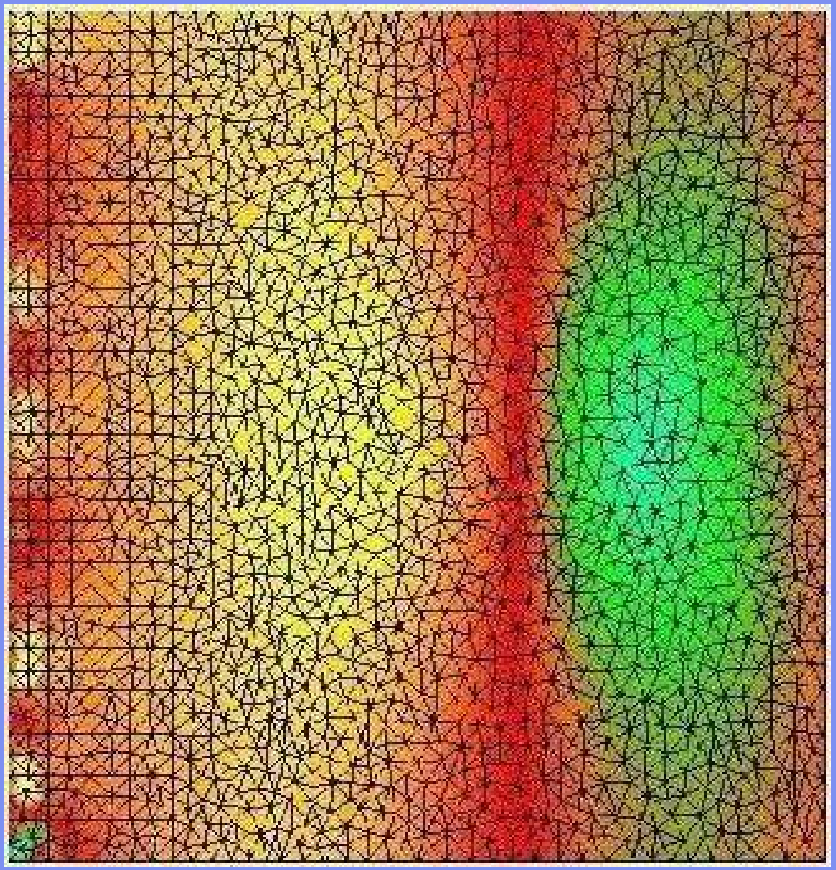


Optimised free surface height at boundary position (x= 320 km, y=0). Case 2: 3D tidal flow without a seamount

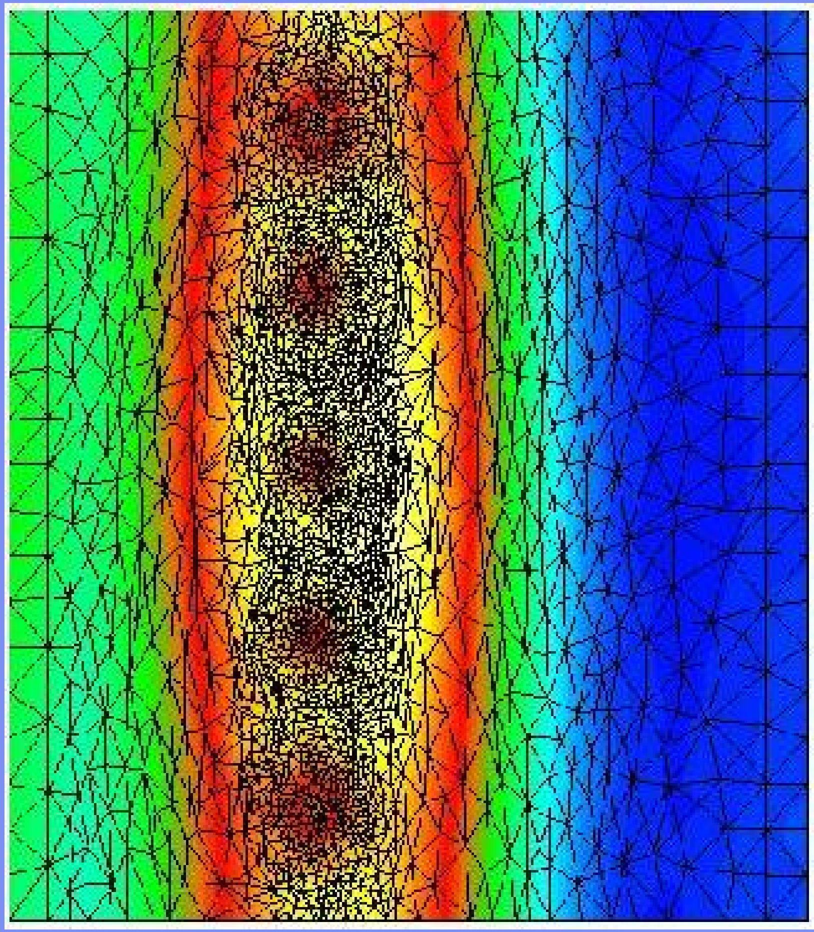


Forward adaptive mesh.
Case 3: 3D tidal flow with a seamount

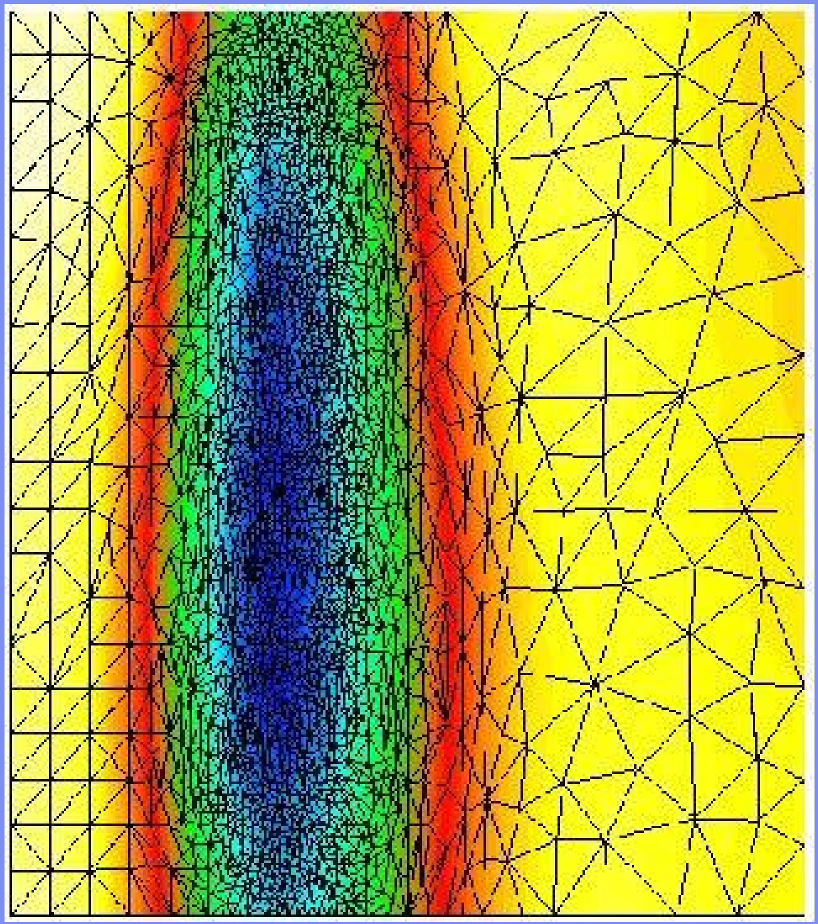




Adjoint adaptive mesh

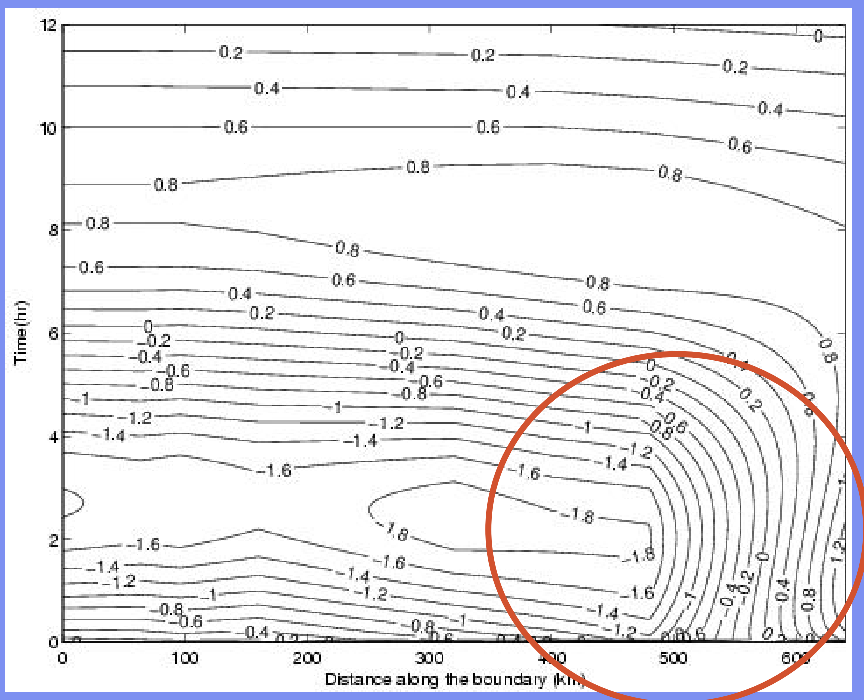


Case 2

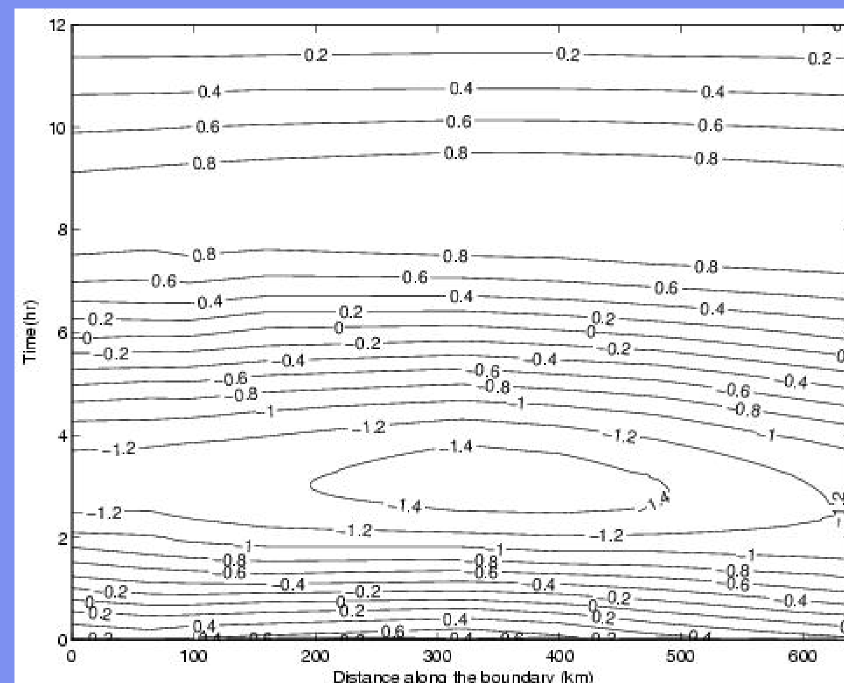


Case 3

Comparison of the optimal results with static and adaptive meshes.
Case 3: 3D tidal flow with a seamount

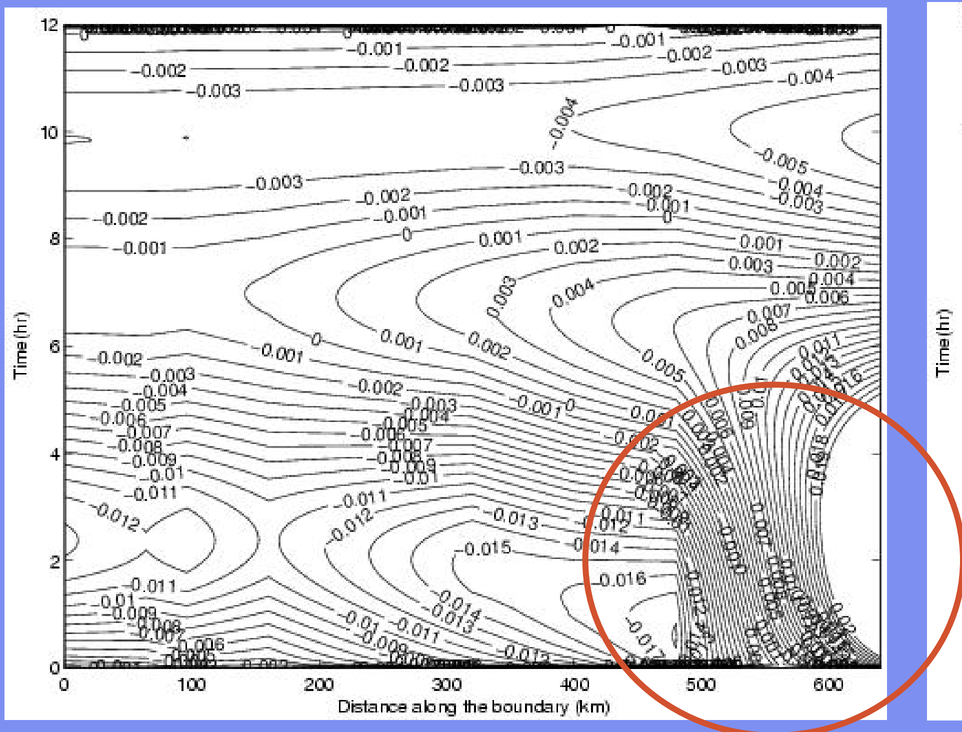


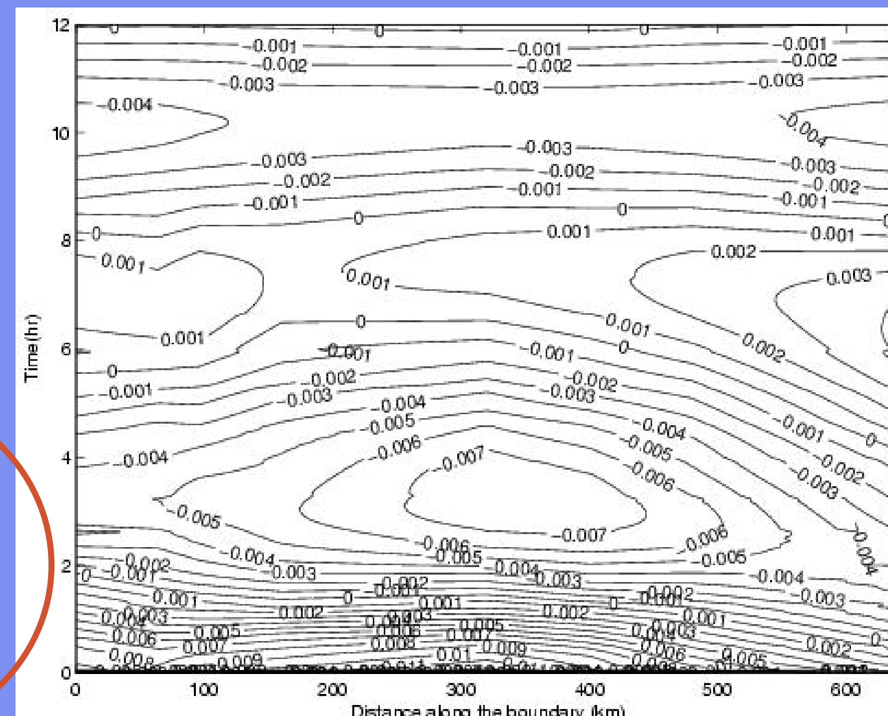
Static meshes



Adaptive meshes

Comparison of the relative error with static and adaptive meshes. Case 3: 3D tidal flow with a seamount





Maximum error: 0.014

Minimum error: 0.001

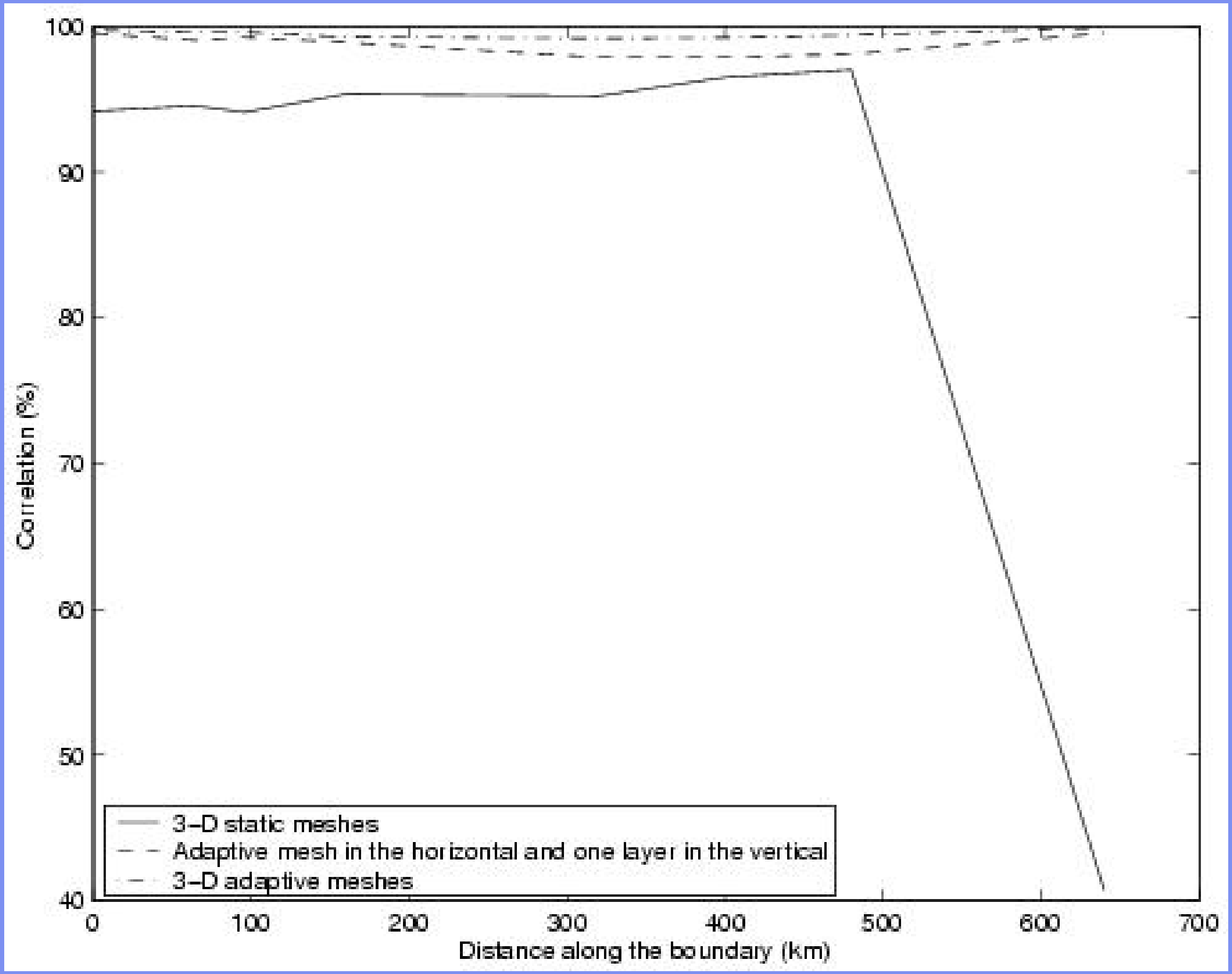
Static meshes

Maximum error: 0.005

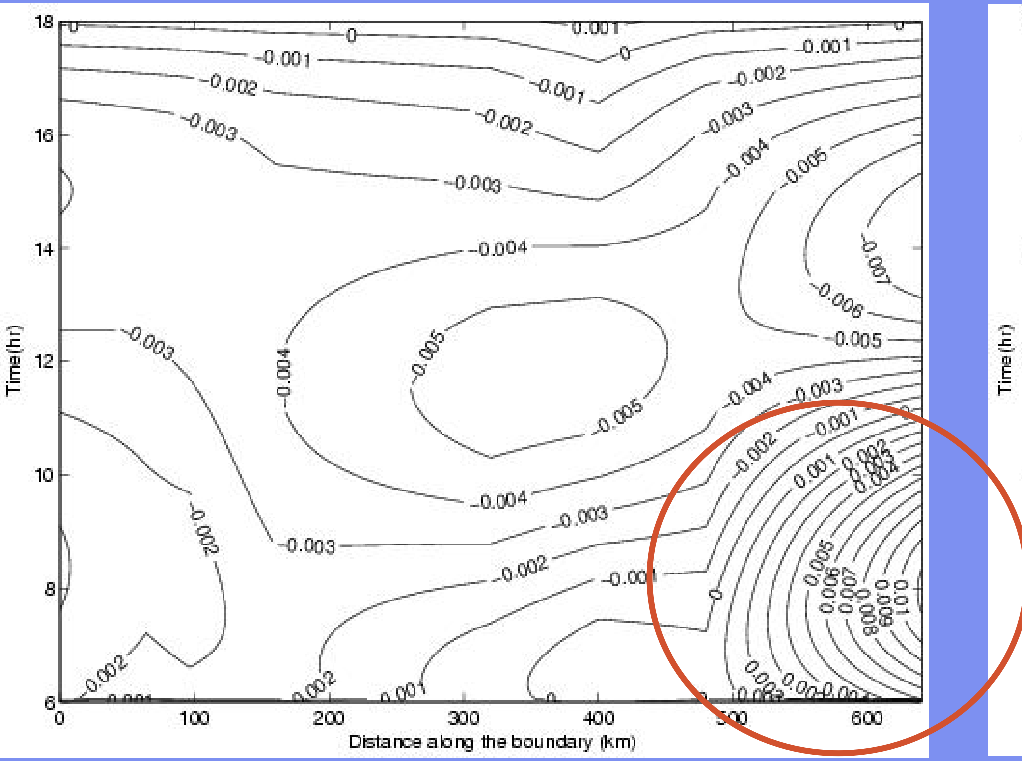
Minimum error: 0.001

Adaptive meshes

Comparison of the correlation with static and adaptive meshes.
Case 3: 3D tidal flow with a seamount



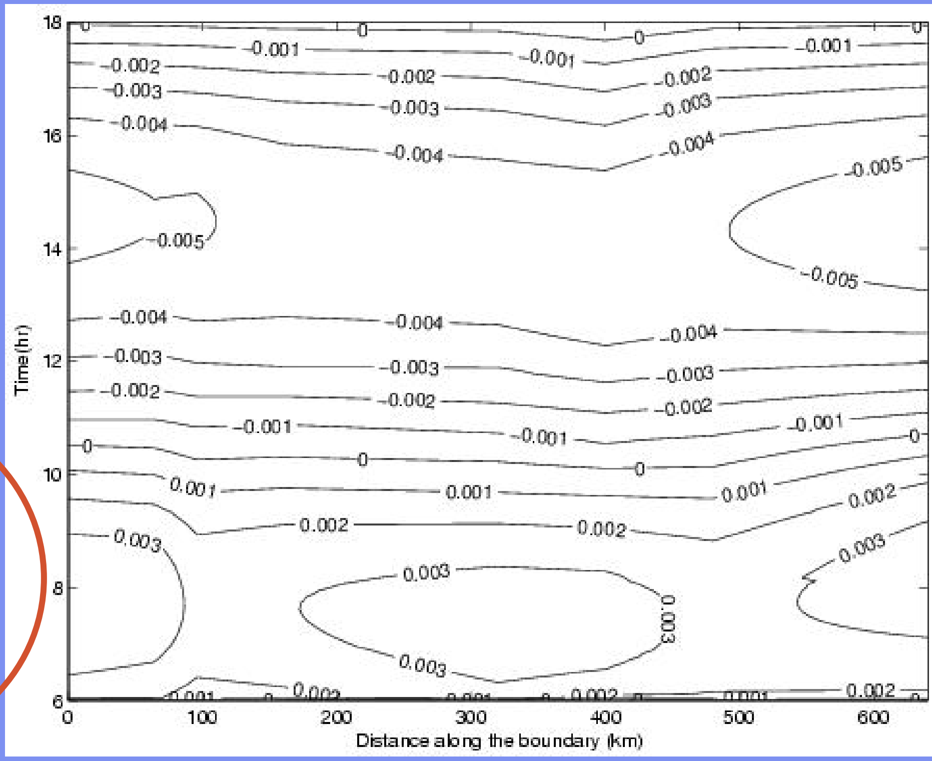
Comparison of the relative error with static and adaptive meshes. Case 2: 3D tidal flow without a seamount



Maximum error: 0.01

Minimum error: 0.001

Static meshes

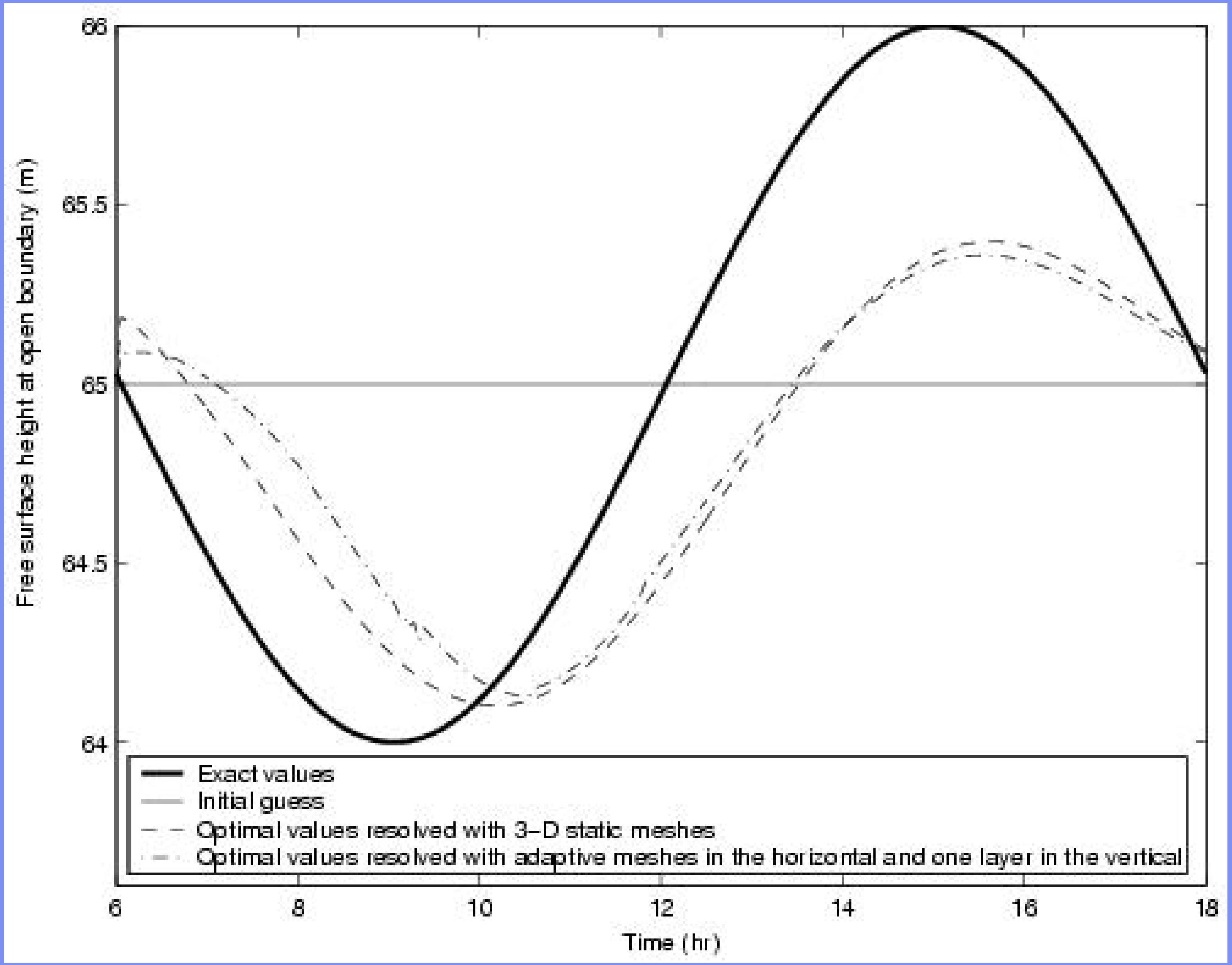


Maximum error: 0.003-0.04

Minimum error: 0.001

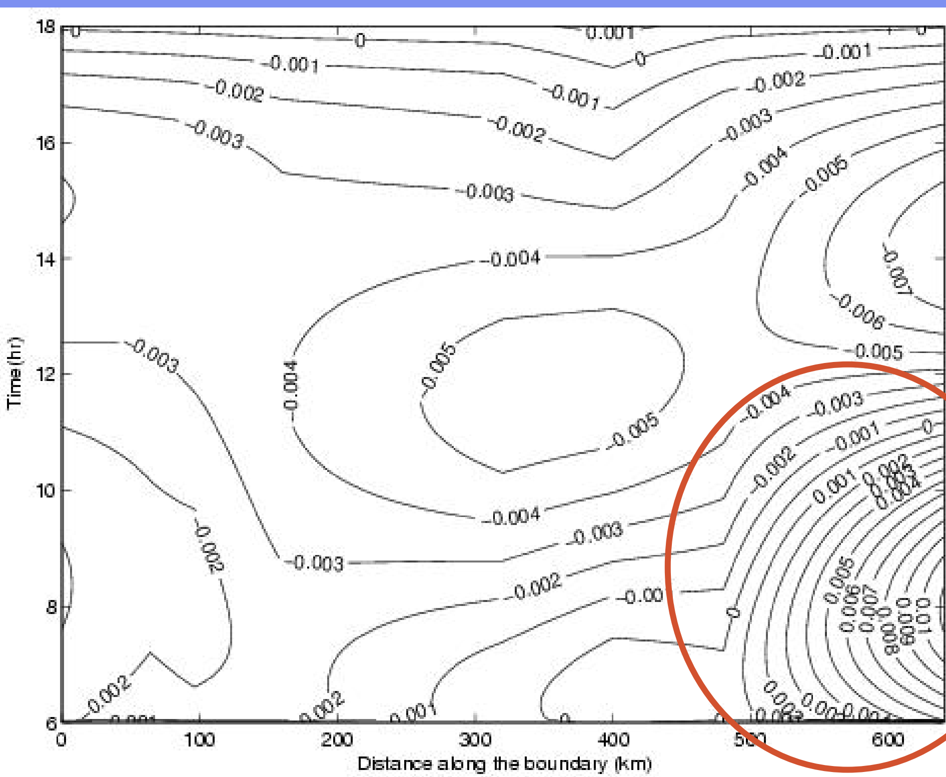
Adaptive meshes

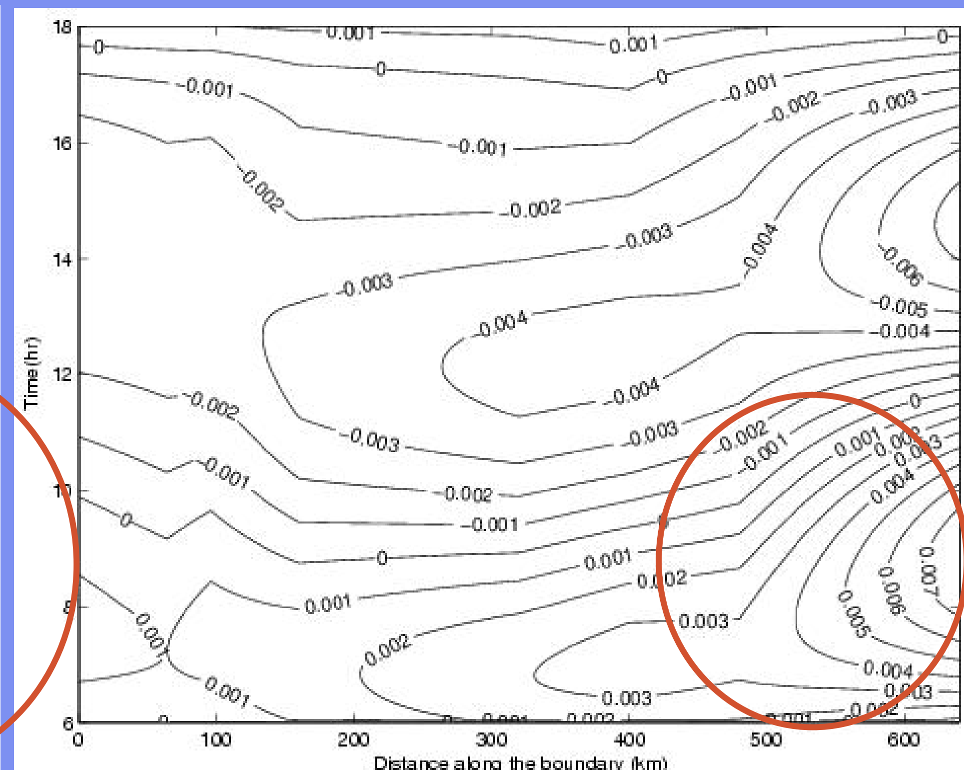
Robustness of the adjoint model



Advantage of the 3D flow model

Comparison between the relative error using 2D and 3D model



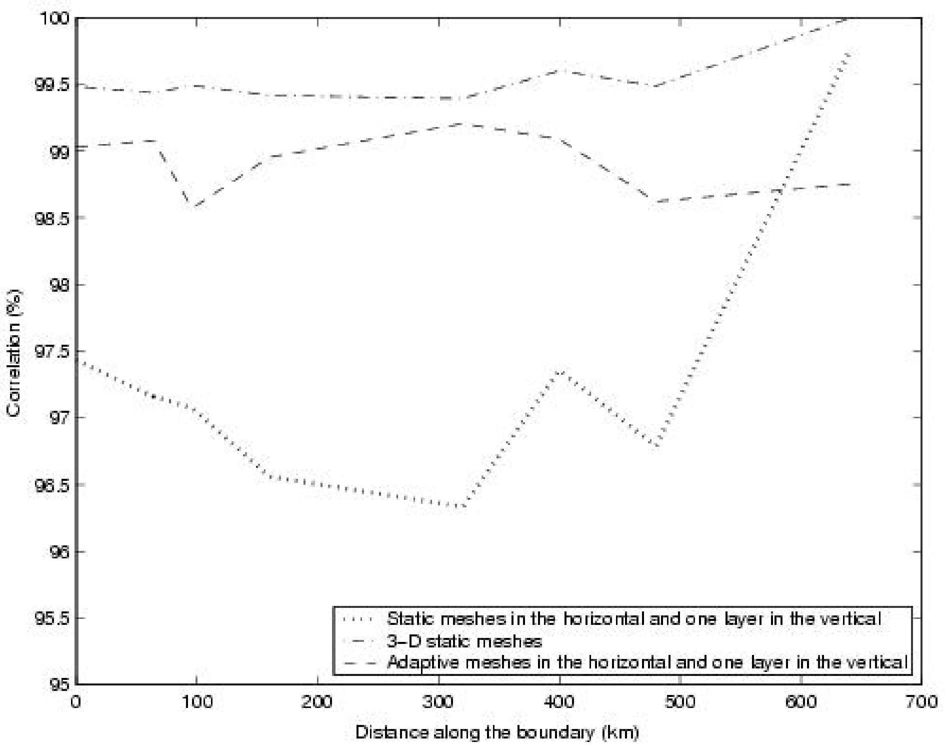


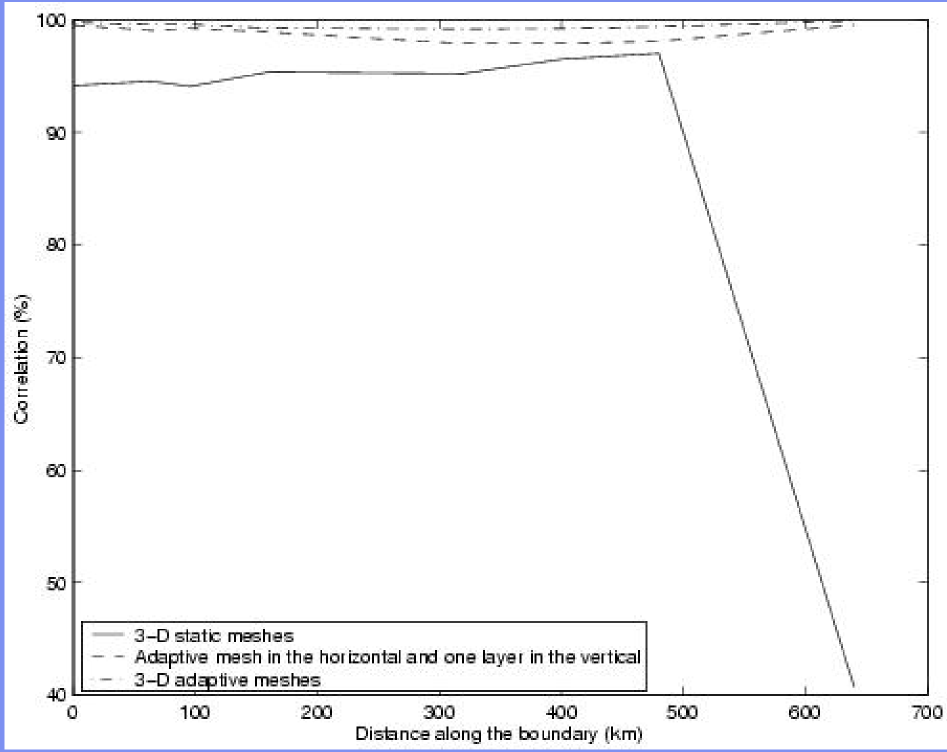
2-D meshes

3-D meshes

Advantage of the 3D flow model

Comparison between the correlation between the optimal and exact values using 2D and 3D model





Discussion

A 3D mesh adaptive adjoint model has been developed and applied to 2D/3D tidal flows.

- **The accuracy of the adjoint model is verified by testing the consistency of the gradient.**
- **The feasibility of using adaptive meshes is evaluated by comparing the numerical results with static and adaptive meshes.**
- **3D ocean model results shown.**

Some conclusions and suggestions can be drawn from this investigation:

- **The accuracy of most of optimal results with adaptive meshes is higher than with static meshes if a suitable set of the parameters for mesh optimisation/adaptivity is chosen.**
- **The use of adaptive meshes can avoid the cumulative dissipation errors which are often seen in the case of static meshes.**
- **To ensure the accuracy of the gradient, it may be necessary to lock the mesh at certain locations and time levels, e.g. the mesh around the boundary.**

Future Work

- **Application to realistic oceanic cases;**

- **Target adaptive observations to optimise data collection and therefore forecast accuracy. The method will use leading SVD's along with an adjoint sensitivity analysis ;**

- ***Duality-based error measures* to guide mesh adaptivity in an inverse model. These methods will be used to optimise the accuracy of the inverse problem. Second order error information will be used which will be obtained from leading Hessian singular vectors of an energy norm in the forecast period reflecting the dynamics and areas in the domain of interest.**