DIRECT NUMERICAL SIMULATION OF COMPRESSIBLE TRANSITION: AN OVERVIEW

M.Y. Hussaini and G. Erlebacher
Institute for Computer Applications in Science and Engineering
NASA Langley
Hampton, VA

ABSTRACT

The last year has seen the completion of several synergistic computational tools to help explore the physics of high-speed transition of wall-bounded flows. In this paper, we briefly describe these tools and how they are used to help clarify some of the physical mechanisms which govern this phenomenon. We conclude with a sampling of interesting results recently obtained at NASA Langley.

INTRODUCTION

After an initially slow start, the last several years have witnessed a continuously increasing dedication to research in the field of compressible transition. As a result, new computational tools have made their appearance, and are already generating numerical data in areas quantitatively explored in the past only by experimenters [1, 2]. These tools include (in order of increasing complexity) numerical methods to solve for mean flows [3], primary instabilities [4], secondary instabilities [7, 8], the parabolized stability equations (PSE) [9, 10], and to perform temporal [11, 12, 13] and spatial [14, 15] direct numerical simulations (DNS). Each stage of a controlled transition process (2-D linear disturbances, onset of three-dimensionality, secondary instabilities, and breakdown) can be simulated using one of the aforementioned methods.

In this overview, we discuss the various techniques, their purpose, how they are used, and how they fit into the overall picture of transition research. We then present pertinent results to illustrate both their capabilities and usefulness.

MEAN FLOW

All detailed studies of the physics of wall-bounded transition require access to mean flow data of high accuracy. Not only must the mean profiles themselves be accurate, but so must their first and second derivatives in the direction normal to the wall (denoted by y). This sensitivity was first brought out by Lees and Lin [16] who related the existence of neutral modes to the presence of a generalized inflection point in the mean flow profiles (a maximum of $\rho dU_m(y)/dy$). Although the analysis was inviscid, and the spatial growth of the mean flow was neglected, this emphasizes the requirement for accurate second derivatives of the mean flow variables.

Mean flow profiles are necessary for any theoretical study of transition (whether under controlled or natural conditions). Traditionally, mean flows intended for stability calculations are computed using at least 4th order accurate finite-difference methods or the more accurate spectral methods [17]. A rule of thumb is that the accuracy of mean flow input data for direct numerical simulation codes should be at least the same as the spatial accuracy of the numerical scheme employed.

There are today many codes available to generate planar boundary-layers profiles. Recently however, research at Langley has begun to focus on the simulation of compressible transition on cylinders and cones to study the effect of curvature on the now standard controlled transition mechanisms. To this end, a spectrally accurate mean flow code which computes spatially growing boundary layers on both cylinders and cones has been developed [17].

PRIMARY INSTABILITY

If

$$q = (\rho, u, v, w, p) \tag{1}$$

denotes the vector of unknowns variables, and a subscript m references the mean flow, q is generally expressed as

$$q(x, y, z, t) = q_m(y) + \tilde{q}(y)e^{i(\alpha x+\beta z-\omega t)} \tag{2}$$

where ω, and α are the complex frequency and streamwise wave number, while β is the (real) spanwise wavenumber. Substitution of q into the Navier-Stokes equations, and linearization with respect to $\tilde{q}$ leads to an eigenvalue problem for either ω or α and β. These stability calculations fall primarily into two categories: temporal or spatial. In the former case, one specifies a real α and β, and computes a complex ω, which is an eigenvalue of the linear set of equations. The sign of the imaginary part of ω determines whether the wave is growing or decaying. Alternatively, the spatial problem involves the solution to a nonlinear eigenvalue problem in α and β, with a prescribed real frequency ω_r. This leads to an indeterminacy in the computation of the spatial wavenumbers. Several remedies have been proposed [18]. Note that although the spatial eigenvalues are computed, the parallel flow assumption is still maintained. To properly compute spatial eigenvalues while taking the growth of the boundary layer into account requires the use of a multiple scales approach [5] or PSE [10].

Linear stability codes are readily available to study a wide variety of stability related problems [4, 19], in both axisymmetric and flat geometries. These codes can compute global spectra, and have local procedures to compute individual eigenfunctions. Some codes (e.g. [19]) can track specific waves along the plate to perform e^n calculations to estimate the onset of transition.

SECONDARY INSTABILITY

Consider a mean flow $q_m(y)$ and a single two-dimensional primary wave. Left to develop according to the full Navier-Stokes equations, this primary wave will often saturate due to nonlinear interactions. A new mean flow formed by the superposition of the parallel mean flow and the spatially periodic saturated primary wave leads to a new stability problem labeled secondary instability. A good summary is contained in [20] where Herbert et al. discuss the use of secondary instability theory to explain the presence of aligned and staggered three-dimensional lambda-shaped vortices in incompressible shear-driven flows. These vorticity patterns are obtained by considering a base flow which is the combination of the previous mean flow and a (typically) two-dimensional eigenfunction. The strong unstable modes emerging from this spatially periodic mean flow are called secondary instability modes. They often travel at the same phase velocity as the primary wave, and thus strong resonances are established which lead to energy exchange. The vorticity structures which emerge are often a precursor to full-fledged transition. Since the work of Herbert, much work has gone into analyzing the properties of secondary instabilities of compressible flows. For example, the work of Ng and Erlebacher [8] studies the secondary instability of primary waves in both subsonic and supersonic flows. The code that was used is an outgrowth of a linear stability code enhanced to handle two-dimensional and oblique primary waves, and is capable of both temporal and spatial eigenvalue and eigenfunction analysis. With the exception of [8] which considers subharmonic, fundamental and detuned modes, all the work to date has considered subharmonic modes only [6, 7].

PARABOLIZED STABILITY EQUATIONS

Next in the line of increased sophistication is to recognize that direct numerical simulation of incompressible flow on a flat plate using the parallel approximation (see below) leads to results which are qualitatively, but not quantitatively correct. In short, the instability waves convect downstream at the phase velocity of the dominant primary waves that are input as initial conditions. Of course, the growth of the boundary layer is not taken into account, but the qualitative agreement suggests that one could derive a marching scheme in the streamwise direction using approximations similar to those used in parabolized Navier-Stokes calculations. These ideas are formulated by Bertolotti [9] for linear compressible flow and nonlinear incompressible flow, and later extended to nonlinear compressible flow by Chang et al. [10]. For simplicity, we restrict this cursory discussion to linearized compressible flows. The idea is express the dependent variables as

$$q(x,y,z,t) = q_m(x,y) + \tilde{q}(x,y)e^{i(\int^x \alpha(y)dy + \beta z - \omega t)} \tag{3}$$

where the amplitude function $\tilde{q}$ is now dependent on the streamwise coordinate x. The exponential expresses the oscillatory behavior of the wave in the x direction, whereas the x dependence of the amplitude functions incorporates effects of boundary layer growth. The parabolization is accomplished by neglecting second and higher order derivatives of q_m, $\tilde{q}$ and α with respect to x. This approximation is valid unless the boundary layer grows too rapidly. Therefore one expects the method to break down near the leading edge of a flat plate, for low Reynolds number flows, and in the advanced stages of transition.

It is possible to generalize the expansion (3) to treat nonlinear wave interactions by replacing the perturbation part of (3) by a double Fourier expansion in t and z [10]:

$$q(x,y,z,t) = q_m(x,y) + \sum_m \sum_n \tilde{q}_{m,n}(x,y)e^{i(\int^x \alpha_{m,n}(y)dy + \beta z - \omega t)} \tag{4}$$

Simulations have been performed using the nonlinear formulation, and qualitative agreement obtained against those of Bertolotti [9].

DIRECT NUMERICAL SIMULATION

Finally, we come to the "brute-force" simulation approach where one strives to solve the full compressible Navier-Stokes equations with a minimum of approximations. At the present time, DNS codes fall into the two standard categories: temporal and spatial. Temporal simulations neglect the effect of mean flow growth, and hence assume that the flow is periodic in the streamwise direction. This assumption allows the efficient implementation of spectral collocation methods which use Fourier expansions in the streamwise direction. This approach has proven very successful in the study of incompressible boundary-layer flows [21]. However, this approximation precludes the study of several types of phenomena such as receptivity and localized roughness elements.

To this end, spatial simulation codes which take full account of the boundary-layer growth are slowly becoming available to transition researchers interested in the study of compressible transition [14, 15]. Unfortunately, a spatial code requires inordinate amounts of computer resources (both memory and CPU time). For example, a temporal simulation of the evolution of a $M_\infty = 4.5$ 2-D second mode wave interacting with two oblique waves on a cylinder takes about 60 time periods of the 2-D primary mode before strong nonlinear effects lead to the final transition stage. An equivalent spatial simulation is quite unthinkable. However, the study of receptivity mechanisms where the spatial dependence of the mean flow and possibly the inclusion of localized disturbances, require a spatial simulation code (or at the very least a PSE code). Fortunately, receptivity-related problems depend only on linear and weakly nonlinear interactions [?]. Therefore the use of spatial DNS is quite realistic.

RESULTS

Parabolized Stability Equations

The parabolized stability equations provide a computational capability which lies between that of linear and nonlinear simulation codes. Already, the PSE approach has been applied to the study of receptivity [22] and secondary instability. To date, both linear and nonlinear computations based on the compressible PSE have been conducted [9, 10]. As expected, there is quite a good agreement between linear PSE and the method of multiple-scales, since the approximations in both approaches essentially depend on the two lowest terms of an expansion in powers of $Re^{-1/2}$. The parabolized method exhibits several features in common with spatial direct numerical simulations, even in the linear regime. First, the slow dependence of the wave amplitudes on x leads to a growth rate which depends on both y and on the dependent variable used to measure it. In the linear regime, we find that the growth rate of the mass flow fluctuation exhibits strong variations in the region of maximum variation. Therefore, if experimentalists are not careful with measurements of growth rate based on hot wire signals, very large disparities in the growth rates could ensue. This is particularly true if the measurements are carried out at the point of maximum signal strength.

The y dependence of the growth rate also results from the nonlinear interaction between waves. To study this effect, and to verify the accuracy of the code, the PSE were solved in a parallel boundary layer and compared with results from Erlebacher and Hussaini [11]. There was good agreement both in the eigenfunctions of the various variables (Fig. 1), and of the growth rate as a function of y (Fig. 2). Both figures were obtained from a nonlinear PSE simulation, starting from a 2-D second mode wave [10] at $M_\infty = 4.5$, $Re^*_\delta = 8000$. Note that, there is a very strong variation of the growth rate in the critical layer region.

Several nonlinear calculations were also carried out for a $M_\infty = 1.6$, $Re = 460$ flow at high resolution (7 temporal modes and 5 spanwise modes) to try and achieve breakdown. One simulation was initiated with a primary wave amplitude of 3%, and two oblique subharmonic waves at 0.019% amplitude (measured with respect to the maximum streamwise primary velocity). The spanwise wavenumber is $\beta/R = 1.52 \times 10^{-4}$. We refer to [10] for full details. A time history of the spanwise vorticity for this case is presented in Fig. 3 for the peak plane. A similar run was also conducted for the fundamental type breakdown [10].

Boundary-Layers on Cylinders

Pruett, Ng and Erlebacher [3] investigate the stability of high-speed, axisymmetric boundary-layers using secondary instability theory and DNS. These are conducted at $M_\infty = 4.5$ under the parallel flow approximation. To date, they have mainly considered a transverse curvature parameter $C = 0.1$, where

$$C = \frac{\delta^*}{R^*}. \tag{5}$$

Here, δ^* and R^* are the boundary-layer displacement thickness and the radius of the cylinder respectively. The effect of C on the shape of the eigenfunctions of the primary wave and on the linear growth rates are about the same as non-parallel effects. In an attempt to quantify the effects of C on the linear stability properties of the flow, they compute the direct and indirect effects of curvature. By direct influence is meant the contributions of the curvature terms in the linearized equations. In contrast, curvature effects also manifest themselves in the streamwise evolution of the mean flow. Figure 4 shows the direct and indirect effects of curvature, obtained by setting either the curvature terms in the linear equations to zero and by using a flat mean profile respectively. These effects are seen to be additive for the case considered.

As for the primary waves, the characteristics of secondary instability are similar to the known properties of flat plates. Numerical experiments indicate that the subharmonic instability at fixed amplitude of the primary wave is rather insensitive to variations of C in the range [0.00-0.13]. However, although the effect of curvature on the primary wave amplitude is small (Fig. 5), the resulting effect on the properties of a subharmonic secondary instability can be substantial due to the strong sensitivity of the secondary growth rate to the amplitude of the primary wave. This could qualitatively change the location of transition onset.

Direct numerical simulations of flow over a cylindrical body at $M_\infty = 4.5$, $Re_{L^*} = 994.6$ and $T_e^* = 110°R$ were compared against results from secondary instability theory [3]. Initial conditions consisted of a mean flow, a second mode primary wave, and two subharmonic secondary waves. The amplitudes of the primary and secondary waves were set to 8.5% and 0.85% respectively (based on the amplitudes of the temperature fluctuations of the primary wave and lowest order secondary wave). Agreement between theory and DNS persisted for over 30 periods (in time) of the primary wave.

The temporal DNS of $M_\infty = 4.5$ flow past a cylinder was continued into the strongly nonlinear stages [13]. Figure 6 shows the growth of selected wave harmonics in time for 55 periods. The symbols represent the growth rates predicted by linear stability theory (LST) (circles) and SIT (squares) respectively. Several indicators suggest that transition has been initiated. For example we observe a saturation of the lower harmonics, a rapid filling of the spectrum in Fourier space, and a sharp increase in the wall friction coefficient C_f. A major result from the simulation is the initial development of strong regions of Reynolds stress in the critical layer (away from the wall). These regions of maximum stress then migrate towards the wall (Fig. 7). Numerical experiments indicate that linear stability theory alone cannot generate the high Reynolds stresses in the critical layer. On the other hand, although LST and SIT together can generate regions of high Reynolds stress in the critical layer, the location of maximum Reynolds stress does not approach the wall fast enough with streamwise distance x (the relationship between x and t is obtained from a Gaster transformation). The conclusion is that the experimentally observed spreading rate of Reynolds stress towards the wall [23] is due to strong nonlinearities in the flow, ariszing from large-amplitude subharmonic secondary instability waves.

Finally, the famous "rope-like" structures observed by several experiments in hypersonic boundary layers [23] are observed to be two-dimensional projections of an array of staggered Λ vortices. This was determined by comparison of Schlieren "photography" generated numerically, correlated with standard three-dimensional contour surfaces of the gradient of density in the wall-normal direction.

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