Non Linear Evolution of a Second Mode Wave in Supersonic Boundary Layers

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Abstract

The non-linear time evolution of a second mode instability in a Mach 4.5 wall-bounded flow is computed by solving the full compressible, time-dependent Navier-Stokes equations. High accuracy is achieved by using a Fourier-Chebyshev collocation algorithm. Primarily inviscid in nature, second modes are characterized by high frequency and high growth rates compared to first modes. Time evolution of growth rate as a function of distance from the plate suggests this problem is amenable to the Stuart-Watson perturbation theory as generalized by Herbert.

1 Introduction

Recent advances in supersonic and hypersonic aerospace technology have led to a renewed interest in the stability and transition to turbulence of high speed flows. The last 30 years have intermittently witnessed some vigorous attempts to understand some of the fundamental routes to transition for incompressible flows. While a fairly comprehensive picture of the initial stages leading to the breakdown of an incompressible laminar boundary layer has emerged (mostly under controlled conditions) [9], the non-linear effects responsible for transition at high speeds are still very much a mystery. However, the current nonlinear incompressible theories, numerical simulations and experiments will, hopefully, serve as a guide in gaining a better understanding of the mechanisms present in the supersonic and hypersonic regimes.

Lees and Lin [13] were the first to study the linear stability of inviscid compressible flows in the 1940's. Their study was restricted to low Mach numbers and the conclusions drawn only apply to the first unstable mode. Mack unraveled some of the intricacies among the many possible modes present when the full stability equations are considered (including viscous and conductivity terms). From 1960 to the present time, Mack has devoted his energies to a thorough numerical investigation of the linear stability characteristics of inviscid compressible flows at both subsonic and supersonic Mach numbers. An extensive review of compressible linear theory appears in his 1984 AGARD report [16]. His research led to the discovery of the higher unstable modes for Mach numbers above 2.2. The importance of the higher modes stems from their high spatial and temporal amplification rates which make them susceptible to noise from the free-stream or from windtunnel walls. One of the main differences between first and second mode waves is their response to cooling. Wall cooling stabilizes first mode perturbation waves, while second mode perturbations are destabilized. At Mach numbers greater than 7, cone experiments have confirmed that the second mode is primarily responsible for the observed instabilities [22, 3]. Therefore, new techniques must be developed to control the growth rates in these regimes.

The existence of an inflectional point in the mean boundary layer profile, a necessary and sufficient condition to insure the instability of inviscid incompressible flows, remains valid at all Mach numbers if the relative Mach number is subsonic everywhere [1]. For compressible flows, the inflection point is replaced by the generalized inflection point which takes into account the mean density variation across the boundary-layer. However, there are also non-inflectional waves which depend on the existence of local pockets of supersonic relative Mach number in the boundary-layer. These waves can be unstable and can exist at all supersonic free-stream Mach numbers. Multiple modes also exist for these non-inflectional waves. The above linear stability characteristics of compressible flows are fully described in Mack's review articles [16, 17].

To date, there are only a handful of experiments that have studied the detailed linear stability characteristics of flat plates [12] and cones [10]. Laufer and Vrebalovich studied the linear stability of a Mach 2.2 flow on a flat plate, and theirs is perhaps the most complete (if not the only) flat plate experiment to date at supersonic velocities. Not only are hot wire measurements difficult to perform and to interpret correctly [19], but the stability characteristics of the boundary layer are affected by free-stream disturbances emanating from the wind tunnel walls. Since the disturbances scale as M_∞^2, it becomes increasingly difficult to obtain good, quantitative data at the higher speeds. The quiet Mach 6 tunnel, currently under construction at NASA Langley Research Center [1] should alleviate some of these concerns.

Among the few numerical codes geared towards the analysis of compressible stability, the best known are perhaps those of Mack [15] and Malik [18]. The former calculates spatial and temporal amplification rates, while the latter is strictly restricted to temporal analysis; a Gaster transformation is then applied to calculate the temporal and spatial growth rates. A new spectral stability code has also been developed at NASA Langley [14]. Until the work by Erlebacher and Hussaini [6, 7], there have been no non-linear simulations. Erlebacher and Hussaini studied the evolution of two 3-D waves superimposed on a 2-D wave. These were first mode waves. They numerically demonstrated the existence of a secondary instability at Mach 4.5. The current incompressible weakly non-linear theory of Herbert [8] has been extended to include compressible flow by El-Hady [4] and Nayfeh [20]. Nayfeh treats both subsonic and supersonic regimes, while El-Hady only presents results up to Mach 1.2. Both studies are based on a quasi-parallel theory and treat subharmonic disturbances.

In this paper, a recent code developed to solve numerically the non-stationary, three-dimensional compressible Navier-Stokes equations applied to wall-bounded flows, is used to study the nonlinear evolution of one two-dimensional second mode at Mach 4.5. A possible application of perturbation analysis to this problem is also discussed.

[1]The relative Mach number measures the difference between the mean velocity in the direction of the wave vector and the phase velocity with respect to the sound speed.

2 Equations

The compressible Navier-Stokes equations for the stability of parallel flows are, in dimensionless form, (Erlebacher and Hussaini [6])

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0 \tag{1}$$

$$\frac{\partial(\rho\vec{v})}{\partial t} + \nabla\cdot(\rho\vec{v}\vec{v}) = -\nabla p + \frac{1}{Re}\nabla\cdot\vec{\vec{\tau}} \tag{2}$$

$$\frac{\partial p}{\partial t} + \vec{v}\cdot\nabla p + \gamma p\nabla\cdot\vec{v} = \frac{1}{RePrM_\infty^2}\nabla\cdot(\mu\nabla T) + (\gamma-1)\Phi \tag{3}$$

where

$$\vec{\vec{\tau}} = \mu(\nabla\vec{v} + \nabla\vec{v}^T) - \frac{2}{3}\mu(\nabla\cdot\vec{v})\,\vec{\vec{I}} \tag{4}$$

is the viscous stress and

$$\Phi = \frac{1}{2}(\nabla\vec{v} + \nabla\vec{v}^T) : \vec{\vec{\tau}} \tag{5}$$

is the viscous dissipation function. The equation of state is

$$\gamma M_\infty^2 p = \rho T, \tag{6}$$

with $\gamma = C_p/C_v$, the ratio of specific heats. At high Mach numbers, temperature variations across the boundary layer become important and the temperature dependence of viscosity must be taken into account. The Prandtl number is assumed equal to 0.7, and Sutherland's law is prescribed for viscosity. Lengths are non-dimensionalized with respect to the displacement thickness δ^*. Velocity, temperature, viscosity, and density are normalized with respect to free-stream values. Pressure is normalized with respect to the total head $\rho_\infty U_\infty^2$.

In incompressible boundary-layer flows, it has been observed that the evolution to laminar breakdown for ribbon-excited Tollmien-Schlichting (TS) waves occupies a space of several TS wavelengths [11]. Apart from numerical considerations, a direct simulation of a 3-D boundary layer flow far into the non-linear regime is not currently practical, even on today's most powerful computers. The situation is worse for compressible flows where the instability mechanisms are attenuated. One possible approximation is to neglect the growth of the mean boundary-layer. The perturbation flow is then periodic and the expense of the simulation is greatly decreased. This has been successful in incompressible theories [8] and numerical simulations [24]. One of the major reasons for this success is that the actual transition process occurs within a very short streamwise extent. At high Mach numbers, the growth rates are attenuated, the transition region is longer, and it is not clear whether this approximation is valid. Nonetheless, in an initial attempt to gain insight into the physics of second mode wave evolution, a temporal simulation is considered. The hope is that the observed features are at least qualitatively correct. The computational domain is restricted to one period in the streamwise distance. Periodicity is also assumed in the spanwise direction. A properly posed initial-value problem is set up, and qualitative comparisons are made with experimental data by relating time (in the simulations) to the streamwise direction (x) in the experiments. In order that the mean parallel boundary layer flow, $(\rho_m(y), U_m(y), T_m(y))$, be a stationary solution to the Navier-Stokes equations, source terms must be added to equations (2)-(3). The x momentum equation must be supplemented by the term

$$\mathcal{F}_{\rho u} = \frac{1}{Re} \frac{\partial}{\partial y} \left(\mu \frac{\partial U_m}{\partial y} \right) \tag{7}$$

while the forcing term

$$\mathcal{F}_p = \frac{1}{RePrM_\infty^2} \frac{\partial}{\partial y} \left(\mu \frac{\partial T_m}{\partial y} \right) + \frac{1}{Re} \left[\left(\frac{\partial U_m^2}{\partial y} \right)^2 \right] \tag{8}$$

must be added to the energy equation (3). In obtaining these relations, the mean pressure is kept constant across the boundary layer. The mean velocity has only a streamwise component.

3 Algorithm

The algorithm described below, has been implemented in 3-D although only 2-D results are presented in this paper. Three-dimensional simulations of first mode wave interactions have been detailed elsewhere [7]. The system of equations (1)-(3) is solved in conservative form under the parallel flow assumption. Periodic boundary conditions in the streamwise and spanwise directions permit a Fourier representation of the primitive variables $\vec{v}$, p and ρ. For example,

$$\vec{v}(x, y, z, t) = \sum_{k=-N_x/2+1}^{N_x/2} \sum_{l=-N_z/2+1}^{N_z/2} \hat{\vec{v}}(y, t)\, e^{\imath(\alpha k x + \beta l z)} \tag{9}$$

where N_x, N_z are the number of collocation points in the streamwise and spanwise directions, and α, β are the wavenumbers in these same two directions. The periods of the physical domain in the streamwise and spanwise directions are respectively $2\pi/\alpha$ and $2\pi/\beta$. In the absence of discontinuities in the solution, spectral collocation methods offer a high degree of accuracy and minimum dispersion and dissipation errors. Periodicity in the two horizontal directions allows the use of Fourier collocation methods. In the direction normal to the plate, Chebyshev collocation is the natural choice. The physical domain $y \in [0, y_{max}]$ in the direction normal to the plate is mapped onto the computational domain $\eta \in [-1, 1]$ through a convolution of two mappings. First, a hyperbolic tangent transformation takes η to an intermediate variable ψ according to

$$\psi + t_\epsilon \tanh\left(\frac{\psi - \psi_0}{\Delta\psi} \right) = \frac{\eta - \eta_0}{\Delta\eta}. \tag{10}$$

In this equation, the hyperbolic tangent concentrates nodes about ψ_0, and $\Delta\psi$ measures the width of the concentration region. t_ϵ measures the degree of influence of the hyperbolic tangent term. If $t_\epsilon = 0$, there is no redistribution of nodes at this level. An algebraic stretching then maps ψ onto the physical variable $y \in [0, y_{max}]$. This mapping takes the form

$$y = \frac{y_{1/2} y_{max} (1 + \psi)}{y_{max} - \psi(y_{max} - 2y_{1/2})} \tag{11}$$

One half the normal nodes are located between $y = 0$ and $y = y_{1/2}$. With both mappings in hand, $\eta_0, \Delta\eta$ ψ_0 and $\Delta\psi$ are readily determined. The user specifies the following: 1) t_ϵ, 2) y_0, the physical location about which the concentration occurs, 3) Δy_0, the concentration width in physical space, 4) y_{max}, the maximum vertical extent of the physical domain, and 5) $y_{1/2}$. Admittedly, the selection is rather tedious, but it is only executed once to optimize the distribution of nodes relative to the initial perturbation wave profiles. The distribution of nodes is fixed during the simulation. Δy_0 and $\Delta\psi$ are related via

$$\Delta\psi = \left.\frac{d\psi}{dy}\right|_{y=y_0} \Delta y_0 \tag{12}$$

while ψ_0 is computed from the algebraic mapping evaluated at y_0. The two remaining unknowns, η_0 and $\Delta\eta$ follow from the conditions

$$\begin{aligned} \eta(\psi)|_{\psi=-1} &= -1 \\ \eta(\psi)|_{\psi=+1} &= +1 \end{aligned} \tag{13}$$

which leads to

$$\begin{aligned} \Delta\eta &= \frac{2}{2+t_\epsilon(T_- + T_+)} \\ \eta_0 &= \frac{t_\epsilon(T_+ - T_-)}{2+t_\epsilon(T_- + T_+)} \end{aligned} \tag{14}$$

where

$$T_\pm = \tanh\left(\frac{1\pm\psi_0}{\Delta\psi}\right) \tag{15}$$

All runs were performed with $y_{max} = 15$, $y_{1/2} = 1$, $t_\epsilon = 0.8$, $y_0 = 1.2$ and $\Delta y_0 = 0.4$.

4 Initial and Boundary Conditions

In this paper, only the evolution of 2 dimensional waves are considered. Therefore, the initial conditions consist of a 2-D waves superimposed on a precalculated mean flow. Mean flow profiles are generated from the spectral solution to the similar compressible boundary-layer equations with zero pressure gradient and heat transfer at the wall [6], while the perturbation wave is a two-dimensional eigenfunction of the linearized compressible Navier-Stokes equations. The stability of flows is studied at a prescribed supercritical Reynolds number $Re(x_0)$. After freezing the streamwise dependence of the mean flow profiles at $x = x_0$, they are extended over the entire plate. Thus, the displacement thickness is constant, and the mean flow is a function only of the coordinate normal to the plate.

The initial condition for u(x,y,z) is therefore

$$u(x,y,z) = u_m(y) + \epsilon_{2D} u_{2D}(y)\cos(\theta_{2D}(y) + \alpha x) \tag{16}$$

with similar relations for the remaining 4 variables (v, w, p, ρ). The 2-D wave is an eigenfunction to the linearized Navier-Stokes equations and is calculated with the spectral linear compressible stability code SPECLS [14]. The angle θ_{2D} is the phase angle of the complex TS wave. The amplitudes u_{2D} is normalized to unity. Therefore, ϵ_{2D} is the amplitude of the two-dimensional perturbation wave with respect to the free-stream velocity. At high Mach numbers, ϵ_{2D} cannot exceed a couple of percent lest the density and temperature maxima (which scale as M_∞^2) become excessive.

Under the assumption of parallel flow, all the variables are periodic in the streamwise and spanwise directions. No-slip conditions are applied to the velocities at the wall which is adiabatic. In the far-field, the variables are either frozen at their initial values (which are vanishingly small) or calculated from zero stress conditions. These approaches both preclude the study of boundary-layer receptivity to incoming sound waves. The frozen boundary conditions also prevent waves from escaping into the free-stream. When the far-field boundary is sufficiently removed from the wall, stability mechanisms which do not involve far-field interactions can still be successfully analyzed.

Zero normal stress boundary conditions lead to an exponential decay of the solution in the free-stream, but not at the proper rate. However, it is plausible that the feedback from the free-stream into the interior of the domain is not as severe as for the frozen far-field conditions. These boundary conditions are derived from a variational formulation of the Navier-Stokes equations. The velocities and temperature are assumed to possess C^1 continuity, while the pressure is only required to be C^0. The natural boundary conditions of this formulation in the far-field are then [21]

$$w_x + u_z = 0 \tag{17}$$

$$w_y + v_z = 0 \tag{18}$$

$$-pRe + 2w_z + \lambda(u_x + v_y + w_z) = 0 \tag{19}$$

$$pw = \frac{1}{\gamma PrReM_\infty^2}(\mu\frac{\partial T}{\partial z}) \tag{20}$$

If $(\tilde{u}, \tilde{v}, \tilde{w}, \tilde{p}, \tilde{T})$ are deviations of (u, v, w, p, T) from free-stream values, the resulting linearized equations become

$$\begin{aligned}
\tilde{u}_z - \iota\alpha\tilde{w} &= 0 && (21)\\
\tilde{v}_z - \iota\beta\tilde{w} &= 0 && (22)\\
-\tilde{p}Re + (2+\lambda)w_z + \lambda(\iota\alpha u + \iota\beta v) &= 0 && (23)\\
\tilde{w} - \frac{1}{\gamma PrReM_\infty^2}T_z &= 0 && (24)\\
\gamma M_\infty^2\tilde{p}_z - \tilde{\rho}_z + \tilde{T}_z &= 0. && (25)
\end{aligned}$$

Derivatives in the vertical direction are evaluated using matrix multiplication. In this way, the unknowns (primitive variables at the last node) can be solved for simultaneously. The alternative are FFT's, which lead to an iterative approach. The interior nodes have already been computed from the explicit time advancement. Therefore, the derivative of u at the far field (node N) is represented by

$$\left.\frac{du}{dy}\right|_{y=y_1} = \sum_{i=1}^{N} D_{ij}u_j \tag{26}$$

where D_{ij} is the collocation derivative matrix [2] and where the only unknown is u_N. The system of equations (21)-(25) reduces to a linear system for the unknowns (u_N, v_N, w_N, T_N). Density has already been calculated through the continuity equation imposed at the outer boundary. A disadvantage comes from the higher truncation errors associated with matrix multiplication when compared to FFT's for large number of collocation points. There are at least two remedies. First, an implicit, rather than an explicit time advancement scheme could incorporate these asymptotic boundary-conditions into the iterative inversion scheme. This could be done solely with FFT's. The second approach is to adopt a multidomain decomposition in the direction normal to the plate. An explicit scheme still requires matrix multiplication for derivative evaluation, but the number of nodes in each subdomain is small, and truncation errors will not affect the numerical results as strongly as in the single domain case. As the Mach number increases, the structure of the perturbation waves is mostly due to the density in a thin region centered about the critical layer whose distance away from the wall is approximately constant in terms of δ^*. Boundary conditions at the interface between two domains of a multidomain methods are continuity of all primitive variables (in the absence of shocks) and normal stress across the interface. A condition similar to zero normal stress can be derived from the energy equation. For best results, the interface conditions should be derived from the actual set of equations solved in the simulation. As yet, the multi-domain approach has not been implemented into the code. A more detailed description of this approach will be given elsewhere. All results presented below are based on the zero stress far-field conditions. inputresults

5 Conclusions

A three-dimensional, fully spectral code developed for the direct simulation of wall-bounded compressible flows has been applied to the nonlinear evolution of a single unstable second mode wave. After a slight initial growth, the growth rate of this fundamental wave decreases in time. This suggests that the second mode is evolving towards a saturated state. Perturbation methods are sometimes well-suited for the study of such states. To ascertain whether such an approach is called for, all streamwise modes above the fundamental were continuously set to zero at every time step. This led to a time evolution of the fundamental waves almost identical with the case where all modes were allowed to interact with each other. This agreement was also found for the structure of the time-dependent instantaneous growth rate of u, even in the critical layer region. The conclusion is that the cubic interaction terms that only involve the fundamental waves are probably responsible for the nonlinearities in the critical layer, and that the density fluctuations play a fundamental role.

A final note. The initial perturbation wave was chosen to maximize its instability. Nayfeh [20] found that stable 2-D second mode waves were susceptible to strong secondary instabilities. It is therefore still not clear which are the actual waves that will be most responsible for the final transition towards turbulence. These issues will be addressed in the future.

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7 Figure Captions

Fig. 1: Amplitude and phase plots of second and first mode eigenfunctions at $M_\infty = 4.5, T_\infty = 110$, $\beta = 0$. First mode: $\alpha = 0.6$, $Re = 10000$. Second mode: $\alpha = 2.25$, $Re = 8000$.
(1a) Second mode amplitudes of u, v
(1b) First mode amplitudes of u, v
(1c) Second mode amplitudes of T
(1d) First mode amplitudes of T
(1e) Second mode phase plot of u, v
(1f) First mode phase plot of u, v, T

Fig. 2: Growth rate and frequency at $t = 3$ periods. Initial u amplitude of 0.1%.
(2a) Growth rate
(2b) Frequency

Fig. 3: Growth rate and frequency at t=0,5,10,15 and 20 periods. Initial u amplitude of 2%.
(3a) Growth rate
(3b) Frequency

Fig. 4 Growth rates of fundamental streamwise velocity due to linear and cubic terms in the transformed streamwise momentum equation (excluding viscous terms). Times t=0,5,10,15,20 periods.
(4a) Linear terms
(4b) Cubic terms

Fig. 5: Amplitude of fundamental modes at t=0,5,10,15,20 periods.
(5a) $\hat{u}_1$
(5b) $\hat{v}_1$
(5c) $\hat{\rho}_1$

Fig. 6: Amplitude of $\hat{v}_0$ (vertical mean flow) at t=0,5,10,15,20 periods.

Fig. 7: Amplitude of first harmonic modes at t=0,5,10,15,20 periods.
(7a) $\hat{u}_2$
(7b) $\hat{v}_2$
(7c) $\hat{\rho}_2$

Fig. 8: Vorticity production terms at t=0,5,10,15,20 periods.
(8a) Baroclinic torque
(8b) Dilatation term
(8c) Convection+barolinic+dilatation

Fig. 9: Perturbation spanwise vorticity at t=20 periods.

References

[1] Beckwith, I.E.; Chen, F.J; and Malik, M.R., Jr.: Design and Fabrication Requirements for Low-Noise Supersonic/Hypersonic Wind Tunnels. AIAA Paper N. 88-0143, presented at the AIAA 26th Aerospace Sciences Meeting, Reno, Nevada, January 11-14, 1988.

[2] Canuto, C.; Hussaini, M. Y.; Quarteroni, A.; and Zang, T. A.: *Spectral Methods in Fluid Dynamics*, Springer-Verlag, Berlin, 1987.

[3] Demetriades, A.: Laminar Boundary Layer Stability Measurements at Mach 7 including Wall Temperature Effects. AFOSR 77-1311, 63 pp., 1977.

[4] El-Hady, N.M.: Secondary Three-Dimensional Instability in Compressible Boundary Layers. Proceedings of the Transonic Symposium, Theory, Application and Experiment, 1988, to appear.

[5] Erlebacher, G: Transition Phenomena over a Flat Plate for Compressible Flows. *Tenth International Conference on Numerical Methods in Fluid Dynamics*, Springer Verlag, F.G. Zhuang and Y.L. Zhu (Eds.), Berlin, 1986.

[6] Erlebacher, G.; and Hussaini, M. Y.: Incipient Transition Phenomena in Compressible Flows over a Flat Plate, ICASE Report 86-39, 1986.

[7] Erlebacher, G. and Hussaini, M.Y.: Stability and Transition in Supersonic Boundary Layers. AIAA paper AIAA-87-1416, presented at the 19th Fluid Dynamics, Plasma Dynamics and Lasers Conference in Hawaii, 1987.

[8] Herbert, T.: Floquet analysis of secondary instability in shear flows, *Stability of Time Dependent and Spatially Varying Flows*, D. L. Dwoyer and M. Y. Hussaini (Eds.), pp. 43-57, Springer Verlag, New York, 1987.

[9] Hussaini, M. Y.: Stability, Transition and Turbulence. *Supercomputing in Aerospace*, NASA Conference Publ. 2454, pp. 211-220, 1987.

[10] Kendall, J. M.: Wind Tunnel Experiments Relating to Supersonic and Hypersonic Boundary Layer Transition. *AIAA Journal*, Vol. 13, pp. 290-299, 1975.

[11] Kovasznay, L.S., Komoda, H. and Vasudeva, B.R., *Proc. of the 1962 Heat Transfer and Fluid Mech.* Inst., pp. 1-26, Stanford University Press, 1962.

[12] Laufer, J. and Vrebalovich, T.: Stability and Transition of a Supersonic Laminar Boundary Layer on an Insulated Flat Plate. *J. Fluid Mech.*, Vol. 9 pp. 257-299, 1960.

[13] Lees, L. and Lin, C. C.: Investigation of the Stability of the Laminar Boundary Layer in a Compressible Fluid. NACA Tech. Note No. 1115, 1946.

[14] Macaraeg, M. G., Streett, C.L. , and Hussaini, M.Y.: A Spectral Multi-Domain Technique Applied to High-Speed Chemically Reacting Flows. Proceedings of the 2nd International Symposium on Domain Decomposition Methods, Los Angeles, CA, Jan. 14-16,1988.

[15] Mack, L. M.: Computation of the Stability of the Laminar Boundary Layer. *Methods in Computational Physics*, Eds. B. Alder, S. Fernbach and M. Rotenberg, Vol. 4, pp. 247-299, Academic, N.Y., 1965.

[16] Mack, L. M.: Boundary-Layer Linear Stability Theory. AGARD, Report 709, 1984.

[17] Mack, L. M.: Review of Linear Compressible Stability Theory. *Stability for Time Dependent and Spatially Varying Flows*, D.L. Dwoyer and M.Y. Hussaini (Eds.), Springer Verlag, New York, pp. 164-187, 1985.

[18] Malik, M.R.: Finite-Difference Solution of the Compressible Stability Eigenvalue Problem. NASA Contractor Report 3584, 1982.

[19] Morkovin, M. V.: Fluctuations and Hot-Wire Anemometry in Compressible Flows. AGARDograph 24, 102 pp., 1957.

[20] Nayfeh, A.H.: Stability of Compressible Boundary Layers. Proceedings of the Transonic Symposium, Theory, Application and Experiment, NASA CP 3020, Vol. I, 1989.

[21] Quateroni, A.: private communication.

[22] Stetson, K. F.; Thompson, E. R.; and Donaldson, J. C.: Laminar Boundary Layer Stability Experiments. AIAA paper AIAA-83-1761, 1983.

[23] Tennekes, H. and Lumley, J.L: *A First Course in Turbulence*. Eighth printing, MIT Press, 1982.

[24] Wray, A. and Hussaini, M.Y.: Numerical experiments in Boundary-layer Stability. *Proc. R. Soc. Lond.*, Vol. 392, pp. 373–389, 1984.

[25] Zang, T.A. and Hussaini, M.Y.: Numerical Experiments on the Stability of Controlled Shear Flows. AIAA paper 85-1698, 1985.