RATE OF STRAIN TENSOR STATISTICS IN COMPRESSIBLE HOMOGENEOUS TURBULENCE

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1. INTRODUCTION

In recent direct numerical simulations of compressible homogeneous shear flow (ref. 1-2), global properties of the flow were examined to determine the effect of compressibility on the growth of kinetic energy and of dissipation. Conclusions of this study indicate that compressibility effects (associated with increased initial turbulent Mach number) decrease the growth rate of the kinetic energy. This reduction in the growth rate was determined in Sarkar, Erlebacher and Hussaini (ref. 2) to be due to enhanced compressible dissipation and pressure-dilatation terms, and due to reduced production by the mean shear in the energy equation. The present study emphasizes the properties of the rate of strain tensor in order to provide insight into the structure of the turbulence at the intermediate and small spatial scales.

Within the last decade, detailed analyses of numerically generated databases have revealed new structural properties of homogeneous turbulence, hitherto unsuspected (ref. 3-7). Most of the work was motivated by the desire to understand the origins of the intermittent dissipative structures.

Very little is currently known about the small scale structure in compressible homogeneous turbulence. Blaisdell (ref. 8) has done some statistical analysis of results from high resolution direct numerical simulations. A significant result is that the vorticity and the dilatation are statistically independent for all time and are therefore decorrelated. This is borne out in our own simulations.

The main objective of this paper is to initiate a systematic investigation of the small spatial structures present in compressible homogeneous turbulence. This is accomplished through an analysis of the rate of strain tensor calculated both from the solenoidal and from the irrotational components of the velocity field. Statistical properties of the tensors' eigenvalues and eigenvectors are presented. We also look into the relationships between the pressure gradient, the vorticity and the eigenvectors of the rate of strain tensors, and how these are affected by the mean shear rate. At low shear rates, we expect only slight departure from incompressible statistics. However, as the shear rate is increased, both the solenoidal and the irrotational velocity components will vary in a $O(S^{-1})$ time, determined by the mean shear. This leads to possible interaction mechanisms between the two components.

2. APPROACH

The homogeneous shear flow data used for the ensuing analysis is obtained from the direct numerical simulation (DNS) of the full compressible Navier-Stokes equations subject to assumptions of spatial homogeneity. The mean flow is unidirectional in the x direction, and has uniform shear, S, in the vertical direction:

$$U(y) = Sy. \tag{1}$$

The mean temperature and density are uniform. The velocity components are expressed in perturbation form which leads to coefficients in the equations which depend on the normal coordinate y. A time-dependent transformation is applied to the physical coordinates to replace this y dependence by coefficients which depend on time. This transformation was first used by Rogallo (ref. 9) in DNS, and is now a standard tool to transform some standard homogeneous flows to a form amenable to Fourier spectral methods.

Velocity, density and temperature are respectively non-dimensionalized by the initial rms velocity, the constant mean density $\overline{\rho}$, and the constant mean temperature, $\overline{T}$. The viscosity μ, is constant, and the Prandtl number is set to 0.7, valid for air.

The coordinate-independence of the flow equations in the (moving) computational domain permits us to express the solution in terms of Fourier basis functions. Thus, the flow is periodic and contained in a cubic box of length 2π. The equations are solved using spectral collocation with dealiasing. The scheme is fully explicit, and the spatial operator is advanced in time with a low-storage third-order Runga-Kutta method. Details of the algorithm and of the initialization process are given in Sarkar et al. (ref. 1).

Contrary to the dynamics of decaying compressible isotropic turbulence (ref. 10,11), in which the effects of initial conditions are never quite eradicated, we expect that, with proper scaling, the structure of the turbulent shear flow is intrinsic to the flow, independent of the initial data. The shear flow simulations are parametrized at initial time by the root mean square velocity $u_{rms} = \sqrt{\overline{u_i u_i}}$, and the degree of initial compressibility, $\chi = \overline{(u^C)^2}/\overline{u^2}$. The superscript C refers to the irrotational component of the velocity obtained from the (unique) Helmholtz decomposition of u_i. The solenoidal component is referred to by a superscript I. Throughout the text, quantities derived from one or the other of these two velocity fields are denoted by one or the other superscript. Further parametrizations of the initial field include the rms fluctuations of two of the three thermodynamic variables. The density and the pressure fluctuations are initialized as described in Sarkar et al. (ref. 2).

The only extensive direct numerical simulations of compressible homogeneous shear flow to date are those of Sarkar et al. (ref. 1) and Blaisdell et al. (ref. 8). They cover different parameter regimes, and obtain results that are generally consistent with each other. Sarkar et al. (ref. 1) perform two series of simulations, differentiated solely by the initial conditions. In the first set of simulations, the initial data is incompressible, i.e. $(\rho_{rms})_0 = 0$, $\chi_0 = 0$, while the initial turbulent Mach number $(M_t)_0$ is varied. In the second class, $(M_t)_0 = 0.3$ while $(\rho_{rms})_0$ and $(\chi_{rms})_0$ are varied. In all these runs, $R_{\lambda,0} = 24$, and $(SK/\epsilon^I)_0 = 7.2$ where $K = u_{rms}^2/2$ is the turbulent kinetic energy and ϵ^I is the solenoidal dissipation. The major result is that increased levels of compressibility, whether measured by an increase in M_t, or by an increase in ρ_{rms} (or χ) decreases the growth rate of kinetic energy.

Sarkar et al. (ref. 1-2) compute the probability distribution functions (pdfs) of velocity, vorticity and dilatation. As expected, the velocities (related to the larger scales) display a Gaussian distribution (characterized by a skewness $\mathcal{S}k \approx 0$, and a flatness $\mathcal{F} \approx 3$. The single exception to this was for the transverse velocity with a slightly higher flatness of 3.3. Results also indicate that the dilatation field is more intermittent than the vorticity field, and that regions of compression are more likely to occur than regions of expansion.

3. STATISTICS

We consider data from DNS on 128^3 grids. A Helmholtz decomposition is applied to the velocity field and the solenoidal and irrotational rate of strain tensors are then computed. To simplify the subsequent analysis of the irrotational rate of strain tensor, it is made traceless by subtraction of $\Theta\delta_{ij}/3$. The derivatives of the velocity are calculated with a 6^{th} order compact scheme rather than with spectral collocation to reduce internal code storage requirements. A description of the algorithm can be found in Carpenter, Gottlieb and Abarbanel (ref. 12). Given the components of the rate of strain tensor, we compute its (real) eigenvalues ($\lambda_i, i = 1, 2, 3$) which are ordered from smallest to largest:

$$\lambda_1 \le \lambda_2 \le \lambda_3. \qquad (2)$$

Along with the eigenvalues, the three principal directions are also computed and normalized to unit length. The i^{th} eigenvector is labeled $\mathbf{f}_i$ with a superscript C or I depending on the rate of strain tensor from which it is derived. When studying the orientation of these unit vectors, we are only concerned with direction, not orientation. Therefore, we take the absolute value of the direction cosines before calculating their distribution.

In what follows, we consider two types of diagnostics. The first type is a simple one-dimensional pdf, sometimes weighted by a second scalar. The second type of diagnostic is a simplified form of conditional averaging. Given two variables $\mathcal{U}_1$ and $\mathcal{U}_2$, we define $\overline{\mathcal{U}_1}(\mathcal{U}_2)$ as a function of $\mathcal{U}_2$. The notation $\overline{\mathcal{U}_1}(\mathcal{U}_2)$ should be interpreted as the average of $\mathcal{U}_1$ over the regions where $\mathcal{U}_2$ lies within a small band $\Delta\mathcal{U}_2$. For all plots, the range of the independent variable is forced to lie within ± 5 rms from its mean value. This range is then divided into 25 equal bins. These then determine the discrete pdf. All pdfs are normalized so that their integral over the independent variable is unity.

We consider results from one simulation with initial data $S = 20$, $M_{t0} = 0.2$, $\nu_0 = 1/150$, $R_{\lambda 0} = 13.5$ and incompressible initial conditions. Most of the plots are shown at $St = 9$ when $R_\lambda = 23.4$ and $M_t = 0.27$.

In this section, we study several properties of the rate of strain tensor to help identify characteristics of the flow which are solely the result of compressibility. Because the flow is anisotropic, it is useful to consider characteristics of the flow both with respect to the fixed laboratory coordinate system, and with respect to a coordinate system attached to, and rotating with an individual fluid element. Global orientation properties of S_{ij} are established by studying how the principal directions of S_{ij} are distributed in the laboratory coordinate system. (In isotropic turbulence, this diagnostic is of course devoid of meaning.) Intrinsic small scale properties, which are hopefully more independent of the large scale conditions, are characterized by considering the relative orientation of vectors such as vorticity and scalar gradients with respect to the principal directions of S_{ij}.

3.1 Intrinsic structure of S_{ij}

As mentioned earlier, in regions of high dissipation, the rate of strain tensor has a preferred shape when the flow is homogeneous and incompressible.

The solenoidal component of S_{ij} retains its preferred shape in the presence of compressibility. This is shown in Figure 1. The pdf of λ_2^I/λ_1^I peaks at a ratio of $\lambda_2^I/\lambda_1^I = -0.2$. When the pdf is weighted with respect to the solenoidal dissipation $S_{ij}^I S_{ij}^I$, the peak of the distribution is sharper, and has shifted to a ratio closer to $\lambda_2^I/\lambda_1^I = -0.25$, which corresponds to the eigenvalues λ_i^I in the ratio $(-4 : 1 : 3)$. This result matches the findings of Ashurst et al. (ref. 3) . This eigenvalue ratio is almost constant across the range of mean shear considered in the DNS.

We now ask whether $\overset{\circ}{S}{}_{ij}^C$ also has some preferred shape in regions of high irrotational strain, measured by the amplitude of $\overset{\circ}{S}{}_{ij}^C \overset{\circ}{S}{}_{ij}^C$. The results are shown in Figure 2 where the contrast between weighted and unweighted distributions is much more pronounced. The maximum value of the weighted pdf is more than twice the maximum of the unweighted distribution, thus indicating that regions of strong compressible dissipation impose a structure different than in the rest of the flow. In Figure 2, the weighted pdf peaks for an eigenvalue ratio of $(-2.2 : 1 : 1.2)$.

We have not been able to establish a definite preferred ratio of λ_2^C/λ_1^C, independent of the mean shear rate, contrary to what is found for λ_2^I/λ_1^I. Although not shown, the pdfs of λ_2^I/λ_1^I corresponding to a compressible, shockless, isotropic simulation (S=0) clearly show that its behavior with respect to weighting by $S_{ij}^I S_{ij}^I$ is identical to that found at $S = 20$ (Figure 1). However, weighting of λ_2^C/λ_1^C with respect to $\overset{\circ}{S}{}_{ij}^C \overset{\circ}{S}{}_{ij}^C$ does not significantly affect its pdf when the flow is isotropic. Furthermore, the shape of the pdf is quite similar to that of λ_2^I/λ_1^I. Thus, the mean and/or the structure of the pressure field must be the major driving force behind the preferred structure of $\overset{\circ}{S}{}_{ij}^C$. Further study is clearly required before a choice is made. Note that for isotropic turbulence, exact solutions to approximations to Euler's equations (ref. 5) have established that the preferred shape of S_{ij}^I is due either to the viscous stress terms, or to the properties of the pressure Hessian ($p_{,ij}$).

To help understand the evolution of $\overset{\circ}{S}{}_{ij}^C$, we consider time histories of the unweighted pdfs of both λ_2^I/λ_1^I and λ_2^C/λ_1^C. In Figures 3-4, we plot the pdfs of λ_2^I/λ_1^I and λ_2^C/λ_1^C at $St = 3, 5, 9, 11, 13, 21$. After an initial transient ($St < 9$), the pdf of λ_2^I/λ_1^I reaches a quasi steady-state with a peak at an eigenvalue ratio close to -0.25. The steady-state reached by λ_2^I/λ_1^I is not as definitive as that of λ_2^C/λ_1^C (Figure 7) judged on the amount of scatter between sucessive times levels. Furthermore, λ_2^C/λ_1^C reaches a quasi steady-state at an earlier time ($St = 5$) than does λ_2^I/λ_1^I. This slight difference of time scales will resurface when we consider the orientation of S_{ij} with respect to the laboratory coordinate system.

3.2 Extrinsic orientation of S_{ij}

By definition, in isotropic turbulence, the principal directions of the rate of strain tensor are randomly oriented in the laboratory coordinate system. However, homogeneous shear turbulence is anisotropic, and we therefore expect the rate of strain ellipsoid to have a preferential orientation. This is obvious, since the mean shear itself imposes a constant strain on the flow, oriented at 45° to the x and y coordinate directions.

Consider the angle θ_{ij} between eigenvector $\mathbf{f}_i$ and the unit coordinate axis vector $\mathbf{x}_i$ from which the pdfs of $|\cos\theta_{ij}^I|$ and $|\cos\theta_{ij}^C|$ are computed. The cosine of θ_{ij} is chosen rather than θ_{ij} so that the probability density function of a Gaussian distribution curve is flat. Because the independent variable is the cosine of an angle, one must be careful when interpreting the magnitude of the pdf maxima. Strong peaks of the $|\cos\theta|$ pdf do not necessarily imply a strong peak of the pdf of θ. The angle at which this peak occurs must also be taken into account.

Figures 5a-c show the pdfs of $|\cos\theta_{ij}^I|$ weighted by $S_{ij}^I S_{ij}^I$ at $St = 9$. In this and the next figure, each subplot corresponds to one eigenfunction (oriented with respect to each coordinate axis). It is evident that the directions of maximum extensional and maximum compressive strain have qualitatively similar distributions. These two eigenvectors have alignments which are most often at angles near 45° to the x and y axes. However, the 45° orientation with respect to the streamwise direction is strongest. This suggests that the mean strain plays the role of a polarizer which forces these two directions to align with itself. The pdf of $|\cos\theta_{2j}^I|$ shows a tendency to align at 53° to each of the three coordinate axes. On the other hand, the weighted pdf of $|\cos\theta_{2i}^I|$ indicates that $\mathbf{f}_2^I$ has equal probability of lying at 53° to each of the 3 axes. Note that this alignment property of $\mathbf{f}_2^I$ is much weaker than that of $\mathbf{f}_1^I$ and $\mathbf{f}_3^I$ as determined by the ratio of maximum to minimum probability.

The pdfs of $|\cos\theta_{ij}^C|$ are markedly different than those of $|\cos\theta_{ij}^I|$ (Figures 6a-c). Once again, the most compressive and the most expansive principal directions have almost identical pdfs. The strongest trend is that $\mathbf{f}_1^C$ and $\mathbf{f}_3^C$ align with the y axis. The intermediate irrotational eigenvector behaves differently. This eigenvector orients itself primarily along the x and z axes. It is also evident, that $\mathbf{f}_2^C$ has a strong propensity to remain normal to the y axis. The analysis has been repeated using data from several turbulent simulations at $M_t = 0.2, 0.4$, shear levels of $S = 10, 20, 30$, and most of the general conclusions remain unchanged. The only noticeable effect of weighting the pdfs of $|\cos\theta_{ij}^C|$ with $\overset{\circ}{S}{}_{ij}^C \overset{\circ}{S}{}_{ij}^C$ is to sharpen the maxima. The qualitative features of the distributions are not affected by the weighting. In summary, the most extensional and most compressive principal directions of S_{ij}^I tend to be at 45° to the coordinate directions, while the principal directions of $\overset{\circ}{S}{}_{ij}^C$ tend to align with the coordinate directions.

Why are the properties of S_{ij}^C so different from those of S_{ij}^I? Although the dynamics of S_{ij}^I are somewhat understood, this is not the case for $\overset{\circ}{S}{}_{ij}^C$. When the flow is incompressible, the vortex stretching term in the enstrophy transport equation is of the form $\omega_i S_{ij}^I \omega_j$. The vorticity stretching is initially in the direction of the maximum mean strain, i.e. at 45° to the streamwise direction. This in turn initially aligns the principal directions of S_{ij}^I so that $\mathbf{f}_3^I$ is aligned with $\boldsymbol{\omega}$. However, the change in u_i^I induces a change in the vorticity, whose interaction with S_{ij}^I serves to reorient S_{ij}^I relative to $\boldsymbol{\omega}$. This process is described by the analytic formulations of Cantwell (ref. 5), Majda (ref. 6), and Vieillefosse (ref. 7), which predict that the vorticity will eventually align itself with $\mathbf{f}_2^I$. The evolution of $\overset{\circ}{S}{}_{ij}^C$, on the other hand, is driven by one of two major effects:

1. The structure of S_{ij}^I acts as a source term or as spatially dependent coefficients in linear equations for $\overset{\circ}{S}{}_{ij}^C$, or
2. The mean shear is primarily responsible for the structure of $\overset{\circ}{S}{}_{ij}^C$.

To investigate which of the two scenarios is the most likely candidate, we plot the distribution of $|\cos\theta_{11}|$ at $St = 3, 5, 9, 11, 13, 21$ in Figure 7. We find that after an initial transient, the pdfs of $|\cos\theta_{1j}^I|$ tend towards a stationary shape. As expected, these distributions are mirrored by those of $|\cos\theta_{3j}^I|$. However, we find that the orientation of $\mathbf{f}_2^I$ with respect to $\mathbf{x}$ was still changing when the numerical simulation ended at $St = 21$. The next figure shows the equivalent distributions for $\mathbf{f}_i^C$ (Figures 8a-b). The existence of a stationary structure of $\overset{\circ}{S}{}_{ij}^C$ is much more definite, and occurs earlier than the equilibration of $\mathbf{f}_i^I$. Therefore, it is likely that the mean shear rather than S_{ij}^I determines the compressible rate of strain tensor to zeroth order.

3.3 Orientation of vorticity

As expected from previous investigations (ref. 3-4) the vorticity orients itself along $\mathbf{f}_2^I$ in regions of high solenoidal dissipation. If the angle between $\boldsymbol{\omega}$ and $\mathbf{f}_i^I$ is denoted by $\theta_i^{I\omega}$, this trend is clearly seen by conditioning $\cos^2\theta_i^{I\omega}$ on $S_{ij}^I S_{ij}^I$. In regions of high dissipation, $\boldsymbol{\omega}$ aligns itself strongly in the direction of $\mathbf{f}_2$, and normal to both $\mathbf{f}_1$, and $\mathbf{f}_3$. This result is consistent with results of Ashurst et al. (ref. 3) and She et al. (ref. 4).

The compressible dissipation has no influence on the relative orientation of $\boldsymbol{\omega}$ and $\mathbf{f}_i^I$. Indeed, the pdfs of $|\cos\theta_i^{I\omega}|$ are flat when conditioned by Θ, as seen in Figure 9. This is consistent with a lack of correlation between the dilatation and the vorticity. We have also found that the average alignment of the vorticity vector with $\mathbf{f}_i^I$ is essentially independent of either Θ or $\overset{\circ}{S}{}_{ij}^C \overset{\circ}{S}{}_{ij}^C$. More details are available in Erlebacher and Sarkar (ref. 10).

3.4 Orientation of the pressure gradient

Also of interest are the properties of the pressure gradient. It is clear that the properties of the pressure gradient and pressure Hessian must be understood before there is any hope of correctly modeling the subtle relationships between dilatation and vorticity. As shown in Erlebacher et al. (ref. 12), the perturbation pressure (in isotropic turbulent flows) can be thought of as a combination of an "solenoidal" (incompressible)component p^I which together with $\mathbf{u}^I$ satisfies the incompressible Navier-Stokes equations, while the difference between the pressure and p^I, i.e. the compressible pressure p^C, is representative of compressible effects. An alternate decomposition of the pressure field is considered in Sarkar (ref. 14), where the incompressible pressure retains all the effects which involve the mean density gradients. This generalization of the pressure decomposition is found to be useful for the analysis of inhomogeneous turbulent flows. Both decompositions have similar characteristics regarding the properties of the pressure components. The incompressible pressure is basically elliptic, i.e. information in one part of the flow is immediately transmitted throughout the flowfield. In contrast, the compressible pressure, p^C has a hyperbolic character, i.e. the pressure and dilatational fields are responsible for the sound waves traveling throughout the flow.

The simultaneous presence of both compressible and incompressible characteristics in the pressure field can be partially extracted from the data by considering the orientation of the pressure gradient with respect to $\mathbf{f}_i^I$ and $\mathbf{f}_i^C$. In Figure 10, we plot the pdf of the pressure gradient alignment with respect to $\mathbf{f}_i^I$. We observe that there is a preferred orientation of ∇p at about 40° to both $\mathbf{f}_1^I$ and $\mathbf{f}_3^I$. In fact, the pdfs of $|\cos\theta_1^{Ip}|$ and $|\cos\theta_3^{Ip}|$ are almost identical, suggesting that with regard to the pressure gradient, the most extensive and most compressive principal directions of the solenoidal rate of strain tensor play similar roles. Ashurst et al. (ref. 3) find that in incompressible homogeneous shear flow turbulence, ∇p tends aligns itself in the direction of $\mathbf{f}_1^I$. This contrasts with our findings which indicate that ∇p is inclined similarly with respect to $\mathbf{f}_1^I$ and $\mathbf{f}_3^I$.

Figure 11 shows the pdfs of $|\cos\theta_i^{Cp}|$. The pressure gradient orients itself along $\mathbf{f}_1^C$ or $\mathbf{f}_3^C$ with equal probability. In addition, there is a strong influence of the compressible dissipation on the alignment of ∇p with $\mathbf{f}_i^C$ (Figures 12). The dilatation Θ was used to condition the distributions of $\overline{\cos^2\theta_i^{Cp}}$. In regions of high $(\overset{\circ}{S}{}_{ij}^{C}\ \overset{\circ}{S}{}_{ij}^{C})$, ∇p is strongly aligned with $\mathbf{f}_1$, while it has quite different orientations in regions of strong expansion and strong compression. When $\Theta > 0$, the alignment is towards $\mathbf{f}_3^C$, while it is in the direction of $\mathbf{f}_1^C$ when $\Theta < 0$. As expected, the solenoidal component of pressure, p^I, responds differently to the dilatation. The orientation of ∇p^I with respect to $\mathbf{f}_i^I$ shows a much weaker dependence on Θ than does ∇p. The gradient of p^I has different mean orientation in regions of compression and expansion. When $\Theta < 0$, ∇p^I is perpendicular to $\mathbf{f}_3^C$, while it is somewhat aligned with $\mathbf{f}_2^C$ in regions of flow expansion (ref. 13).

4. CONCLUSIONS

Compressible turbulence is a difficult subject due to the number of parameters which control the flow. The purpose of this work was to determine the leading order effects of compressibility on the small spatial scales in homogeneous compressible shear flow turbulence.

We find that the qualitative statistical properties of the vorticity and solenoidal rate of strain tensor are affected very little by compressibility effects. This is due in part to the lack of correlation between the dilatation and the vorticity fields. The only major compressibility effect is on the orientation of the pressure gradient In regions where the flow is compressed, ∇p aligns with $\mathbf{f}_1^C$, while it is oriented along $\mathbf{f}_3^C$ in expansion regions. This effect is also seen in compressible isotropic turbulence, but is much weaker.

In shear flow, the eigenvectors of S_{ij}^I and $\overset{\circ}{S}{}_{ij}^{C}$ have a preferred orientation with respect to a frame of reference fixed in the laboratory. While the principal directions of S_{ij}^I tend to orient themselves at angles of $30° - 50°$ to each of the three coordinate directions (with the strongest alignments along the x axis), those of $\overset{\circ}{S}{}_{ij}^{C}$ are primarily aligned along the axes of laboratory coordinate system. Both tensors have an orientation which tends to a stationary distribution for large time. However, the irrotational rate of strain tensor "equilibrates" on a faster time scale, which suggests that it is the mean shear that is responsible for its properties, not the fluctuating rate of strain S_{ij}^I.

While there are an infinite number of ways to combine variables, an equal number of ways to display them, not to mention interpret what is displayed, we feel that our initial attempt has succeeded in opening a veritable Pandora's box of new questions to be answered. For example, are regions of strong vorticity and strong dilatation correlated at a distance. Perhaps, regions of strong vorticity, together with the mean shear, induce the irrotational velocity field to acquire a particular structure. What are the roles played by p^I and p^C in the dynamics of these structures. Can the structure of the irrotational rate of strain tensor be explained by linear theory, or are nonlinear effects the primary driving force. These are but some of the questions which come to mind.

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