DEVELOPMENT OF WAVELET ANALYSIS TOOLS FOR TURBULENCE

A. Bertelrud*, G. Erlebacher**, Ph. Dussouillez***, M.P. Liandrat[1]
J. Liandrat***, F.Moret Bailly***, Ph. Tchamitchian****
* Experimental Flow Physic Branch, NASA LaRc
Hampton, VA 23665, USA
** ICASE, MS 132C, NASA LaRc, Hampton, VA 23665, USA
*** IMST, 12 avenue général Leclerc 13003 Marseille, France
**** CPT-CNRS Luminy, case 907-13288 Marseille, France

ABSTRACT

This paper is devoted to the presentation of the general framework and the initial results of a joint effort to derive novel research tools and easy to use software to analyze and model turbulence and transition.

After a brief review of the issues and a summary of some basic properties of wavelets, we present our preliminary results. Both the technical aspects of the implementation and the physical conclusions reached at this time are discussed.

Current developments are summarized in the last section.

[1]Research was supported in part by the National Aeronautics and Space Administration under NASA Contract No. NAS1-18605 while the author was in residence at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA 23665

1. INTRODUCTION

The goal of this work is, through a common effort of people from various fields (Turbulence theory, Experimental and Numerical simulation, Signal processing, Numerical analysis, Harmonic analysis), to generate innovative research tools devoted to the analysis, understanding and modeling of turbulence.

The starting point of this program is the *wavelet decomposition* and the initial algorithms derived by some of the authors (Liandrat and Moret-Bailly 1990, Liandrat and Tchamitchian 1990, Moret-Bailly et al. 1991) to study and model turbulence, and more generally, nonlinear phenomena. Compared with more classical analysis tools, e.g. Fourier Transforms, this new family of algorithms is very flexible. However, it rapidly became apparent that before these algorithms become really useful to the fluid dynamics scientific community, they must first be organized, and translated into an understandable and easily usable form.

In this interim report, we present the first kernel of a program devoted to this task. The first applications and the ultimate goal of this work are also briefly presented. We wish to emphasize that the goal of the work is to greatly extend the now classical wavelet decomposition algorithms. A hierarchy of tools is being developed that will provide quantitative results together with the basic elements for the modeling of variables within the context of nonlinear dynamics, transition and turbulence. Currently, several algorithms developed for the study of boundary-layer transition on a rotating disk have been implemented and are being tested in other configurations.

Continuous interaction between all the collaborators of this program (i.e. the authors of this report) should not only bring the already existing algorithms to a level of easy applicability, but will also provide two additional benefits. First, a subset of these algorithms will evolve into methodologies which will be directly related to the modeling of the studied phenomena. An example of this is the relation between wavelet analysis tools and Large Eddy Simulation. The second benefit of this interaction will be the give birth to a new generation of algorithms, more powerful than the last.

2. REVIEW OF WAVELET TRANSFORM BASICS AND NOTATIONS

Since the basic motivation of this work is the wavelet decomposition, some basic properties of one-dimensional wavelets, together with the conventions used in the report are first reviewed.

A complete review on the current state of wavelet theory is not available, but various tutorials, book conference or reviews can be found in the literature (Combes et al. 1989, Rioul and Vetterli 1991, Farge 1992)

From an analysis function $\psi(x)$ (Figure 1), the family of functions used to decompose a signal $u(x)$ is generated by dilations and translations following the formula:

$$\psi_{ab}(x) = \frac{1}{\sqrt{a}}\psi(\frac{x-b}{a}) \tag{1}$$

where (a,b) belongs to a 2D continuous or discrete space. (a,b) belongs to $R^{*+} \times R$ in the so-called continuous case and to a countable subspace in the discrete case. If in addition, ψ_{ab} form an orthogonal basis, one has: $a = 2^j, b = k2^{-j}$ with $(j,k) \in \mathbb{Z}^{2}$ and the dilated and translated wavelet is denoted ψ_{jk} instead of ψ_{ab}.

One defines the wavelet coefficients ($T(b,a)$ or T_{jk}) of the function $u(x)$ as:

$$T(b,a) = \int_{-\infty}^{+\infty} u(x)\psi^*_{ab}(x)dx \tag{2}$$

or equivalently as:

$$T(b,a) = \int_{-\infty}^{+\infty} \hat{u}(\omega)\hat{\psi}^*_{ab}(\omega)d\omega \tag{3}$$

where $u^*(x)$ and $\hat{u}(\omega)$ stand respectively for the conjugate and the Fourier transform of the function $u(x)$.

One recovers the original function $u(x)$ through summation formulas coming from the identity decomposition (see Combes et al. (1989) for details).

In the continuous version, one recovers the original function from

$$u(x) = \frac{1}{C(\psi)}\int_{0}^{+\infty}\int_{-\infty}^{+\infty} T(b,a)\psi_{ab}(x)\frac{dadb}{a^2} \tag{4}$$

and its energy (L^2 norm) from

$$||u||^2 = \int_{-\infty}^{+\infty} u(x)u^*(x)dx = \frac{1}{C(\psi)}\int_{0}^{+\infty}\int_{-\infty}^{+\infty} T(b,a)T(b,a)^*\frac{dadb}{a^2} \tag{5}$$

where $C(\psi)$ is a constant depending only on $\psi(x)$.

For the orthogonal decomposition one gets:

$$u(x) = \sum_{jk} T_{jk}\psi_{jk} \tag{6}$$

and

$$\int_{-\infty}^{+\infty} u(x)u^*(x)dx = \sum_{jk} T^2_{jk} \tag{7}$$

The wavelet theory is "constructive" in the sense that the construction of the wavelets brings along with it the algorithms needed to compute the wavelet coefficients of the decomposition. Most of these algorithms have an asymptotic operation count at most equal to that of the Fast Fourier Transform which is $O(N \log N)$. If the wavelet is of compact support, $O(N)$ algorithms exist. Here, N stands for the number of points in the signal). Details on the algorithms can be found in the book of Combes et al. (1987) and in the papers of S. Mallat (1988) and Perrier and Basdevant (1989).

As stated in the introduction, the wavelet plane (i.e the family of the wavelet coefficients T_{ab}) is assumed to be the *zero level* output of the algorithms. This 2D field becomes the starting point for the *first level* algorithms.

3. SPECIFIC PROBLEMS OF TURBULENCE AMENABLE TO WAVELET ANALYSIS

If one had to summarize the initial motivation for the use of wavelets in the field of turbulence, *localization* and *scaling* would be two key words for both wavelets and turbulence. These two properties can be used to good advantage to help decompose turbulence into its natural scales without loss of spatial information, and also to compress turbulent data by throwing away information which is not directly relevant to the observed turbulent signals.

But, as stated earlier, the fundamental question addressed here is the following:

What quantitative information can be extracted from the wavelet decomposition of a data field?

Within that context, a good choice of the wavelet family is important since the "quantitative information" must refer to the analyzed signal and not to the analyzing wavelet. In that regard, it is known that localization in both physical and Fourier spaces is necessary. Numerically, spline wavelets of sufficiently high degree (say $m \geq 4$) are satisfactory (Liandrat and Tchamitchian 1990).

After a choice for the family of wavelets is made, we turn our attention to the wavelet plane. Energetic information (Moret Bailly et al. 1991) is first derived and a new plane of coefficients is built. This energy plane now forms a new platform upon which *second level* algorithms are constructed. Different representations and decompositions of this plane are then available. Depending on there definitions, they can refer for instance to coherent structures or optimal decomposition of the signal. These issues are at the cutting edge of data interpretation in turbulence.

Finally, it appears that a large number of specific problems should be addressed by this program. Among them are:

1. The study of the relations between the global spectrum (or scale decomposition) and singularity distributions in a turbulent signal. This problem relates to the fractal or multifractal properties of turbulence as well as the notions of complex singularities in the theory of partial differential equations (Sulem et al. 1983).

2. The characterization of processes undergoing transition using local scale decompositions and as a consequence, of the development of models for turbulence.

3. The definition and extraction of typical (coherent) events and the quantification of the vortex modeling of turbulence.

Answers to these issues would lead to a better understanding of the mechanisms intrinsic to the dynamics of the flow. This understanding will in turn lead to a new generation of modeling concepts for the turbulence cascade, and for transition and turbulence in general.

4. ALGORITHMS ALREADY IMPLEMENTED

In this section, the algorithms that have already been implemented along with some significant results are presented. One should refer to the quoted papers for more details.

4.1. Basic wavelet decomposition algorithms (*Zero Level*)

Two kinds of decomposition algorithms have been implemented that provide:

- A decomposition on periodic, even order spline orthogonal wavelets (for $N = 2^p$ data points, $a = 2^{-j}$and $b = k2^{-j}$ where $0 \leq j \leq p-1$ and $0 \leq k \leq 2^j - 1$)

- A decomposition on periodic, even order spline wavelets (for $N = 2^p$ data points, $a = \frac{2n}{N}$,$b = k$ where $1 \leq n \leq \frac{N}{2}$ and $0 \leq k \leq N-1$) (see [Perrier and Basdevant 1989])

This last algorithm provides a fully redundant decomposition in the position parameter b. The ratio of the number of voices per octave can be adapted using available interpolation routines.

In both algorithms, the scale limits can be arbitrary chosen.

4.2 *First Level* Algorithms

Starting from the wavelet plane, the energy plane is constructed. Following equation (5), the energy at the point (b, a) is (Moret-Bailly et al. 1991):

$$(8) \qquad E(b,a) = \frac{1}{C(\psi)} \int \frac{|T(b,a)|^2}{a^2} \left[\frac{1}{a}\chi\left(\frac{b'-b}{a}\right)\right] db'$$

where $\chi(x)$ is a bump function that represents the envelope of $\psi(x)$ and satisfies

$\int \chi(x)dx$=1. This leads to $\int E(b,a)dadb = ||u||^2$.

Using a different normalization at each point, the local energy density probability (also called scale decomposition) is defined as

$$D_b(a) = \frac{E(b,a)}{\int E(b,a)da} \tag{9}$$

4.3 *Second Level* Algorithms

The analysis of the energy plane $E(b,a)$ in terms of the characterization of local or averaged scale decomposition. Over a subspace S of points, the averaged scale decomposition is given by

$$D_S(a) = \frac{\sum_{b\in S} E(b,a)}{\int (\sum_{b\in S} E(b,a))da} \tag{10}$$

Starting from this scale decomposition, one defines a mean scale as:

$$\bar{a}_s = \int \log(a) D_s(a) da \tag{11}$$

where the subscript s referes to a point b or a subset S. Notice that the integration is made in function of $log(a)$ and not in function of a. Indeed, $log(a)$ is the scale that corresponds to the dilation factor a.

A normalized standard deviation w is then introduced as:

$$w_s = \sqrt{\overline{(log(a) - \bar{a}_s)^2}} \tag{12}$$

The mean scale $\bar{a}_s$ is the scale around which the active scales gather at point b (or in the subspace S) while the normalized standard deviation w_s quantifies the dispersion of the active scales around $\bar{a}_s$.

The scale decomposition averaged over the entire energy plane is easily related to the Fourier energy spectrum thanks to equation (3). A case of special interest for turbulence occurs when the energy spectrum exhibits a $\omega^{2\alpha}$ behaviour at large frequency. Then, the averaged scale decomposition behaves as $a^{-2\alpha-2}$. (see example 5.2).

The local scale decomposition characterizes the scaling behaviour of a function at a given point (Tchamitchian and Holschmeider 1989). For a Hölder exponent α at point b the scale decomposition at the point b behaves as $a^{2\alpha+1}$. (see example 5.3).

4.4 *Ultimate Level* Algorithms

Derived for a specific application, these algorithms make the connection with the physical modeling of the phenomenon under study. The existing algorithms are being applied to the study of the transition on a rotating disk and to the analysis and characterization of wall structures. One must refer to Moret-Bailly et al. (1990) for details.

5. FIRST RESULTS

In the following section the graphical environment of the existing version of the code is presented along with some illustrations of the algorithm outputs at each level.

5.1 Test case of modulated waves (see Figure 2)

For this test case (Figure 2.a), the local mean scale (Figure 2.b), standard deviation (Figure 2.c) and local scale decomposition (Figure 2.d) plots are presented. The two flat zones of the mean scale decomposition represent the local waves, while the local peaks in the standard deviation reveals the singular regions. The different local scale decompositions can be used to characterize each part of the signal.

5.2 $\omega^{2\alpha}$ energy spectrum signal

A signal computed from a random phase $\omega^{-\frac{5}{3}}$ energy spectrum has been constructed and wavelet transformed (Figure 3.a). The averaged scale decomposition (Figure 3.c) is compared to the Fourier spectrum (Figure 3.b). The local scale decompositions (Figure 3.d) clearly show that no local scaling occurs in the signal (see Figure 3.a).

5.3 Test case of local singularities

The analyzed signal exhibits a x^2 local behaviour around $x = 512$ (Figure 4.a). This local scaling is revealed by the local scale decompositions (Figure 4.b) which gives an a^5 spectrum.

5.4 Rotating disk boundary layer velocity signal

Figures 5 show some results obtained from the analysis of a hot film velocimeter signal (Figure 5.a) in a transitional boundary layer over a rotating disk. As a first step, the variations of the standard deviation (Figures 5.b and 5.c) are used to discriminate turbulent bursts from smoothly oscillating parts of the signal. Then, a full decomposition of the signal parametrized by the value of the standard deviation is performed.

A comparison of this decomposition at different Reynolds numbers is used to modelize the transition from an oscillating regime all the way to turbulence (Moret-Bailly et al. 1991).

5.5 Analytical signals to model the compressible boundary layer

A computer code that generates transitional signals has been written for use in connection with a digital signal processing (DSP) system under development at NASA Langley. This system will be part of hypersonic flight tests (Bertelrud 1991). The output signal is deterministic and has built into it intermittency, simulated anomalies and a variety of periodic or turbulent quantities undergoing a series of discrete oscillations. The signal chosen for the current analysis consisted of either

1. A single frequency in the laminar regime (like a wind tunnel fan might yield)
2. A series of frequencies with an amplitude distribution corresponding to the $-1/3$ and $-5/3$ slopes.

The full signal contains a DC as well as an AC component. However, the DC component was removed from the signal displayed in Figure 6.a.

The analysis is performed in term of mean scale (Figure 6.b) and standard deviation (Figure 6.c).

The intermittency function that discriminates the different parts of the signal (6.d) is obtained from the study of the standard deviation. A comparison between the averaged scale decompositions in each part of the signal with an adapted windowed Fourier transform is presented in Figure (6.e). Here, the averaged scale decomposition is plotted versus the frequency instead of the dilation parameter a (one reminds that the bump shape of the Fourier transform of a wavelet allows to associate to each value of a a frequency ω).

6. TECHNICAL IMPLEMENTATION

6.1 Goals

The technical implementation of the algorithms and the associated graphics should be clear, interactive, well-suited to incorporate new algorithms, and easily portable across computer platforms. Among the possible solutions to this challenge, three types of interfaces are under study:

- A full graphic user interface with buttons, sliders, ...
- An interconnected family of modules
-A graphic system governed by a specific command language.

6.2 Existing version

At this time, our prototype has the first type of user interface. Typical screens can be seen on figure 7 with the different windows standing for the signal representation (1), the wavelet or energy plane (2), the local or mean scale decomposition (3), the second level output window (4) and the choice and control window (5). Although this software is already quite satisfactory, it does not satisfy all the previously stated objectives. However, we think that this version can already help answer basic questions and will serve as an elementary module for the next release of the program.

7. FUTURE DEVELOPMENTS AND CONCLUSIONS

Together with the technical implementation, new algorithms have been constructed and tested. On the one hand, the second level algorithms described in this report are being refined by the introduction of a better description of the local or averaged scale decompositions. An algorithm devoted to the detection and characterization of complex singularities is being implemented and tested.

Third level algorithms based on the optimal L^2 decomposition (Karhünen-Loéve decomposition) of the energy plane are under study in the framework of coherent structures and transition modeling.

On the other hand, among the algorithms already available, the class of ridge and skeleton algorithms derived by Toressani and Tchamitchian (1991) and the class of fractal characterization algorithms developed by Arneodo et al. (1991) should be implemented numerically in the near future. They clearly have applications (even if it is not yet completely demonstrated) in the fields of turbulence and transition.

REFERENCES

ARNEODO, A., BACRY, E., MUZY, J.F. :" Wavelet analysis of fractal signals- Direct determination of the singularity spectrum of fully developed turbulence data", Centre Paul Pascal preprint (1991).

BERTELRUD, A.:private communication (1991).

FARGE, M.: "The wavelet transform and its applications to turbulence", to appear in Annual Review of Fluid Mechanics (1992).

HOLSCHNEIDER, M., TCHAMITCHIAN, Ph.:"On the wavelet Analysis of Riemann's function", preprint CPT (1989).

MEYER, Y.: "Ondelettes et Opérateurs 1: Ondelettes" Herman (1990).

COMBES, J.M, GROSSMANN, A., TCHAMITCHIAN, Ph.: " Wavelets, time frequency methods and phase space", proceedings of the 1st International Conference on Wavelets, Marseille, Springer Verlag, (1987).

LIANDRAT, J., MORET BAILLY, F.: "The wavelet transform;some applications to fluid dynamics and turbulence", European Journal of Mechanics B/Fluids vol 9, $N^o 1$, p 1,(1990).

LIANDRAT, J., TCHAMITCHIAN, Ph.: " Resolution of the 1D regularized Burgers equation using a spatial wavelet approximation: Algorithm and Numerical results", ICASE Report N^o 90-83 (1990).

MALLAT, S.: "Multiresolution representations and wavelets" PhD, MS- CIS-88-68, grasp lab 153, University of Pennsylvania, Philadelphia, PA 19104 (1988).

MORET-BAILLY, F., LIANDRAT, J., CHAUVE, M.P., TCHAMITCHIAN, Ph: "Décomposition en ondelettes d'un signal de vitesse dans une expérience de transition laminaire-turbulent sur un disque tournant", C.R.Acad.Sci, Paris, Série 2 (1991).

PERRIER, V., BASDEVANT, C.: "La décomposition en ondelettes périodiques, un outil pour l'analyse de champs inhomogénes. Théorie et algorithmes," la Recherche Aérospatiale, No. 3, pp. 53-67 (1989).

RIOUL, O., VETTERLI, M.:"Wavelets and Signal processing", IEEE SP MAGAZINE October 1991.

SULEM, C., SULEM, P.L., FRISCH, H.:" Tracing complex singularities with spectral methods", Journal of Computational Physics, Vol. 50, N^o 1, April 1983.

TCHAMITCHIAN, P., TORRESANI, B.:" Ridge ands Skeleton Extraction from the Wavelet Transform", in Wavelets and their applications, Ed. Jones and Barlett (1992).