THE PLANETS

By

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Pages

The planets

The stars form patterns that remain fixed, although the sky as a whole seems to rotate from east to west. For example, a distinctive group of seven stars shaped like a saucepan is known as the Big Dipper in the west and as <u>Bei Tou</u> (north ladle) in China. But there are five visible exceptions: the planets. Let us start with the brightest.

Venus

You might look for Venus one night and look in vain, but after a few nights you glimpse it shortly after sunset when the sky has just become dark enough for it to be seen. It is just above the western horizon and sets almost immediately. The next night it sets a little later and can be seen for a little longer. The length of time for which it is visible increases each night for three or four months but then begins to decrease and eventually Venus disappears. This phase, Venus as "evening star", lasts about nine months. Venus then remains invisible for a period which varies from 2 days to 20. Then one morning you see it just before sunrise. It is almost immediately swamped by the glare of the rising sun. The next morning it rises a little earlier and can be seen for a little longer. The length of time for which it is visible increases and then decreases to zero. Venus is the "morning star" for about nine months. It remains invisible for about 50 days. Then it reappears in the morning and the cycle of appearances and disappearances is repeated. The technical name for this cycle is <u>synodic cycle</u> and the time taken by it (which varies slightly) is a <u>synodic period</u>.

(Venus behaves in this way because it it circuits the sun in an orbit smaller than the earth's, and in a plane very close to the plane of the earth's orbit. It is never more than about 47° from the sun.)

The Incas said the the sun, as lord of all the stars, commanded Venus to be near him, sometimes in front and sometimes behind, because it was the most beautiful star.

The very early Greeks thought that Venus was two planets: Eosphorus in the morning and Hesperus in the evening.

Venus is the planet for which we have the earliest known details. The famous Venus tables of Ammisaduqa from ancient Babylon well before 1000 B.C. gave dates of appearances and disappearances. Historians have tried to match these with dates calculated from modern data (impossible to do exactly: we don't know the state of the sky nor the keenness of the observer's eyesight). The probable dates have been narrowed down to three, called the high, middle and low chronologies.

Five pages in a Mayan codex deal with Venus. Along the bottom of each page are the Mayan numerals for 236, 90, 250, 8. These are roughly the number of days in the four phases of the synodic cycle, and they add up to 584, which is the number of days in the average synodic cycle to the nearest whole day. (The Mayas dealt only in whole days). J. E. S. Thompson had an ingenious theory of a way in which the Mayas could have modified the table for use over a long period, necessary because the average period is not quite 584 days. You will find details in Anthony Aveni's <u>Skywatchers</u>, pages 189 and 190, and in my <u>Early astronomy</u>, on pages 119 and 120.

The later Babylonians, from about 300 B.C., treated Venus and the other planets mathematically in detail.

The planets do not wander all over the sky. In a planetarium, in which the visible sky is represented as a hemispherical dome, the path of the sun is a semicircle. If the dome were completed to a whole sphere the path would be a circle; in fact, a <u>great</u> circle. This is the circle that you would get if you cut a spherical ball in half by a cut through the centre. (If you miss the centre, you get a small circle). The technical name for the path of the sun on the sphere is the <u>ecliptic</u>. The planets move along the ecliptic, never deviating far from it. The Babylonians ignored deviations from the ecliptic.

They divided the ecliptic into twelve equal segments: the signs of the zodiac. They often used Sumerian (just as mediaeval Europeans used Latin, or Hindus used Sanskrit). The Sumerian names of the signs (from west to east) are:-

<u>hun</u> <u>múl</u> <u>mash</u> <u>kushu</u> <u>a</u> <u>absin</u> <u>rín</u> <u>gír</u> <u>pa</u> <u>másh</u> <u>gu</u> <u>zib</u>

(mash and másh look similar in modern spelling, but the cuneiform symbols are quite different.)

The Babylonians divided each sign into thirty equal parts called ush (we use the same unit today, calling it a degree). They used these to define positions on the ecliptic. The modern name for angular distance round the ecliptic is celestial longitude and I shall translate Babylonian positions into longitudes, treating hun 1 ush as longitude 1°, so mul 5 ush is 35°, zib 30 ush is 360° (or 0°), and so on.

The sun moves steadily eastward through the signs giving rise (through the Greeks and the Romans) to phrases like "born under Aries".

When Venus first appears in the evening it is east of the sun along the ecliptic and as time passes it moves further east, then it reverses its motion. The reverse, westward, motion is called retrogression. The four appearances and disappearances of Venus and the beginning and end of retrogression are called synodic phenomena. The Babylonians investigated the times when these phenomena occurred and the the longitude of Venus then.

Venus itself moves quite irregularly, but each individual phenomenon moves much more regularly. For example, the beginning of retrogression occurs when Venus is just past its greatest angular distance east of the sun, and this angular distance does not vary much. So this phenomenon moves more or less in step with the sun, whose motion through the signs is only slightly irregular.

The Babylonians devised (for all the planets) theories that enabled them to calculate the time and longitude of an occurrence of a phenomenon from the time and longitude of its previous occurrence, and they produced tables giving times and longitudes of successive occurrences.

One table for Venus is particularly simple. Each occurrence of the phnomenon is 215°30' further round the ecliptic than the previous one, and $19\frac{139}{180}$ months later. (I have translated the Babylonian sexagesimal fractions into modern ones.) The synodic period of Venus is over a year but less than two years, so between two occurrences Venus covers 360° + 215°30', which is equal to 1151/720 revolutions. Then in 720 synodic periods the phenomenon makes 1151 revolutions, which takes 1151 years. So the table appears to be based on the relation 720 synodic periods = 1151 years.

Another tablet is a little more complicated. From it we can deduce five synodic arcs: 210°30', 214°30', 212°, 224°10', and 216°20'. Other tablets give slightly different figures.

The Chinese, in contrast, tracked Venus itself through its synodic cycle, giving changes in longitude in Chinese degrees and the time taken for each change in days. A Chinese degree is the angular distance covered by the sun in one day. The early Chinese took the year to be $365\frac{1}{4}$ days, so there are $365\frac{1}{4}$ Chinese degrees to a complete circle.

The Si fen almanac, about A.D. 100, gave the following figures.

Days	91	91	46	8	10	10	10	8	46	91	91	82 + 562/23320
Chinese degrees	113	106	33	0	-6	-8	-6	0	33	106	113	100+

The table starts with a first appearance in the evening. Venus then covers 113 Chinese degrees while the sun covers 91, so it is getting further away from the sun, as explained earlier. Then Venus slows down, slightly at first. It covers 246 Chinese degrees in 246 days (the first five entries) so it is back at the same angular distance from the sun and on the point of disappearing. It remains invisible for 10 days, then reappears in the morning on the other side of the sun. The last column gives the second period of invisibility. The reason for the gargantuan fraction is that the Chinese had an estimate that 2915 synodic periods take 4661 years, and calculated the number of days in a synodic period precisely.

Before I describe how later astronomers, starting with the Greeks, treated Venus, let us see how the Babylonians dealt with the other planets. (The Chinese treated them all in the same way.)

Mercury

Mercury behaves like Venus except that it is nearer the sun and much less bright. It has the same synodic phenomena.

We are lucky enough to have found a tablet that explains how the Babylonians dealt with Mercury.For a first appearance in the morning, MF (the abbreviation is due to van der Wærden), they divided the ecliptic into three zones: 121° to 236° to 60° to 121°. If an MF occurs in the first zone, to find the longitude of the next MF, add 106°. If this takes us past the end of the zone, multiply the portion past the end by 4/3. (If this takes us into a third zone, another step will be needed.) If an MF occurs in the second zone, add 141°20'; multiply any portion past the end by 2/3. If an MF occurs in the third zone, add 94°13'20"; multiply any portion past the end by 9/8. (Note. 141°20' is 4/3 of 106°; 94°13'20" is 2/3 of 141°20'; 106° is 9/8 of 94°13'20".) The Babylonian description is quite terse, and this is an interpretation rather than a translation. However, we have found a tablet that gives the longitudes of MFs of Mercury, and if we apply the procedure above to it we get the right result whenever two successive longitudes are preserved.

To find the date of an MF take the increase in longitude (in degrees) between it and the previous MF and add to it 03 30 39 04 20. This is a sexagesimal fraction: the 30 means 30 sixtieths, the 39 means 39 sixtieths of a sixtieth, and so on. Interpret the result as thirtieths of a month and add it to the date of the previous MF.

ML and EL are not found by using zones; they are deduced from MF and EF respectively. The longitude of ML is found by adding to the longitude of MF an amount that depends on the longitude of MF. (It varies from 12° to 44°.) The longitude of EL is found by adding to the longitude of EF an amount that depends on the longitude of EF. (It varies from 14° to 46°). The dates of ML and EL are found similarly by adding varying amounts to the dates of MF and EF.

Oddly enough, the Babylonians had a second system for Mercury that worked the other way round. ML and EL were computed using zones and MF and EF were deduced from them. We do not have a Babylonian description of this system, but we do have tablets giving successive longitudes and, knowing the general method, we can deduce the zones that fit (DIO, to be published).

The tablets were arranged in columns. One pair of columns gave longitudes and dates of successive occurrences of MF, the next pair ML, then EF, then EL. So each column gives successive longitudes and dates of one phenomenon; each row gives data for all four phenomena in chronological order.

Mars

Mars behaves quite differently from Venus. One morning it rises long enough before the sun to be seen briefly before being swamped by the glare of the sun. This first appearance is denoted by MF. Mars rises a little earlier each day and can be seen for longer. This continues until Mars rises at sunset and sets at sunrise, and can be seen the whole night from dusk, when the sky becones dark enough for Mars to be seen, until dawn, when the sky becomes too light. Mars is then opposite the sun in the sky. This is called opposition, OP. Eventually Mars rises just after sunrise and sets just after sunset. It can be seen briefly from dusk until it sets. It then becomes invisible, EL.

If you watch Mars against the background of the stars, you will see that at opposition it is moving westwards along the ecliptic: it is retrogressing. After a while retrogression ends, RE. After Mars reappears it is moving east, but after a while retrogression begins again, RB. OP, RE, EL, MF, and RB are the synodic phenomena for Mars and form its synodic cycle, which takes about 780 days.

Jupiter and Saturn behave similarly, their cycles taking about 400 and 380 days respectively.

These planets behave like this because they circle the sun in orbits larger than the earth's. They are the outer planets, Mercury and Venus are the inner planets.

For the MF, RB, and EL of Mars the Babylonians divided the ecliptic into six zones and used the same procedure as for Mercury. The boundaries of the zones are 30°, 90°, 150°, 210°, 270° and 330°. The arcs added in the first step are 45°, 30°, 40°, 60°, 90° and 67°30'. To get the date of an occurrence add 23 57 52 plus the increase in longitude to the date of the previous occurrence.

So far, the Babylonians have ignored the arc RB-OP-RE, the arc of retrogression. They deduced the longitude of OP from the longitude of RB and we have found four different ways in which this could be done. The length of the arc depends on the longitude of RB. In two of the systems it varies from 6° to 7°12', in the other two from 6° to 7°30'. The arc from OP to RE is one-and-a-half times the arc from RB to OP.

Jupiter

For Jupiter there were several systems. One had two zones, with added arc 30° from longitude 85° to 240° and 36° for the rest of the ecliptic. We have found tablets using this for RB, OP, RE, and EL. One tablet gives the increases in date between occurrencces as well as the actual dates. In the first zone the interval between successive dates is 12 months plus 42 05 10 thirtieths. (It is the thirtieths that are tabulated.) In the other sector the number is 48 05 10. In each case we add 12 05 10 to the increase in longitude (plus 12 months) to get the interval of time.

The Babylonians also had several systems using four zones and one using six.

But besides all these the Babylonians had also an entirely different system for Jupiter that does not use zones. Otto Neugebauer called it system B.

Let us call the increase in longitude between one occurrence and the next a synodic arc. Specifically, it is the synodic arc corresponding to the longitude of the first occurrence.

In system B there is a fixed amount by which the synodic arc changes from one occurrence to the next. There is also a fixed maximum and a fixed minimum. The arc starts by increasing; when it reaches the maximum it decreases; when it reaches the minimum it increases again, and so on.

The maximum and minimum do not appear in the tablet because they occur between entries, but they are easily deduced. How this is done is most easily explained by an example.

In ACT 620 the change from one entry to the next is 1°48'. The last entry before the maximum is 36°54'. The arc increases from there to the maximum and then decreases to 37°22'. The total change is 1°48' so the maximum must be at 38°02'. The minimum is 28°15'30".

The synodic period behaves in the same way; it increases or decreases by 1°48' at a time. The maximum and minimum are 50°07'15" and 40°20'45", giving the same difference as for the synodic arc.

We can make an interesting deduction. One synodic period causes a change of 108' in longitude. The total change from maximum back to maximum is 1173'. This corresponds to one revolution round the ecliptic and so takes one sidereal period. So (dividing both numbers by 3), 391 synodic periods equal 36 sidereal periods.

Relations of this sort seem to underlie Babylonian theory. They can be found by noting when a synodic phenomenon is repeated at the same longitude and counting the number of synodic cycles and the number of circuits of the ecliptic between the two occurrences.

They could also be built up from data that do not take so long to find. (391 synodic periods take over 400 years.) For example, ACT 812 shows that in 65 synodic periods Jupiter circles the ecliptic 6 times plus 6°. ACT 813 shows that in 11 synodic periods Jupiter circles the ecliptic once less 5°. Combining five of the first with six of the second, we find that in 391 synodic periods Jupiter circles the ecliptic exactly 36 times; the odd degrees total zero.

Saturn

The same system is used in a table for Saturn. The change from one entry to the next is 12'. The maximum and minimum synodic arcs are 25 32 03 07 30 and 22 41 23 07 30 degrees, so the total change from maximum back to maximum is 341'20". So (multiplying both figures by 3/4) we find that 9 sidereal periods equal 256 synodic periods.

Planets in general

Similar relations underlie the theory that uses zones. For example, one tablet says that for Mars 133 synodic periods equal 151 sidereal periods (and take 284 years).

Such relations determine the average synodic arc. If X synodic periods equal Y sidereal periods, the average distance round the ecliptic covered by the planet in one synodic period is Y/X revolutions, and the average synodic arc is the fractional part of this. So the average synodic arc for Saturn is 9/256 revolutions, for Jupiter it is 36/391, for Mars it is the fractional part of 151/133, which is 18/133. This, to the nearest minute of arc, is 48°43', a figure that is actually given in ACT 811a.

This gives rise to an interesting piece of mathematics. The Babylonians seem to have regarded the different added arcs as representing the different speeds at which the synodic phenomena progress through the various zones. If a body goes at speed u for a distance a, speed v for distance b, and so, the total time taken is a/u + b/v + ... and the total distance is a + b + ... so the average speed is a/u + b/v + ... divided by a + b + ...

For the Babylonians, u, v,... are the added arcs, and a, b, ... are the lengths of the zones. Then a + b + ... = 1 (one whole revolution), so the average "speed", i.e. the average synodic arc, is 1 divided by a/u + b/v + ...

For the tablet for Jupiter that uses two zones, a = 155°, b = 36°, u = 30° and v = 36°, so a/u + b/v = 155/30 + 205/36 = 186/36 + 205/36 = 391/36, giving an average synodic arc 36/391, revolutions, just as we found from ACT 620.

Not only that, but one of the systems that uses four zones has zones 120°, 53°, 135°, 52° and added arcs 30°, 33°45', 36°, 33°45' respectively. This gives the same result.

For Mars, the zones of 60° and the arcs of 45°, 30°, 40°, 60°, 90° and 67°30' yield 133/18, as found earlier.

If we apply these calculations to Mercury, we find four different synodics arcs for the phenomena MF, ML, EF and EL.

Geometrical treatment

The earliest details that we have are from Eudoxus, shortly after 400 B.C. He treated all the planets in the same way.

He pivoted one transparent sphere inside another. If the inner sphere were set rotating relative to the outer sphere and if the outer sphere were set rotating at the same speed in the opposite direction about the same axis, the motions would cancel and the inner sphere would remain still. But Eudoxus had the two axes at a slight angle to each other and a point on the inner sphere half-way between the pivots moved round a figure-of-eight. Greek geometry was capable of proving this.
Eudoxus pivoted these spheres inside a third sphere that rotated parallel to the ecliptic so that the figure-of-eight carrying the planet moved round the ecliptic. He then pivoted this sphere inside a fourth sphere which rotated parallel to the equator. This accounted for the rising and setting of the planets.
This ingenious piece of geometry has astronomical defects. It has the planet on the ecliptic twice in a synodic period (which is presumably the time taken to go round the figure-of-eight). If the planet moves round the figure-of-eight fast enough relative to the speed at which the figure-of-eight moves round the ecliptic, that will account for retrogression. But if the angle between the axes is chosen to give the right maximum deviation from the ecliptic it will give the wrong length to the distance covered in retrogression, and vice versa. The description that has come down to us (from Aristotle) has no numerical details, but if we try real periods, latitudes and retrogressions, it just does not work.

Aristotle built Eudoxus's model not merely into a complete solar system, but into a complete universe. He started with a sphere rotating parallel to the equator carrying the stars.

This sphere serves as the outermost sphere for Saturn. To the innermost sphere of Saturn he pivoted three spheres rotating in the opposite direction to the spheres of Saturn and cancelling their motion, ending with a sphere rotating parallel to the equator, which serves as the outermost sphere for Jupiter. And so on. Callippus tried to improve the model by adding extra spheres but this led nowhere and later Greek theories were quite different.

We have very little information about Greek treatment of the planets between the work of Eudoxus and the encyclopaedic <u>Syntaxis</u> written by Klaudios Ptolemaios (usually called Ptolemy: not one of the pharaohs of that name) about 150 A.D. The only earlier work that he mentioned is a geometrical theorem by Apollonius which enables us to find when retrogression begins and ends.

Ptolemy pointed out that a planet exhibits two anomalies (departures from regularity), one depending on the position of the sun, the other on the longitude of the planet. His aim was to show that these can be accounted for by regular circular motions. (As we shall see, he failed.)

In a traditional philosophical dogma, a point moving at constant speed round a fixed circle is moving regularly, and so is a point moving at constant speed round a circle whose centre is moving regularly.

Ptolemy then listed some numerical data. He was clearly thinking of retrogression being produced by the planet moving round a circle (an epicycle) because he referred to a synodic period as a return of anomaly, made up of 360 degrees of anomaly.

His data, in which revolutions are revolutions in longitude, are:-

Saturn
57 returns of anomaly = 59 years plus about $1\frac{3}{4}$ days = 2 revolutions plus 1°43'

Jupiter
65 returns of anomaly = 71 years less about $4\frac{9}{10}$ days
= 6 revolutions less $4\frac{5}{6}°$

Mars
37 returns of anomaly = 79 years plus $3 + \frac{1}{6} + \frac{1}{20}$ days
= 42 revolutions plus $3\frac{1}{6}°$

Venus
5 returns of anomaly = 8 years less $2 + \frac{1}{4} + \frac{1}{20}$ days
= 8 revolutions less $2\frac{1}{4}°$

Mercury
145 returns of anomaly = 46 years plus about $1\frac{1}{30}$ days
= 46 revolutions plus 1°.

Ptolemy noted that for an outer planet the number of years is always the number of returns of anomaly plus the number of revolutions.

Ptolemy's data are clearly corrections to the well-known Babylonian relations

Saturn: 57 synodic periods = 59 years = 2 sidereal periods
Jupiter: 65 synodic periods = 71 years = 6 sidereal periods
Mars: 37 synodic periods = 79 years = 42 sidereal periods
Venus: 5 synodic periods = 8 years = 8 sidereal periods
Mercury: 145 synodic periods = 46 years = 46 sidereal periods.
(These are found in texts called "goal-year texts".)

In later chapters of the _Syntaxis_ Ptolemy compared observations of his own with observations made some 400 years earlier and computed the following results, in which I have converted Ptolemy's Egyptian years plus days into days and revolutions in anomaly into degrees (and minutes).

Saturn: 12671°27' of anomaly take $133079\frac{3}{4}$ days
Jupiter: 124305°45' of anomaly take 137733 days less about an hour
Mars: 69181°43' of anomaly take $149881\frac{2}{3}$ days approximately
Venus: 92138°25' of anomaly take 149452 days
Mercury: 456726°53' of anomaly take 147013 days $13\frac{1}{2}$ hours.

Calculations from these data give the following results (rounded to the nearest 60th of a day).

Saturn
57 returns of anomaly take 21551;18 days = 59 years 1;42 days

Jupiter
65 returns of anomaly take 25927;37 days = 71 years less 4;31 days

Mars
37 returns of anomaly take 28857;41 days = 79 years plus 3;56 days

Venus:
5 returns of anomaly take 2919;40 days = 8 years less 2;18 days

Mercury
145 returns of anomaly take 16802;24 days = 45 years 1;03 days.

Ptolemy said that it was from these calculations that he made his corrections to the periods of Mercury and Venus.

Ptolemy reduced his first set of data to days, rounded to the nearest 60th, and degrees of anomaly:-

Saturn 20520° of anomaly take 21551;18 days
Jupiter: 23400° of anomaly take 25927;37 days
Mars: 13320° of anomaly take 28857;53 days
Venus: 1800° of anomaly take 2919;40 days
Mercury: 52200° of anomaly take 16802;24 days.

A correct calculation gives 28857;42 for Mars and 16802;23 for Mercury.

Although the calculations from the data that Ptolemy cited later do not agree very well with the intervals in years and days in the first set of data, they agree four times out of five with this second set. Only Mars is not exact. (All versions of the Syntaxis give 53 for the fraction of a day. Calculation from the first and second sets of data give 42 and 41, Toomer amends the 53 to 43, claiming incorrectly that it is the rounding of 42;18. Manitius, Halma and Taliaferro leave it as 53.)

Next Ptolemy said that he divided the number of degrees of anomaly by the number of days and obtained the following daily increases in anomaly, from which he built up his tables.

Saturn	0°57	07	43	41	43	40
Jupiter	0°54	09	02	46	26	00
Mars	0°27	41	40	19	20	58
Venus	0°36	59	25	53	11	28
Mercury	3°06	24	06	59	35	50

This is true for Saturn, Venus and Mercury, but for Jupiter and Mars division does not give the result cited. For each of the planets, when Ptolemy cited the later data he said that he obtained the daily increase by dividing the number of degrees there by the number of days. This is false all five times.

Ptolemy then explained that we can find the daily increase in longitude of an outer planet by subtracting the daily increase in anomaly from the daily increase of the mean sun; there is no need to reduce the revolutions in longitude to degrees and divide by the number of days. However, if we do reduce the revolutions in longitude for Jupiter in the first set of data to degrees and divide by the number of days in the second set, we do get the daily increase in longitude to the precision cited. So it looks as though this is what Ptolemy did.

This technique does not work for Mars. But if we change the fraction of a day from the 53 in the Syntaxis, not to either of the correctly-calculated values 41 and 42 but to the 43 suggested by Toomer, this technique does give the right result.

Next Ptolemy described his theory of motion for the planets. Each planet moves at constant speed once in its synodic period round a small moving circle (the epicycle) whose centre C moves round a larger circle (the deferent) whose centre D moves round the earth T at the rate of precession (so the line TD, which I call the axis, keeps pace with the stars: Ptolemy regarded the stars as moving and the equinox points as fixed). C moves at constant angular speed round a point E (the equant) on the axis; D bisects TE. C completes a revolution in a longitudinal period.

While investigating longitude Ptolemy took everything to be in the plane of the ecliptic, saying that this will not make a noticeable difference.

Mercury is an exception because the centre of its epicycle reaches perigee (the point on its orbit nearest the earth) twice in each revolution. Let Z be the point on the axis for which ZE = ET. The centre D of the deferent moves round Z in a circle whose radius equals ZE in the opposite direction from the motion of the epicycle and at the same angular speed. This is illustrated in diagram 1.

The angle TCE is called an equation of ecliptic anomaly. If P is the planet, the angle PTC is the elongation. Although Ptolemy did not say so here, it is clear from what he wrote later that the longitude of C is the longitude of the mean sun: the mean sun is an imaginary body that moves round the ecliptic at constant speed having the same longitude as the sun at apogee and perigee. Ptolemy could compute the longitude of the mean sun at any given instant from his tables (which were out by just over a degree).

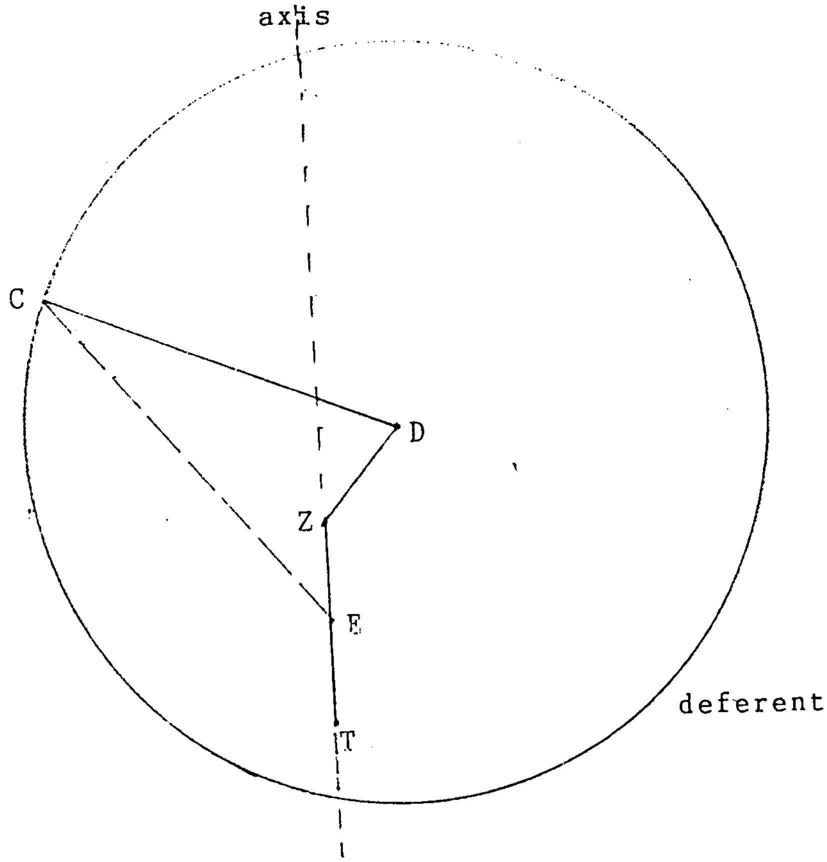


Diagram 1

Ptolemy's theory for Mercury (not to scale).

T: the earth
E: the equant
Z: the point on the axis for which ZE = ET
D revolves round Z. DZ = ZE .
D is the centre of the deferent.
C: the centre of the epicycle.

How would we find a greatest elongation? We might measure the longitude of Mercury or Venus each night, recording the time and date, compute the longitude of the mean sun, and subtract. This gives the elongation. When the elongation stops increasing and starts to decrease we have a greatest value. But this is not the greatest value that Ptolemy needed; he needed the greatest value for a given position of the epicycle. He would need many observations of the elongation listed against the corresponding longitude of C. He would have to pick the greatest elongation for the longitude of C that he is using. If we represented this graphically we would get diagrams like the ones computed by Dennis Duke (my diagrams 2 and 3, from DIO volume 11, page 64, in which evening observations are eastern, and western elongations are displayed as negative).

Ptolemy remarked that such combinations are rare.

Ptolemy started his work on the parameters with Mercury. To find the direction of the axis he looked for two equal greatest elongations, one eastern and one western. He maintained that they must be symmetrical about the axis. He had earlier proved the converse: that if they are symmetrical they must be equal, but not that if they are equal they must be symmetrical. Diagram 2 shows that if we have, say, a western elongation of 21° and look for an eastern elongation of 21° there will be two. If one is symmetrical, the other will not be.

Ptolemy found an eastern elongation $21\frac{1}{4}$° at longitude $309\frac{3}{4}$° and an equal western elongation at 70°. These made the axis run from $189\frac{7}{8}$° to $9\frac{7}{8}$°. Elongations of $26\frac{1}{2}$°, the eastern at $70\frac{1}{2}$° and the western at 310°, made the axis run from $190\frac{1}{6}$° to $10\frac{1}{6}$°. Ptolemy concluded that the axis runs between about 190° and 10°. (According to Duke's figures the western elongation never reaches $26\frac{1}{2}$°). From observations about 400 years earlier Ptolemy found a western elongation of $25\frac{5}{6}$° at longitude $318\frac{1}{6}$°. He could not find an equal eastern elongation, but he found $24\frac{1}{6}$° at longitude $29\frac{1}{2}$°

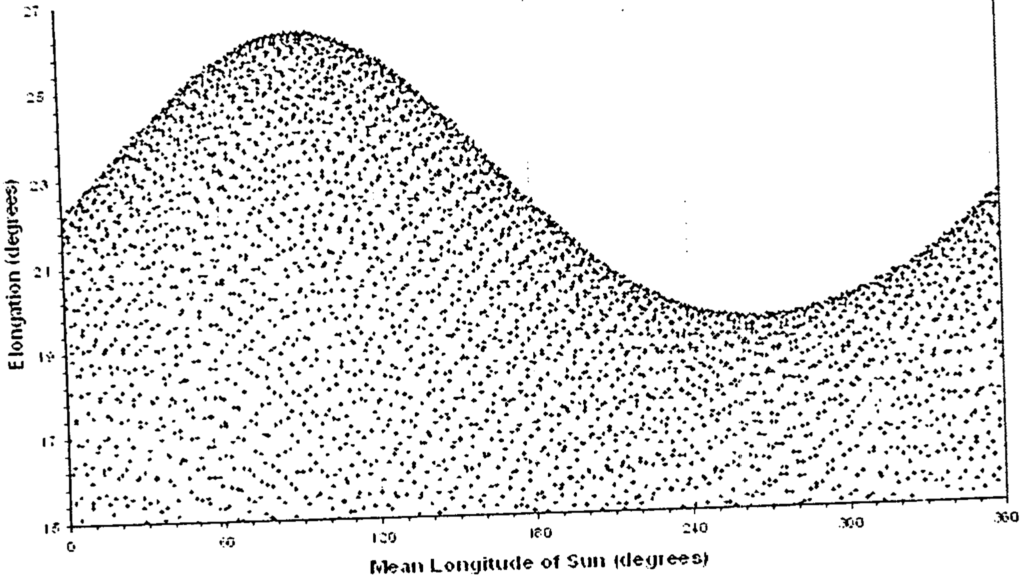


Diagram 2

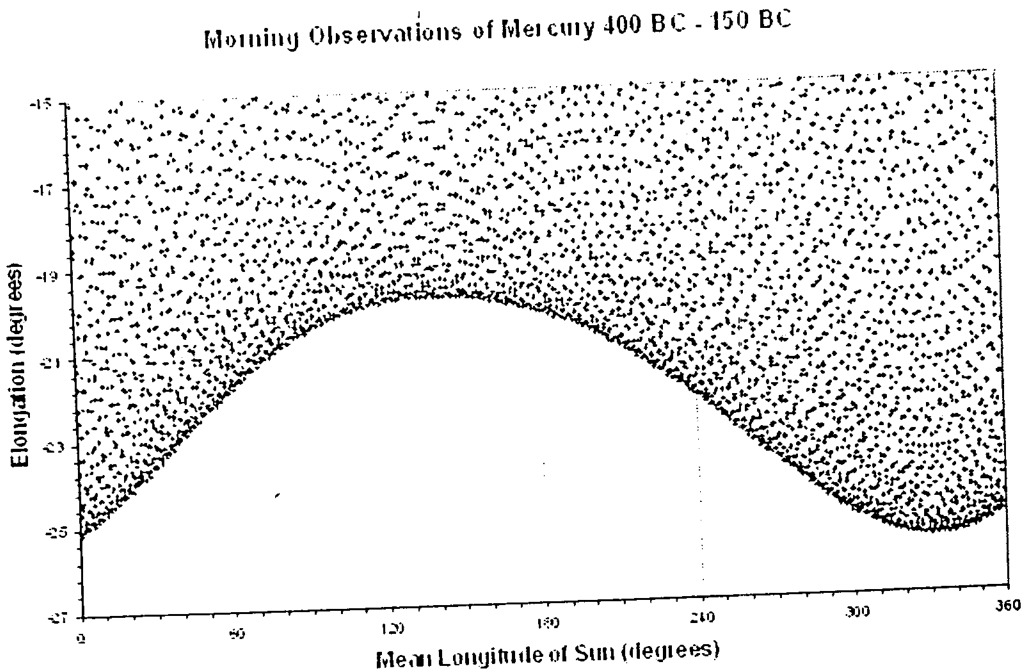


Diagram 3

and $26\frac{1}{2}$° at longitude $62\frac{5}{6}$°. Assuming that the elongation increaseed at constant rate between these two longitudes he calculated that it would be $25\frac{5}{6}$° at longitude $53\frac{1}{4}$°. This makes the axis run from $185\frac{5}{6}$° to $5\frac{5}{6}$°. (Here again Ptolemy's logic was faulty. The elongation rose to a maximum and fell to $25\frac{5}{6}$° at about longitude 125°. This would have made the axis run from about $41\frac{1}{2}$° tp $221\frac{1}{2}$°.) A similar calculation with an eastern elongation of $21\frac{2}{5}$° at $147\frac{5}{6}$° and western elongations of 21° at $215\frac{1}{6}$° and $22\frac{1}{2}$° at $234\frac{5}{6}$° makes the axis run from 186° to 6°. How the earlier astronomer recognised these elongations as greatest elongations is not clear.

This is the reason for Ptolemy's statement that the axis rotated with the stars under precession.

From elongations $19\frac{1}{20}$° (western) at $170\frac{1}{3}$° and $23\frac{1}{4}$° (eastern) at $11\frac{1}{12}$° Ptolemy concluded that the apogee is at 190°, not 10°.

The sum of the eastern and western elongations for a given position of the epicycle gives the apparent size of the epicycle as seen from the earth, so the greater the sum the nearer the epicycle. The elongations of $21\frac{1}{4}$° and $26\frac{1}{2}$° at 310° and again at 70° (rounding off the $309\frac{3}{4}$° and $70\frac{1}{2}$°) give a sum of $47\frac{3}{4}$°. This is greater than the sum at 10° (the point opposite apogee), which is $46\frac{1}{2}$° (twice $23\frac{1}{4}$°, the eastern elongation at $11\frac{1}{12}$°, which Ptolemy evidently assumed was not appreciably different from the value at 10°. On the axis the two elongations are equal). This is the reason for Ptolemy's statement that Mercury reaches perigee twice in a revolution.

Ptolemy then calculated the parameters for his model, citing just enough observations of the right kind. The details can be found in the Syntaxis and, slightly modernised, in R R Newton's The crime of Claudius Ptolemy and my Early astronomy.

Ptolemy finished by citing the observations from which he corrected the synodic period and by computing the longitude and degrees of anomaly at the early date from which he started his tables (the first day of the Egyptian year in the first year, 747 B.C., of the reign of the Babylonian Nabu-nasir). We call this the epoch.

Note: for each elongation Ptolemy cited the date but not the time though the time is needed to calculate the longitude of the mean sun as precisely as he did.

Next, Ptolemy dealt with Venus. He cited two greatest elongations of $47\frac{1}{4}°$, the eastern at $344\frac{1}{4}°$, and the western at $125\frac{3}{4}°$. These make the axis run from 235° to 55°. Another pair, $47\frac{16}{30}°$ at $197\frac{26}{30}°$ (western) and $272\frac{1}{15}°$ (eastern) make the axis run from $234\frac{29}{30}°$ to $54\frac{29}{30}°$, which round to 235° and 55°.

Elongations of $44\frac{4}{5}°$ (western) at $55\frac{2}{5}°$ and $47\frac{1}{3}°$ (eastern) at $235\frac{1}{2}°$ show that 55° is the apogee and 235° the perigee.

Ptolemy added that everywhere on the ecliptic the sum of the elongations is between the values at 55° and 235° and concluded that the deferent is fixed, though the most that can be deduced logically is that he did not have to make it move as he did for Mercury.

There is something badly wrong with these elongations. The greatest eastern elongations at $235\frac{1}{2}°$ and $274\frac{1}{15}°$ were cited at dates 37 days apart. This simply cannot happen.

Ptolemy took as his unit of length 1/60 of the radius of the deferent. From greatest elongations at 55° and 235° it is easy to calculate the distance TD, $1\frac{1}{4}$, and the radius of the epicycle, $43\frac{1}{6}$.

Next Ptolemy found the position of the equant, using greatest elongations $43\frac{7}{12}°$ (western) and $48\frac{1}{3}°$ (eastern) at longitude $325\frac{1}{2}°$, very close to halfway between apogee and perigee.

In diagram 4 T is the earth, E the equant, C the centre of the epicycle. EC and TQ point to the mean sun (considered far enough away that they are parallel). V and W are the two positions of Venus. VTQ and QTW are the elongations.

$VTC = \frac{1}{2}VTW = \frac{1}{2}(VTQ + QTW) = 45°57\frac{1}{2}'$.

$QTV - QTW = QTC + CTV - (CTW - CTQ) = 2CTQ$, so $CTQ = 2°22\frac{1}{2}'$.

$ECT = CTQ$. Then $ET = CT \sin 2°22\frac{1}{2}'$. $CT = CV/\sin 45°57\frac{1}{2}'$. $CV = 43\frac{1}{6}°$, from the previous calculation. Then $ET = 2\frac{1}{2}$, or very slightly less. This is twice TD.

This calculation is very susceptible to errors in measurement. An error of 1° in the longitude of the mean sun could change TE from $2\frac{1}{2}$ to $4\frac{3}{10}$ or 2/5.

Ptolemy finished by correcting the synodic period and computing the longitude and degrees of anomaly at epoch.

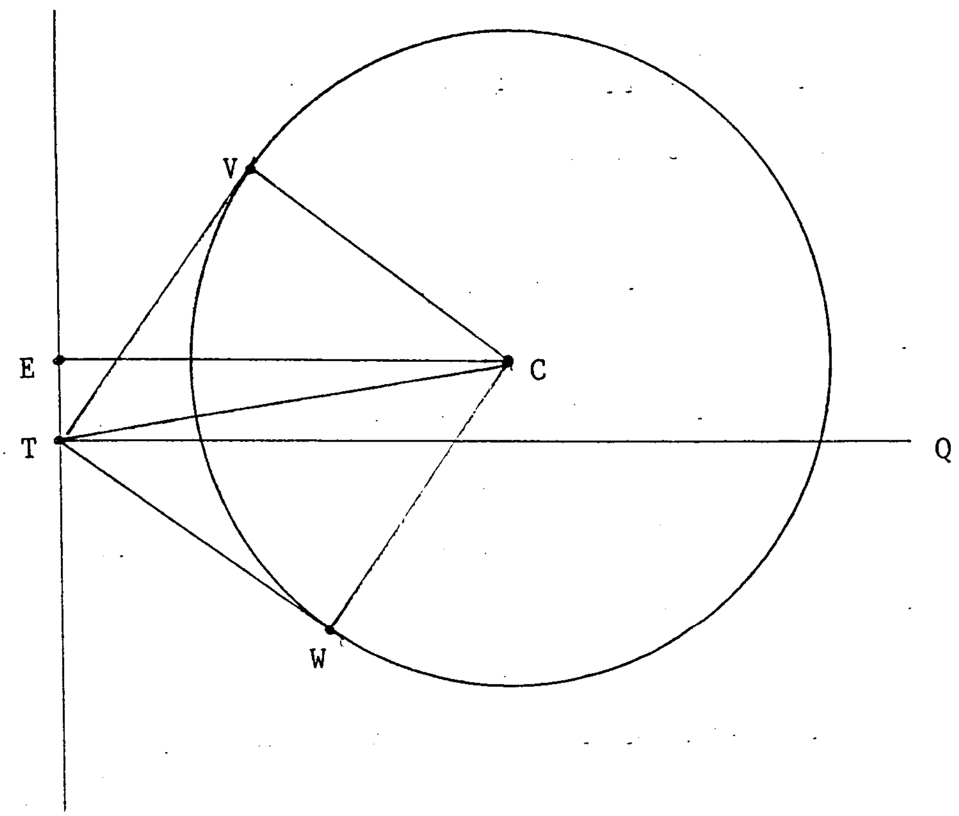


Diagram 4

Ptolemy treated the three outer planets alike. He assumed that the axis kept pace with the stars, like Mercury, and that D bisects TE as it does for Venus. His reason: the eccentrcity found from the greatest equation of ecliptic anomaly is about twice the eccentricity found from the sizes of the retrograde arc at apogee and perigee.

The outer planets do not have greatest elongations. Instead Ptolemy used oppositions. At an opposition the planet P and the centre C of its epicycle have the same longitude. CP then points to the mean sun. Ptolemy stated that it always did, which is equivalent to saying that P revolves round the epicycle in a year. The epicycle revolves round D in the sidereal period.

To find the distances TD and TE and the longitudes of D and E Ptolemy used the following technique. If we have three points X, Y, Z on a circle and we know the angles subtended by them at the centre E and at a point T, we can find the ratio of ET to the radius of the circle and the angles between TE and EX, EY, and EZ. He showed how to do this using Greek geometry and his table of chords.

To apply this to the planets, let the circle shown in diagram 5 have radius 60 and centre E. Let C_1, C_2, and C_3 be the positions of the centre of the epicycle on the deferent at three timed oppositions.

We know the angles subtended at E by X, Y and Z because they are the same as the angles subtened by the C_i. We do not know the angles subtended by X, Y and Z at T, but we do know the angles subtended by the C_i from the observed longitudes.

Ptolemy took the angles subtended by the C_i at T as first approximations to the angles subtended by X, Y and Z. From these he calculated a first approximation to the length and direction of of TE. This gave him a first approximation to the motion of the planet so he could, from the times of the observations, calculate the angles subtended at T by X, Y and Z. This second approximation is better than the first and from it Ptolemy calculated a second approximation to the length and direction of TE. And so on.

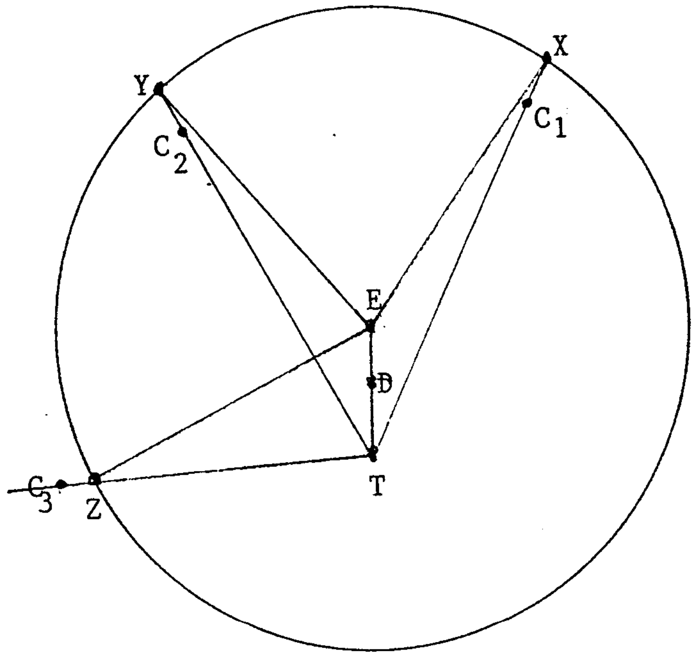


Diagram 5

The successive approximations to the length of TE are:-

Mars	$13\frac{7}{60}$	$11\frac{5}{6}$	12
Jupiter	$5\frac{23}{60}$	$5\frac{1}{2}$	
Saturn	$7\frac{2}{15}$	$6\frac{5}{6}$.	

These are suspicious. Anyone who has used successive approximation (perhaps using the Newton-Raphson method in elementary calculus) will know that as approximations proceed they become less round. Ptolemy's become rounder. It looks as though he were working backwards from the answer (as he did when calculating the length of the year).

From a timed longitude not at opposition Ptolemy calculated the radius of the epicycle. He finished by correcting the synodic period and finding the longitude and degrees of anomaly at epoch.

From his models Ptolemy could calculate the longitude of a planet at any given time.

Ptolemy then investigated latitudes, but without much success. The main reason for this is probably that he had the planes of the deferents contain the earth, whereas in fact the planes of the orbits contain the sun.

In a later work (Hypotheseis ton plamenon) Ptolemy formed the planets into a solar system. He had, from calculations using parallax, a fairly good estimate of the greatest distance of the moon from the earth. He took this to be the least distance of Mercury. His theory for Mercury told him the greatest distance. He took this to be the least distance of Venus. He fitted the sun, Mars, Jupiter and Saturn in the same way.

We have no information about Greek astronomy after Ptolemy, but a theory using epicycles was used by Hindu astronomers, from about 500 A.D. onwards. It appeared in a number of slightly different forms. One of the earliest was described by Aryabhata, who was born in A.D. 476, in the Aryabhatiya. This is the version that I shall describe. A very slightly different version, also by Aryabhata, was described by Brahmagupta in the Khandakhadyaka. He also described a system of his own in the Brahmasphuta siddhanta. Hindu coordinates, like the Babylonian and the Chinese, were sidereal; not tropical like the Greek.

The Hindus called the centre C of the epicycle madhya graha (literally mean planet) It moves at constant speed round the earth T. In 4320000 years it revolves round the earth 4320000 times for Mercury and Venus, 2296824 times for Mars, 364224 times for Jupiter and 146564 times for Saturn. All longitudes were assumed to be zero at dawn on a day that has been identified as 3202 B.C. February 18th in our Julian calendar, so these figures enable us to calculate the longitude of C at any given time. C is the centre of two epicycles.

The manda (slow) epicycle carries a point M. The direction of M from C is called the longitude of the mandocca; it changes very slowly. Aryabhata listed the longitudes in his time. The mandakendra is the longitude of C minus the longitude of the mandocca, so it gives the position of M on the epicycle. The radius of the epicycle depends on the mandakendra and varies as follows, on a scale on which the radius of the orbit of C is 360:

Mercury,	from $22\frac{1}{2}$ to $31\frac{1}{2}$
Venus,	from 9 18
Mars,	from 63 to 81
Jupiter,	from $31\frac{1}{2}$ to 36
Saturn	from $40\frac{1}{2}$ to $58\frac{1}{2}$.

The *sighra* (fast) epicycle carries a point S. The direction of S from C is the longitude of the *sighrocca*. The *sighrakendra* is the longitude of the *sighrocca* minus the longitude of C, so it gives the position of S on the epicycle. The radius of the epicycle depends on the *sighrakendra*, and varies as follows:

Mercury,	from	130½	to	139½
Venus	from	256½	to	265½
Mars	from	229½	to	238½
Jupiter	from	67½	to	72
Saturn	from	40½	to	46.

In 4320000 years S revolves round C 17937020 times for Mercury, 7022288 times for Venus, 4320000 times for each of the outer planets, (We recognize that these figures give the heliocentric sidereal periods of Mercury and Venus. Possibly early Hindu astronomers thought that these two revolved round the sun. If so, later astronomers gave this idea up, just as later Greeks gave up the ideas of Aristarchus.)

The longitude of a planet is found by applying two adjustments to the longitude of C. One source spoke of heavenly bodies at S and M pulling on the *madhya graha* with cords of wind. Although Aryabhata did not say so, it is clear that the *manda* adjustment is the angle MTC and the *sighra* adjustment is the angle CTS.

Aryabhata's description is concise. For an outer planet:

> For the *mandocca* and *sighrocca* half is taken
> negatively or positively for the planet and
> the *manda*. A true mean planet should be known
> from the *mandocca* and the true planet from the
> sighrocca, (Literal translation.)

With the help of later commentaries we can interpret this as follows:-

> Apply to C half of the *manda* adjustment, shifting it to U.
> Apply to U half of its *sighra* adjustment, shifting it to V
> Apply to C the *manda* adjustment for V, shifting it to W.
> Apply to W its *sighra* adjustment.
> This gives the longitude of the planet.

For Mercury and Venus the first adjustment is omitted.

In A.D. 629 Bhaskara commented "It is really curious. Tradition has it, so it must be respected". (Translation by D.A. Somayaji)

Aryabhata's second system applied the first two adjustments the other way round: sighra first. And it used the same method for all five planets. In a commentary written in the eleventh century Varuna gave a complete calculation.

(1) The longitude of the madhya graha	325°01'10"	
(2) The longitude of the mandocca	127°	
(3) The longitude of the sighrocca	168°01'55"	
(4) The mandakendra	198°01°10"	
(5) The sighrakendra	203°00'45"	
(6) The corresponding sighra adjustment	32°07'37"	negative
(7) The first adjusted mandrakendra	181°57'22"	
(8) The corresponding manda adjustment	22'50"	positive
(9) The second adjusted mandakendra	182°08'47"	
(10) The corresponding manda adjustment	25'	positive
(11) The third adjusted mandakendra	198°26'10"	
(12) The corresponding sighra adjustment	31°47'35"	negative
(13) The mandrakendra of the planet	166°38'35"	
(14) The longitude of the planet	293°38'35".	

Items (1) and (3) are calculated from the time of the observation, (2) is listed, (4) and (5) are found by subtraction. The adjustments are found by interpolation from tables. Adjustment (6) reduces the longitude of C and therefore the mandrakendra by half of 32°07'37". This gives (7). Adjustment (8) increases the mandrakendra by half of 22'50", giving (9). Adjustment (10), which is applied to C, gives (11) from(3). Adjustment (12) then gives (13). Adding (2) gives the final result.

Varuna showed how to calculate the sighra adjustment. Aryabhata listed the increases in sighra adjustment corresponding to increases in sighrakendra. From these we can (and presumably the Indians did) produce the following table:-

sighrakendra	0	28	60	90	121	135	148	164	173	180	187	196	212
adjustment	0	11	23	33	40	40½	-37½	-25½	-12½	0	-12½	-25½	-37½

225	239	270	300	332	360
40½	40	33	23	11	0

To interpolate for the *śīghra* adjustment corresponding to a *śīghrakendra* s after the entry s_0, let the previous and following increases in *śīghrakendra* be x and y, and let the previous and following increases in the adjustment be u and v.

Step 1. If $\frac{1}{2}(v + uy/x)$ is larger in magnitude than v, subtract $\frac{1}{2}(v + uy/x)(s - s_0)/y$; if smaller, add.

Step 2. Multiply by $(s - s_0)/y$ and add to the adjustment for s_0.

Interpolating for 203°00'45" we have $s_0 = 196$, $x = 9$, $y = 16$, $u = -13$, $v = -12$. Then $\frac{1}{2}(v + uy/x) = -$ 17°33'20", which is larger than 12. Multiply by $(s - s_0)/y$, i.e. by (7°00'45")/16, getting -2°26'06". Subtraction gives -15°07'14". Multiplying by $(s - s_0)/y$ yields -6°37'37". Adding to $-25\frac{1}{2}$° yields -32°07'37".

The distance of the planet from the earth is the distance TS multiplied by TM and divided by TC.

Āryabhaṭa built up a complete solar system. A yojana is 8000 times the height of a man. The circumference of the sky is ten times the number of minutes of arc in 57753336 revolutions, the number of revolutions made by the moon in 4320000 years. The length of the orbit of a planet is the circumference of the sky divided by the number of revolutions of the planet in 4320000 years.

Later Hindu astronomy was not substantially different from Āryabhaṭa's. The next people to tackle the motions of the planets were the Moslems: mostly Arabs, but also Persians, Uzbeks and others. Their theories were firmly based on Ptolemy's but with some important differences.

Around A.D. 1000 Ibn al-Haytham wrote a treatise whose title has been translated as *Doubts about Ptolemy*, objecting to motions that could not be produced by a combination of regular circular motions. (Motion produced by an equant is not regular).

The first model without an equant, by Mu'ayyad al-Dīn al-ʿUrḍī (who died in 1266), is shown in diagram 6. (the dashed lines are only for comparison with other models.) D is the centre of the deferent, whose radius is R. E is distant 2r from D and K is the midpoint of DE. L revolves round K in a circle of radius R, making one revolution in the longitudinal period of the planet. C revolves round L in a circle of radius r at the same rate relative to KL as L round K, so angle CLK is always equal to angle EKL. Then CE is parallel to LK. But LK revolves uniformly; therefore so does CE , and al-ʿUrḍī has produced uniform motion round E using only regular circular motions. If angle EKL is θ, the distance DC is $\sqrt{(R^2 + 4r^2\sin^2\theta)}$, so C is outside the deferent except at apogee A and perigee P. For Venus , r is 1/96 of R, so the greatest value of DC is 1.0002R: al-ʿUrḍī has very nearly produced motion of C round the deferent. This model, like the following two, and, indeed, like Ptolemy's, applies to four planets but not to Mercury.

Naṣīr al-Dīn al-Tūsī (who died in 1274) has M revolving round E at a distance R in the longitudinal period, as shown in diagram 7. N revolves round M at a distance r at the same rate relative to EM, so angle EMN is always equal to angle AEM. C revolves round N at a distance r at a rate that keeps NC parallel to AE. This ensures that C is on the line EM. In fact, it coincides with al-ʿUrḍī's C in diagram 6.

The linkage MNC, by which two circular motions combine to give motion in a straight line, has come to be known as a Tusi couple. It was used also in models for the moon and for Mercury. And it was used later by Copernicus.

Ibn al-Shāṭir (who died in 1375) had a point Q revolving round the earth T, which was distant 2r from D, in a circle of radius R in the longitudinal period, as shown in diagram 8. L revolves round Q at the same rate in the opposite direction (which keeps QL parallel to ET) at a distance 3r. This makes L coincide with al-ʿUrḍī's L in diagram 6. C revolves round L in the same way as in al-ʿUrḍī's model. The advantage of this, from the point of view of Arabic astronomers, is probably that it starts with Q revolving round a real point T, anchoring Q and the rest to reality. By contrast, the earlier models start with revolutions round imaginary points E and K; al-Shāṭir's model has been described as "without eccentrics".

This brings us to the start of modern investigations of the planets. Copernicus had them orbiting the sun. Kepler found their orbits to be ellipses. Newton found physical laws governing their motions. And now we have space-craft visiting Mars.

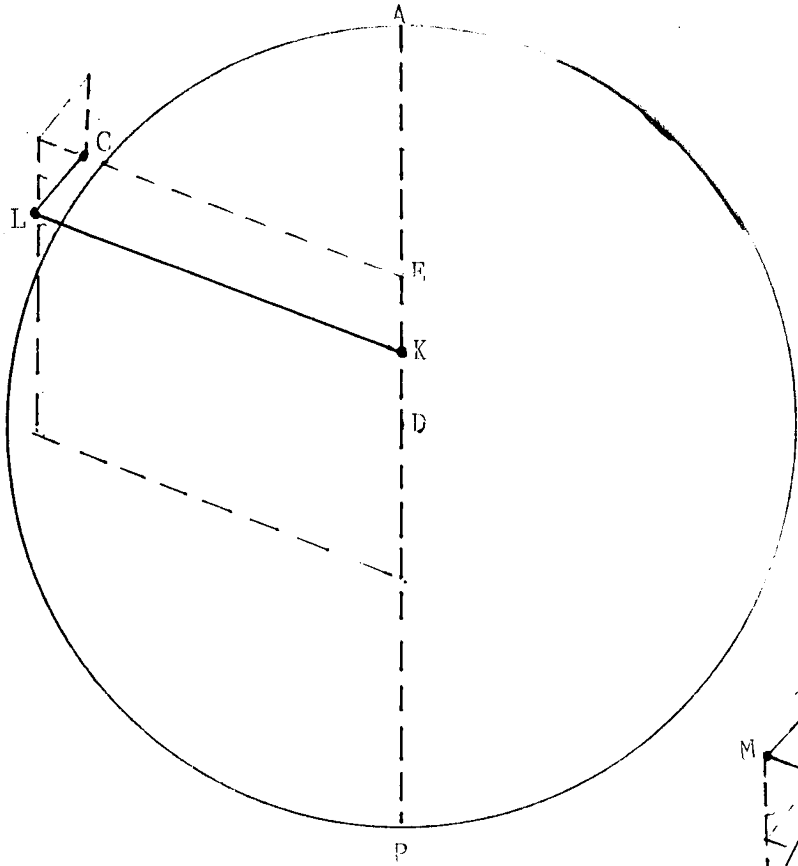
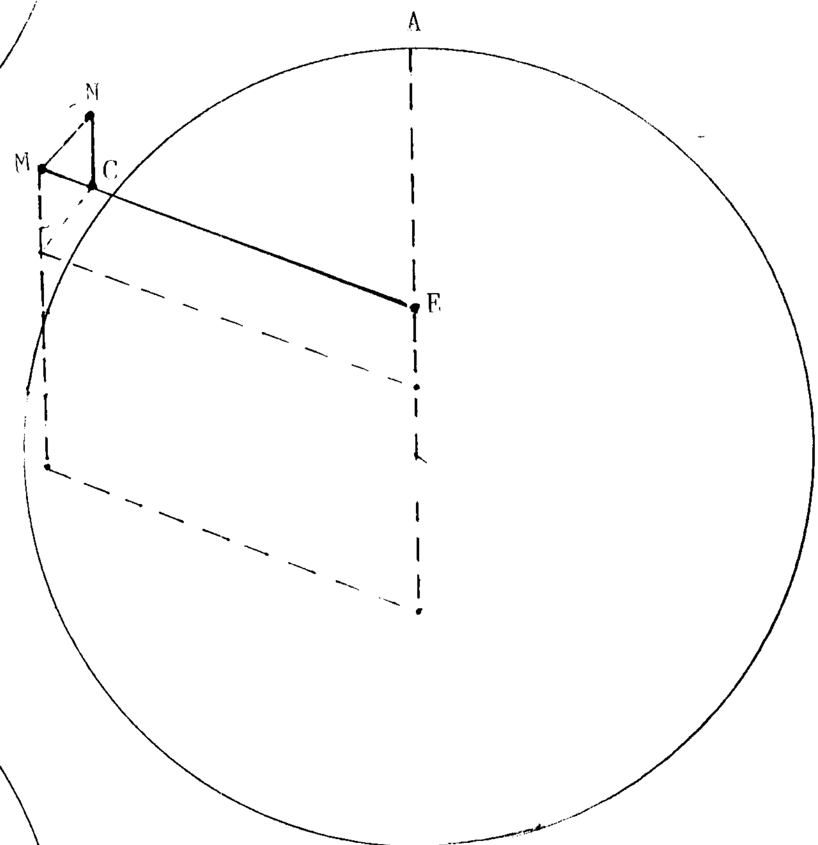


Diagram 6

Diagram 7



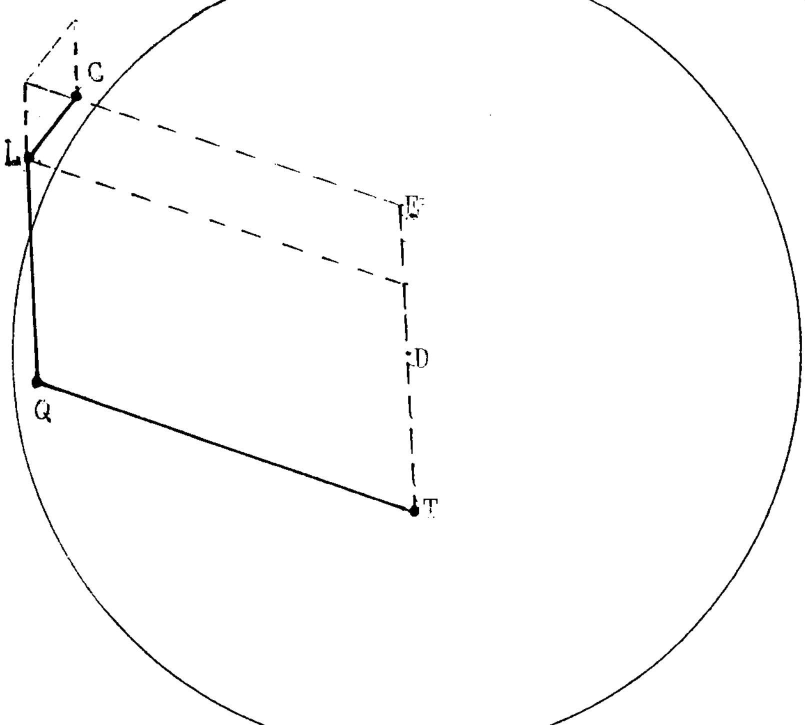


Diagram 8

Modern comments

Because of the inconsistencies and implausibilities in Ptolemy's treatment of the planets and because of the importance of the Syntaxis in the development of western astronomy, many writers have made criticisms and suggestions, the most extreme criticism being by R R Newton, who claimed that every observation made by Ptolemy was fabricated and that the longitudes for the planets were substantially less accurate than they should have been.

It is fairly obvious why Ptolemy's longitudes were inaccurate. Irregularity has two causes: the irregular motion of the planet and the irregular motion of the earth from which the longitudes are measured. Ptolemy made motion of the planet round the epicycle regular and tried to account for all the irregularity by irregular motion of the epicycle. For real accuracy the sun should have an equant, as Kepler discovered.

Dennis Duke investigated the apparent size of the epicycle of Mercury (diagram 9, from DIO volume 11.3) and found that it does not reach perigee twice in a revolution. Ptolemy's complication was not needed.

As explained earlier, it seems unlikely that Ptolemy found that D bisects TE for Venus. Perhaps he assumed it (because he had reason to believe it for the outer planets) and fudged his measurement to give this result.

James Evans suggested that the bisection could be deduced (most easily for Mars) by comparing the irregularity deduced from the sizes of the retrograde arcs with the irregularity deduced from their spacing.

Dennis Duke suggested that Ptolemy found TE as he described and found TD by using elongations at apogee and perigee (DIO 11.3, page 58). This would make TE about twice TD.

Dennis Rawlins, perhaps taking a hint from Ptolemy's use of successive approximation, starting with three oppositions, for the outer planets, suggested that Ptolemy could have used successive approximation for Venus, starting with three greatest elongations.

Sum of Greatest Elongations of Mercury 400 BC - 150 BC

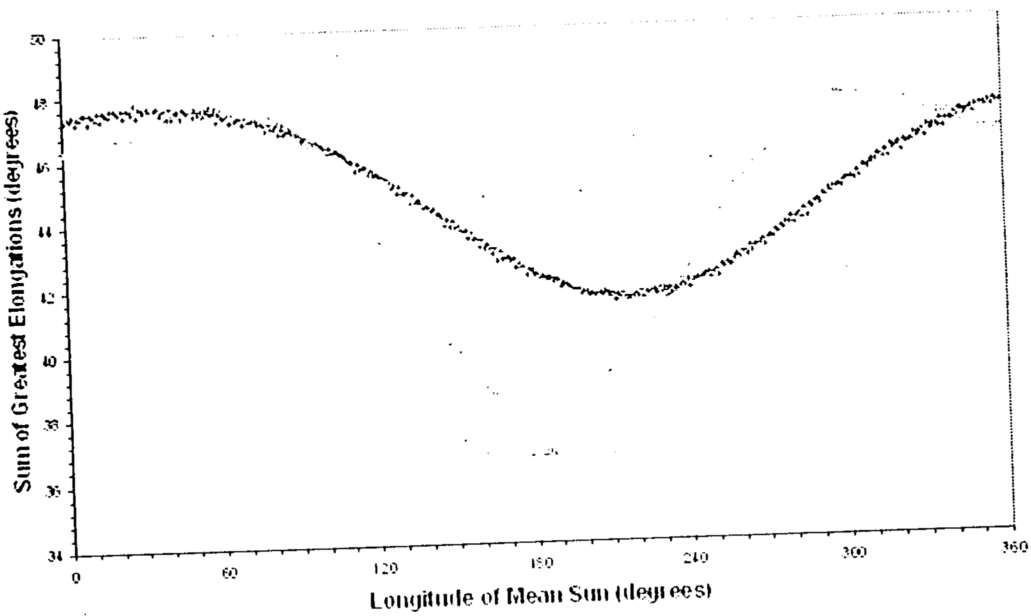


Diagram 9

In diagram 10, T is the earth, D is the centre of the deferent, C is the centre of the epicycle, V is Venus at greatest western elongation, DQ is the direction of zero longitude, DM is perpendicular to TV, TZ to DQ, ZN to TV, ZH to DM, and OF to CV. α is the angle between DC and DF, and β is the angle between DQ and TV.

Where a modern geometer would use trigonometrical functions, an ancient Greek used a table of chords: chdθ is the length of a chord that subtends an angle θ at the centre of a circle of diameter 120.

Angle ZTN = $90° - \beta$. ZN subtends an angle 2ZTN at the centre of the circumcircle of the triangle ZTN, whose diameter is ZT, so ZN = (ZT/120)chd2ZTN = (ZT/120)chd($180° - 2\beta$). Similarly, from the triangle HZD, HD = (DZ/120)chd2HZD = (DZ/120)chd2β. From the triangle CDF, in which CD = 60, CF = $\frac{1}{2}$chd2CDF = $\frac{1}{2}$chd2α. Then CV = CF + FV = CF + DM = CF + HM − HD = CF + ZN − HD

$$= \tfrac{1}{2}\mathrm{chd}2\alpha + (ZT/120)\mathrm{chd}(180° - 2\beta) - (DZ/120)\mathrm{chd}2\beta.$$

Given three greatest western elongations, if we knew the three angles α and the three angles β, we would have three equations between CV, ZT, and DT, which can be solved, giving the radius of the epicycle, the distance TD, and the longitude of D.

We know each β: it is the observed longitude of Venus. We do not know α, but because TD is small compared witn DC, it is close to the angle between EC and EF and we can calculate this from the longitude of the sun and the time of the observation. So we use successive approximation.

Step 1. Take the angle between EC and EF as a first approximation to α and calculate a first approximation to the geometric parameters.

Step 2. From this approximation, calculate the angles α; this will be a second approximation.

Step 3. From this second approximation, calculate a second approximation to the geometrical parameters.

And so on.

This suggestion is an effective rejoinder to apologists for Ptolemy who maintain that he was forced to use the (fraudulent)

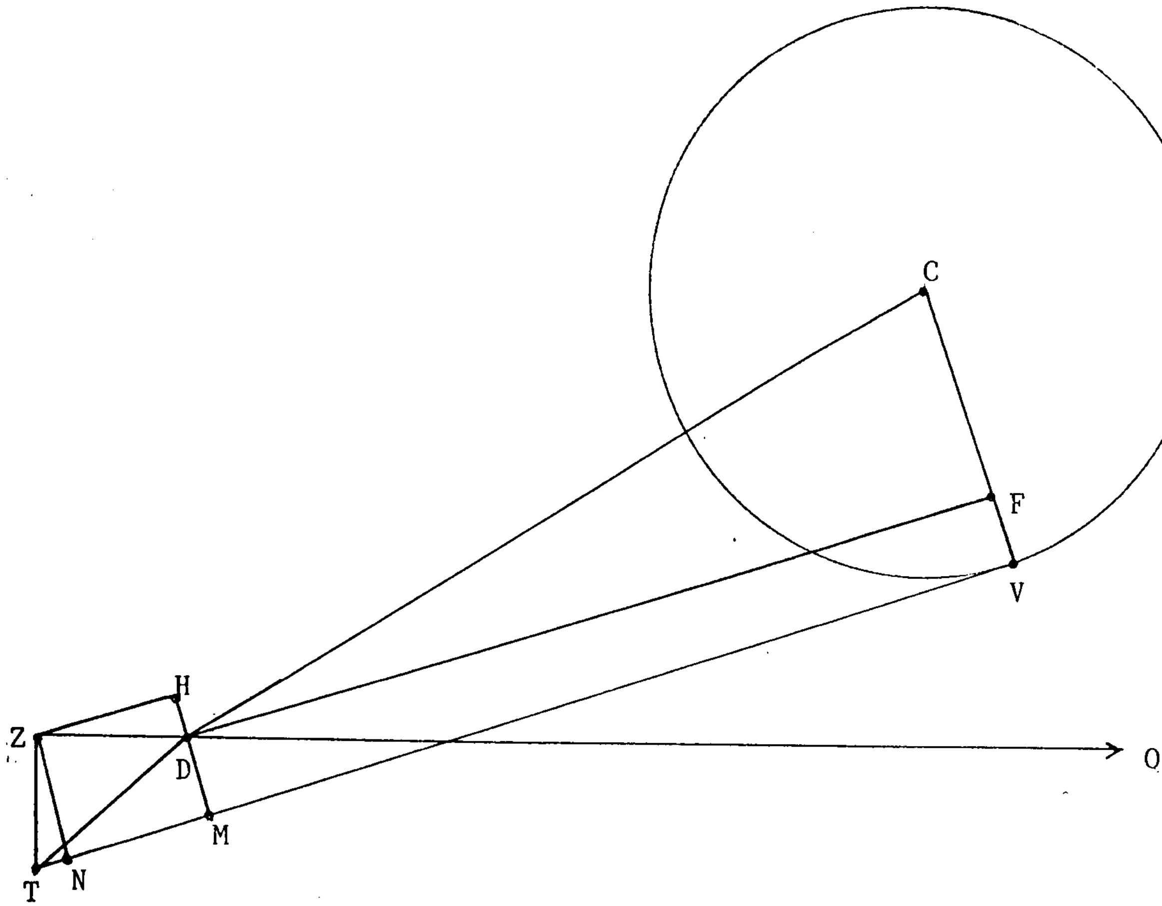


Diagram 10

T: earth

D: centre of the deferent

C: centre of the epicycle

V: Venus

DQ: direction of zero longitude

method that he described (for example Owen Gingerich, *Isis*, volume 93, number 1 (2002) page 71: Ptolemy demonstrated his ingenuity when orbital constraints made it impossible to observe the preferred configurations).

Pliny (*Natural history* 2.6.38) attributed to Timaeus a value of 46° for the greatest elongation of Venus. If Ptolemy knew this and took it to be the value when the epicycle is at its average distance, 60, from the earth, this would make the radius of the epicycle $43\frac{1}{6}$, the value that Ptolemy cited. Similarly, Pliny attributed to Sosigenes the value 27° for Mercury, which would again yield Ptolemy's result.

There are other ways to find the parameters for an outer planet. From the sidereal period and any one timed opposition we can calculate the position of C at any given time. We find the greatest and least longitudes of the planet for a fixed position of C; their difference gives the apparent size of the epicycle. The position of C when this is least gives the longitude of E. (James Evans suggested using the lengths of the arcs covered during retrogression for this purpose.)

Once we have found the longitude of E a timed opposition gives us the direction of C from E and the observed longitude gives the direction of C from T, so we know all the angles of the triangle CET. If we assume that D bisects ET and DC = 60, an easy calculation gives us DT.