The Equant in India: the Mathematical Basis of Ancient Indian Planetary Models

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Introduction

The planetary models of ancient Indian mathematical astronomy are described in several texts.[1] These texts invariably give algorithms for computing mean and true longitudes of the planets, but are completely devoid of any material that would inform us of the origin of the models. One way to approach the problem is to compare the predictions of the Indian models with the predictions from other models that do have, at least in part, a known historical background. Since the Indian models compute true longitudes by adding corrections to mean longitudes, the obvious choices for these latter models are those from the Greco-Roman world. In order to investigate if there is any connection between Greek and Indian models, we should therefore focus on the oldest Indian texts that contain fully described, and therefore securely computable, models. We shall see that the mathematical basis of the Indian models is the equant model found in the *Almagest*, and furthermore, that analysis of the level of development of Indian astronomy contemporary to their planetary schemes strongly suggests, but does not rigorously prove, that the planetary bisected equant model is pre-Ptolemaic.

The Indian models

The earliest fully described Indian planetary models are two sets from the writer Aryabhata, both of which probably date from the early 6th century A.D. One model, called the sunrise system after its epoch date of sunrise on 18 February –3101, is conventionally attributed to a line of development called the Aryapaksa, and first appears in Aryabhata's *Aryabhatiya*, although our complete understanding of the model is not established until the commentaries of Bhaskara, which date from about 628.[2] The second, called the midnight system after its epoch date of midnight on 17/18 February –3101, is conventionally attributed to a line of development called the Ardharatrikapaksa, and first appears for us in Latadeva's *Suryasiddhanta* included in Varahamahira's *Pancasiddhantika*, which appeared in the second half of the 6th century, and then later in more detail in Brahmagupta's commentary of 665.[3] Both systems set their local meridian to Lanka, longitude about 76° east, latitude 0°.

For the five planets, the models recognize both the zodiacal and solar anomalies. The zodiacal, or manda, correction is computed using

$$q(\alpha) = \sin^{-1}(-2e\sin\alpha)\,.$$

Here $\alpha = \bar{\lambda}_P - \lambda_A$, $\bar{\lambda}_P$ is the mean longitude of the planet, λ_A is the longitude of the apogee, and the anomaly scale is given as $2e$ in anticipation of the equant. The solar, or sighra, correction is computed from

$$p(\gamma) = \tan^{-1}\left(\frac{r\sin\gamma}{1 + r\cos\gamma}\right),$$

where r gives the scale of the correction, and γ is the argument of the sighra anomaly. For the outer planets, the Indian models specify directly the mean motions in sidereal longitude of the planet, and like the Greek models, the longitude of the epicycle radius with respect to Aries 0° is the mean longitude of the Sun. Thus if $\bar{\lambda}_P$ is the mean longitude of the planet and $\bar{\lambda}_S$ is the mean longitude of the sun, then for an outer planet the sighra argument is

$$\gamma = \bar{\lambda}_S - \bar{\lambda}_P .$$

For the inner planets, the sighra argument uses not the mean longitude of the planet, which would be just the mean longitude of the Sun, but instead the absolute longitude λ'_P of the sighra epicycle radius,[4] so we have

$$\gamma = \lambda'_P - \bar{\lambda}_S ,$$

while the manda argument is given by

$$\alpha = \bar{\lambda}_S - \lambda_A .$$

The sunrise system has an additional feature, pulsating correction scales $2e$ and r, but for the purposes of this paper these pulsations are irrelevant.

The algorithm for combining the manda and sighra corrections to compute the true longitude is a bit intricate, and slightly different in the two systems. In the following, for an outer planet let $M = \bar{\lambda}$ and $S = \bar{\lambda}_S$, while for an inner planet $M = \bar{\lambda}_S$ and $S = \lambda'_P$. In the midnight system, the steps in the algorithm are:

(1) with sighra argument $\gamma = S - M$ and longitude of apogee λ_A compute $\nu_1 = \lambda_A - \frac{1}{2}p(\gamma)$.

(2) with manda argument $\alpha = M - \nu_1$ compute $\nu_2 = \nu_1 - \frac{1}{2}q(\alpha)$.

(3) with manda argument $\alpha = M - \nu_2$ compute $\nu_3 = M + q(\alpha)$.

(4) with sighra argument $\gamma = S - \nu_3$ compute the true longitude $\lambda = \nu_3 + p(\gamma)$.

Notice that in the first two steps the corrections are scaled by a factor ½ and are applied with opposite their usual signs.

In the sunrise system, for an outer planet:

(1) with manda argument $\alpha = M - \lambda_A$ compute $\nu_1 = M + \frac{1}{2}q(\alpha)$.

(2) with sighra argument $\gamma = S - \nu_1$ compute $\nu_2 = \nu_1 + \frac{1}{2}p(\gamma)$.

(3) with manda argument $\alpha = \nu_2 - \lambda_A$ compute $\nu_3 = M + q(\alpha)$.

(4) with sighra argument $\gamma = S - \nu_3$ compute the true longitude $\lambda = \nu_3 + p(\gamma)$.

For an inner planet one omits the initial manda correction, and the algorithm is:

(1) with sighra argument $\gamma = S - M$ compute $\nu_1 = \lambda_A - \frac{1}{2}p(\gamma)$.

(2) with manda argument $\alpha = \nu_1 - \lambda_A$ compute $\nu_2 = M + q(\alpha)$.

(3) with sighra argument $\gamma = S - \nu_2$ compute the true longitude $\lambda = \nu_2 + p(\gamma)$.

It is clear that while the sunrise and moonrise models are not mathematically equivalent, they are nevertheless closely related. Indeed, Aryabhata appears to have drawn heavily on sources from an even earlier school, the Brahmapaksa, first documented for us in the very imperfectly preserved *Paitamahasiddhanta*,[5] believed to originate no earlier than the 5th century A. D., and later, although likely with significant changes to the models, in Brahmagupta's *Brahmasphutasiddanta*, which was written in 628.

The numerical values of the parameters assigned to the scales of the manda and sighra corrections make it clear that they are analogous to the eccentric and epicycle corrections found in Greek models. For example, for Jupiter the *Almagest* gives $2e$ = 5;30 and r = 11;30, while the midnight system gives $2e$ = 5;20 and r = 12 (in conventional units with R = 60). Indeed, since the very earliest investigation of the Indian models by Western scholars it has been presumed that the models are somehow related to a double epicycle system, with one epicycle accounting for the zodiacal anomaly, and the other accounting for the solar anomaly (retrograde motion).[6] This perception was no doubt reinforced by the tendency of some Indian texts to associate the manda and sighra corrections with an even older Indian tradition of some sort of forceful cords of air tugging at the planet and causing it to move along a concentric deferent.

Since our goal in this paper is to investigate the nature of any connection with ancient Greek planetary models, it is only important to accept that the models appear in Indian texts that clearly pre-date any possible Islamic influences, which could, at least in principle, have introduced astronomical elements that Islamic astronomers might have derived from Greek sources.

Comparison of Indian and Greek Planetary Models

In 1956 Neugebauer presented schematic arguments supporting the idea that the multi-step Indian algorithms were approximating an underlying Greek geometrical model, which he thought was an eccentre plus epicycle model.[7] In 1961 van der Waerden showed, on the basis of the first few terms in power series expansions in e/R and r/R, that the equant model and a model very similar to the sunrise model (the difference being that the order of the first two steps is interchanged) are closely related, and in particular that the factor of ½ used in the initial steps of the Indian models is directly related to the bisection of the equant.[8] However, van der Waerden published no numerical comparisons of the Indian models and the bisected equant, and so it was perhaps not entirely clear whether or not his conclusion was solid. Regardless of the reasons, van der Waerden's result appears to have been systematically ignored in the principal Indian[9] and western literature.[10] Indeed, apart from a few references in van der Waerden's own later papers, the only references to his paper that I can find are two papers which primarily mention the paper's existence,[11,12] and a paper that usefully discusses van der Waerden's power series argument and points out, correctly, that it is not very accurate for Venus.[13]

As mentioned earlier, the parameters for the *Almagest* and Indian models are closely related but not identical, and if one compares the predictions of the *Almagest* model for, say, Jupiter, with the predictions of the sunrise model, using the given parameters for both, then as shown in Figure 1 the *Almagest* model is a superior predictor of Jupiter's positions.[14]

[Figure 1 goes here]

Indeed, since the models are mathematically different, there is no reason to expect that the optimum parameter values should be the same. However, if the apparent mathematical difference is only masking a deeper relationship, then the relevant question to ask is how the models compare when using the *same* parameter values.

Figure 2 shows the differences between the longitude of Jupiter computed using modern theory and the predictions of (a) the sunrise theory, (b) a Greek eccentre plus epicycle theory, and (c) the *Almagest* equant, this time using the *Almagest* parameters for each ancient theory.[15]

[Figure 2 goes here]

Figure 2 shows that the sunrise theory and the *Almagest* equant are, for the *Almagest* parameters of Jupiter, effectively the same, and indeed, as shown in Figure 3, the maximum difference of the two theories never exceeds 0.042° (the midnight theory is virtually identical with both and so is not shown).

[Figure 3 goes here]

For Saturn the agreement is even closer (see Figure 4),

[Figure 4 goes here]

but for Mars the agreement is not as good, as shown in Figure 5.

[Figure 5 goes here]

The close numerical agreement between the *Almagest* equant and the Indian models, when evaluated using the exactly the same parameters, is quite striking,[16] and obviously requires an explanation.

The History of the Equant

The first question is to ask is why do the *Almagest* equant and the Indian models agree so well? Since both models clearly attempt to explain both the zodiacal and solar anomalies in planetary motion, it seems extremely likely that both models were originally intending to solve the same astronomical problem. We can then usefully distinguish two possibilities:

(1) the equant and the Indian models are versions of the *same* solution to the same astronomical problem, or
(2) the equant and the Indian models are *independent* efforts to solve the same astronomical problem.

The principal evidence suggesting that option (1) is the most plausible option is that, as shown above, for small to moderate values of the parameters e/R and r/R the equant and Indian models are numerically indistinguishable. Furthermore, it is shown in the appendix that one can, by means of a sequence of well-defined approximations, in fact analytically derive the sunrise theory from the equant. While the derivation presented there is likely to differ from the method used originally, what is important is that it is possible to establish a precise mathematical relationship between the two theories, and that relationship allows us to understand exactly why the approximation is very close for Jupiter and Saturn and not nearly so close for Mars. Also, it is clear from Figures 2, 4, and 5 that the sunrise system is much closer to the equant than to the eccentre plus epicycle model, thus establishing beyond any doubt that the Indian schemes are *not* approximations to the eccentre plus epicycle model.

In addition, the derivation very strongly suggests that the sunrise model is derived from the equant, and not, for example, invented by some sort of iterative tinkering, starting from, say, an eccentre plus epicycle model and trying to create a tinkered version that (a) decouples the anomalies for computational convenience, and (b) corrects its deficiencies. The reason is that a theory arrived at by such tinkering need not be analytically derivable starting from the equant, a property that is demonstrably true for the sunrise theory.

Option (2) basically requires that some inventor first found a theory other than, and independent of, the equant that explains the data, and then for computational convenience derived what we now know as the Indian models as an approximation to this alternate theory. It is easy for us to imagine candidate theories: perhaps some variant of Kepler motion, or one of the approximations to the equant that respect uniform motion invented by Islamic astronomers. What is difficult to establish is that any such theory is historically plausible.

So compared to the consequences of option (2), it seems far more plausible to simply assume option (1), that the equant and Indian models are versions of a single solution to the same astronomical problem, and that it is furthermore very likely that of the two models, the equant came first and the Indian models are an approximation to it. The discussion in the Appendix shows how the Indian models might have been derived, starting from the equant, and while we can only admire the ingenuity of whoever did it first, we can be sure of the motivation – trading numerical accuracy for ease of computation by decoupling the zodiacal and solar anomalies. The same tradeoff is made in the *Almagest*, and the tabular interpolation scheme described by Ptolemy manages to maintain more accuracy at the cost of significantly more bother in computation.

The crucial question now becomes, which came first – the *Almagest*, or some other unknown Greek text that was the ultimate source of the *Paitamahasiddhanta* and hence Aryabhata's models? Unfortunately, our documented knowledge of Greek planetary theory prior to Ptolemy is quite limited. We have Ptolemy's remarks in *Almagest* 9.2 that Hipparchus had shown that the planetary observations available to him were not consistent with the models of the astronomers at that time, and Ptolemy's further comment that the planetary models developed in the post-Hipparchan years "…by means of eccentric circles or by circles concentric with the ecliptic, and carrying epicycles, or even by combining both….for these representations have been employed by almost all those who tried to exhibit the uniform circular motion by means of the so-called 'Aeon–tables', but their attempts were faulty and at the same time lacked proofs; some of them did not achieve their object at all, the others only to a limited extent".[17] Essentially the only other primary source materials for planetary models are the remarks by Pliny and those in *PMich.* 149, both of which indicate that models were developed that attempted to explain both the zodiacal and solar anomalies, but do not make any recognizable reference to the equant.[18]

This leaves trying to date the equant in astronomical history in terms of the other elements of mathematical astronomy present in India at the time as the equant. The fact that no Indian text gives the slightest hint regarding the origin of their models or the empirical basis of their model parameters is not helpful, but except for the *Almagest*, the same is largely true of ancient western astronomical texts.[19] Thus we must take a more circumstantial approach, reasoning under the conventional Western sight that while the astronomy of the Indian texts is thought to be based largely on material imported from Greco-Roman sources in the 2nd through 5th centuries,[20] it is impossible not to notice that all the sophisticated elements of mathematical astronomy included in Ptolemy's *Almagest* are completely missing in any known Indian astronomy until late in the first millennium. Examples include

- *The equation of time*. Throughout the first millennium the Indians used an abbreviated version which includes only the effect of the zodiacal anomaly of the Sun, and neglects the effect of the obliquity of the ecliptic.
- *Obliquity of the ecliptic*. When used in spherical trigonometry, the Indians use either 24° or 23;40°, both associated with Hipparchus, but never the Eratosthenes/*Almagest* value 23;51,20°.
- *The second lunar anomaly*. The Indians did not discuss evection until the beginning of the second millennium, and then in a form different from that used by Ptolemy.
- *Accurate discussions of parallax*. The Indians were aware of parallax and used it for computing eclipses, but always used various approximations.
- *Decoupling the anomalies*. The scheme used by the Indians works well for moderate eccentricities and epicycle sizes, such as those appropriate to Jupiter, but as we have noticed it breaks down for larger values, such as those appropriate to Mars. Presumably they would have used Ptolemy's more accurate tabular interpolation scheme if they had known about it.

- *Trigonometry scales*. The Indians used a variety of values for the radius of the reference circle, and mostly the value $R = 3438$ in the earliest texts. This is a value used by Hipparchus but apparently abandoned by the time of Ptolemy.
- *Retrograde motion*. When mentioned at all in connection with the multi-step models we are discussing, the Indians quoted specific values of the sighra anomaly that correspond to first and second station. There is no mention of the variation in the size of retrograde arcs with zodiacal position.
- *Model of Mercury*. Unlike Ptolemy, who used a complicated crank mechanism to generate a pair of perigees for Mercury, the Indians used the same model for Mercury and Venus, which is also often the same or closely related to the model used for the outer planets. The basis of all of these models is the equant.
- *Determination of orbit elements*. While the bulk of the *Almagest* is devoted to explaining how to determine orbit elements from empirical data, it is not at all obvious that any comparable derivation is even possible in the context of the Indian approximation schemes.
- *Values of orbit elements*. The values used in the Indian schemes for e, r, and A are generally different from the values found in the *Almagest*. Except for Mercury, the resulting Indian model predictions for true longitudes are generally inferior to those in the *Almagest*.
- *Star catalog*. The Indian coordinates for star positions are generally inaccurate, and bear no relation to those found in the *Almagest* star catalog.
- *Zodiacal signs*. The Indian texts routinely divide circles such as epicycles into 30° segments and refer to them in terms of the zodiacal signs. The only other known use of this practice is in Hipparchus' similar description of circles of constant latitude in the *Commentary to Aratus*.

The list above is no doubt incomplete, but is already long enough to illustrate the point that the astronomy surrounding the equant in India is generally far less developed than that found in the *Almagest*. This has led to an essentially universally accepted (except among some Indian scholars, see ref. 20) view that the astronomy we find in the Indian texts is pre-Ptolemaic.

How do we reconcile the appearance of planetary theories in India clearly derived from the equant with this mass of circumstantial evidence? Once again we have two principal options. Either

(1) the Greek source that introduced the approximation of the equant into India was influenced by Ptolemy and the *Almagest*, or
(2) the Greek source pre-dates the *Almagest*, so there was no such influence, and hence Ptolemy did not invent the equant.

The circumstantial evidence listed above strongly supports the long prevailing view that the Greek sources that are the basis of the Indian texts are definitely pre-Ptolemaic, and there is no compelling reason to overturn that conclusion based only upon the fact that the equant must pre-date the *Almagest*. In this case, we have only to conclude that Ptolemy did not invent the equant.

The simplest alternative would be that while all or most of Indian astronomy is based on Greek sources that pre-date Ptolemy, somehow the planetary theory that got to India was based on an approximation to Ptolemy's equant that was developed after the publication of the *Almagest*. The more radical alternative is that the bulk of the Greek material that reached India is post-Ptolemy.

In either case, however, there must be a significantly different relationship between the Indian models and ancient Greek astronomy than the one universally accepted until now.

Acknowledgement
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Appendix

It is interesting to consider how the Indian schemes might have been derived from the bisected equant. The discussion below generally follows van der Waerden, but adds a number of clarifying steps.[21] Along the way we will need to quantify the degree of agreement between the models at various stages of approximation. To this end imagine a table of 46 rows and 46 columns. To each row corresponds a value of $\alpha = \bar{\lambda} - \lambda_A$ (in degrees) from the sequence 0,6,12,…,84,90,93,96,…,180, and to each column corresponds a value of $\bar{\gamma} = \lambda_S - \bar{\lambda}$ from the same sequence. Then to compare two models *A* and *B* we compute

$$(\lambda - \bar{\lambda})_A - (\lambda - \bar{\lambda})_B$$

for each cell in the table and as a figure of merit quote the 'cpair' (MAE,XAE), where MAE and XAE are the mean and maximum of the absolute values of the table entries (in degrees). For example, using $2e = 6/60$ and $r = 12/60$ and comparing the equant and the sunrise schemes, one finds the cpair (0.015, 0.042), while comparing an eccentric model with the sunrise scheme gives (0.189, 0.745).

[Figure 6 goes here]

The task facing the ancient analyst was to efficiently compute the longitude of a planet using the equant model. In Figure 6, ED = ET = e, DC = 1, and CP = r. Given the angles α and $\bar{\gamma}$, one strategy begins as follows:

(1) compute $\alpha_1 = \alpha + q_1 = \alpha - e\sin\alpha$

(2) compute $q \simeq -2e\sin\alpha_1$

The value of q computed in step (2) is indeed an approximation, but comparing the correctly computed equant with one using this strategy gives the cpair (0.003, 0.006), thus no larger than the errors caused by the trig tables of the time.

The problem for the analyst occurs in the next step:

(3) compute $p = \tan^{-1}\left(\dfrac{r\sin\gamma}{\rho(e,r) + r\cos\gamma}\right)$

(4) compute $\lambda - \bar{\lambda} = q + p$

In step (3), $\rho(e,r)$ = TC is a function of both variables e and r, and that is the obstacle to efficient computation of the equant.

One way to overcome that obstacle is to consider a point Z on the line CT (or its extension) that is unit distance from point C. Then to first order in e we have

$$CT = CZ + ZT = 1 + e\cos\alpha_1 \tag{A.1}$$

and we can now compute the angle $p' = CZP$ as a function only of r:

$$p' = \tan^{-1}\left(\frac{r\sin\gamma}{1 + r\cos\gamma}\right).$$

It remains to compute the angle $\delta = ZPT$ and use that to compute $p = p' - \delta$. It is clear that δ will under all circumstances be a small angle, and we will incur no noticeable error by approximating $\sin\delta \simeq \delta$. Then using the law of sines on triangle PZT we get

$$\delta = \frac{e\sin p'\cos\alpha_1}{PT} \simeq \frac{e\sin p'\cos\alpha_1}{1 + r\cos\gamma}.$$

Comparing the correctly computed equant with the approximation

$$\lambda = \bar{\lambda} - 2e\sin\alpha_1 + p' - \delta \tag{A.2}$$

gives the cpair (0.008, 0.041), so this is indeed a very good approximation to the equant that effectively decouples the two anomalies and enables efficient computation.

The sunrise scheme is, however, even easier to compute, and almost as good an approximation, since its cpair is (0.015, 0.042). We can see the analytic connection by evaluating the scheme as follows:

(1) compute $\alpha_1 = \alpha + \dfrac{-2e\sin\alpha}{2} = \alpha + q_1$

(2) compute $\mu_2 = \alpha_1 + \dfrac{p(\bar{\gamma} - q_1)}{2} = \alpha_1 + \dfrac{1}{2}p_1$

(3) compute

$$\begin{aligned}\mu_3 &= \alpha - 2e\sin(\alpha_1 + \frac{1}{2}p_1) \\ &= \alpha - 2e\sin\alpha_1\cos\frac{p_1}{2} - 2e\cos\alpha_1\sin\frac{p_1}{2} \\ &\simeq \alpha - 2e\sin\alpha_1 - e\cos\alpha_1\sin p_1 \\ &= \alpha + q - \delta'\end{aligned}$$

(4) compute $\lambda = \bar{\lambda} + q + p(\bar{\gamma} + 2e\sin\alpha_1 + e\cos\alpha_1\sin p_1) - \delta'$. (A.3)

In steps (2) and (4) we have, of course, the usual sighra convention, $p(\gamma) = \tan^{-1}\left(\frac{r\sin\gamma}{1 + r\cos\gamma}\right)$, and in step (3) we have $\delta' = \delta(1 + r\cos(\bar{\gamma} - q_1))$. To complete the comparison, at least from our modern perspective, it is straightforward, if tedious, to show that $p' - \delta$ in equation (A.2) is a very good approximation to $p - \delta'$ in equation (A.3): their cpair is (0.010, 0.025).

If indeed an ancient analyst followed this path from the equant to the sunrise scheme, then the important steps seem to be

(1) the idea to divide the line CT into a unit line and a remainder of approximate length $e\cos\alpha_1$,

(2) realization and use of the fact that the angle $\delta = p - p'$ is always small,

(3) realization that the term $e\cos\alpha_1\sin p_1$ can be combined with the term $2e\sin\alpha_1$ to give $2e\sin(\alpha_1 + \frac{p_1}{2})$, which is constructed in step (2) of the sunrise scheme.

The original derivation by the ancient analyst would no doubt have included a large amount of trial and error, numerical checks of the approximations at the intermediate steps, the geometric reasoning would probably have proceeded by decomposing all the triangles into sums of right triangles, and our sines and cosines would be specific sides of right triangles, but it seems plausible that an expert analyst – certainly one at the level of Archimedes or Apollonius – could have constructed the sunrise scheme along the lines suggested above.

Figure Captions

Figure 1. The difference between the longitudes of Jupiter predicted by modern theory and those predicted by the equant model (solid line) using the *Almagest* parameters and the sunrise model (dotted line) using Aryabhata's parameters.

Figure 2. The difference between the longitudes of Jupiter predicted by modern theory and those predicted by the equant model (open circles), the eccentre plus epicycle model (solid circles), and the sunrise model (solid line), using the *Almagest* parameters for all three ancient models. The equant and sunrise models are very nearly coincident in the figure, while the eccentre model differs significantly from both.

Figure 3. The difference between the equant and sunrise models using identical *Almagest* parameters for Jupiter.

Figure 4. As in Figure 2 except for Saturn.

Figure 5. As in Figure 2 except for Mars. Note that for Mars the sunrise model now differs noticeably from the equant, but is an even poorer match to the eccentre.

Figure 6. The geometry of the sequence of approximations used to derive the sunrise model from the equant.

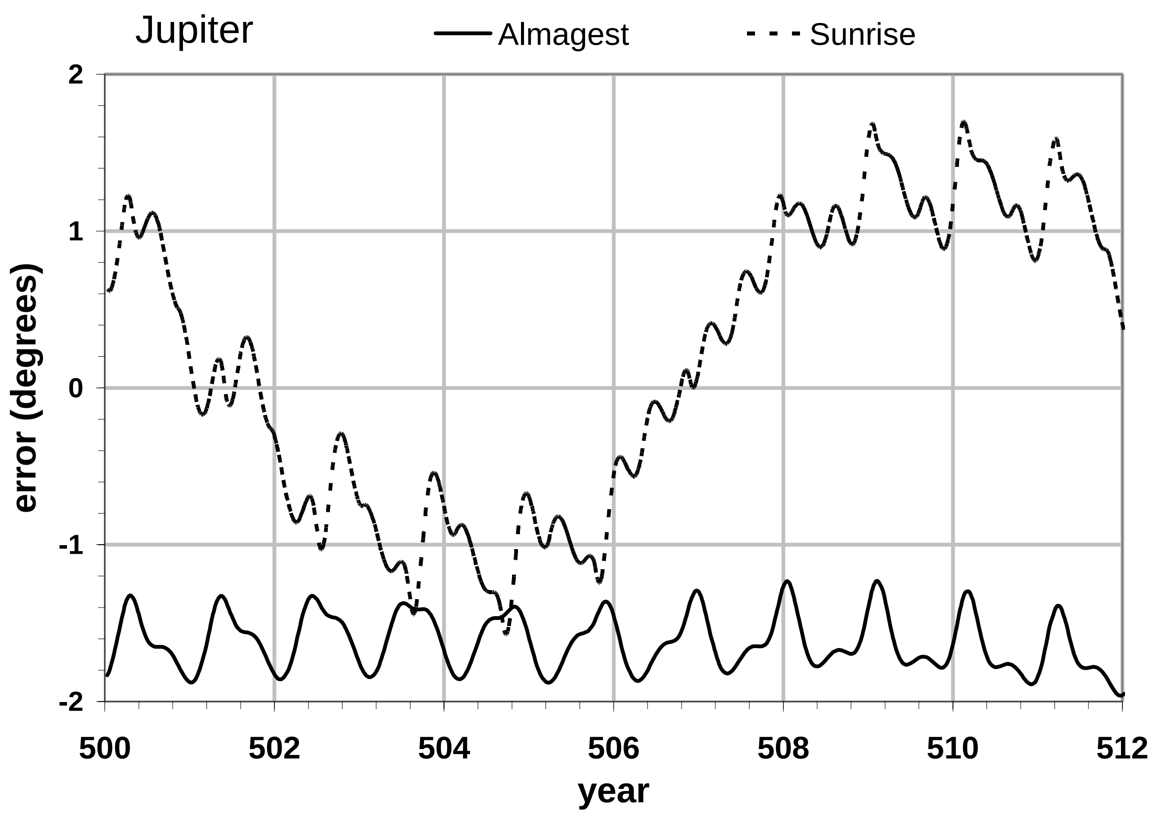

Jupiter
Almagest
Sunrise
error (degrees)
2
1
0
-1
-2
500
502
504
506
508
510
512
year

Fig 1

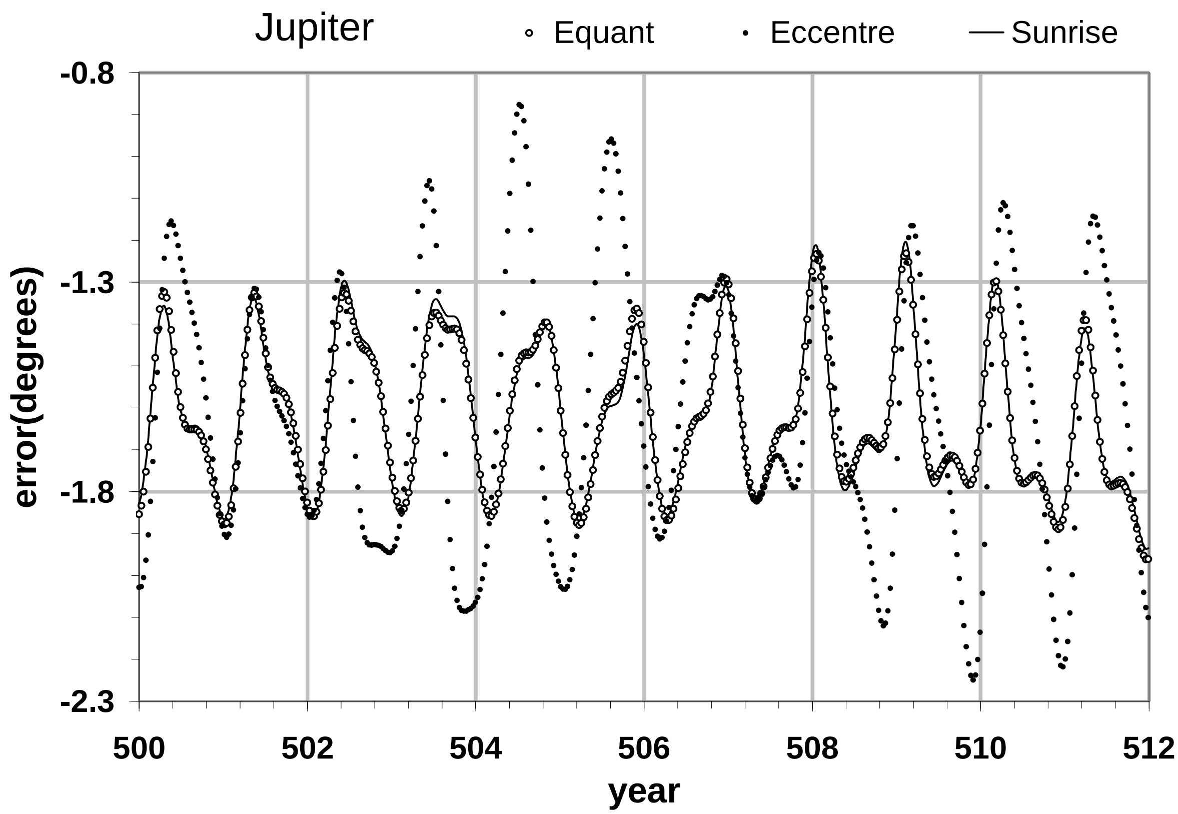

Jupiter
Equant
Eccentre
Sunrise
-0.8
-1.3
-1.8
-2.3
error(degrees)
500
502
504
506
508
510
512
year

Fig 2

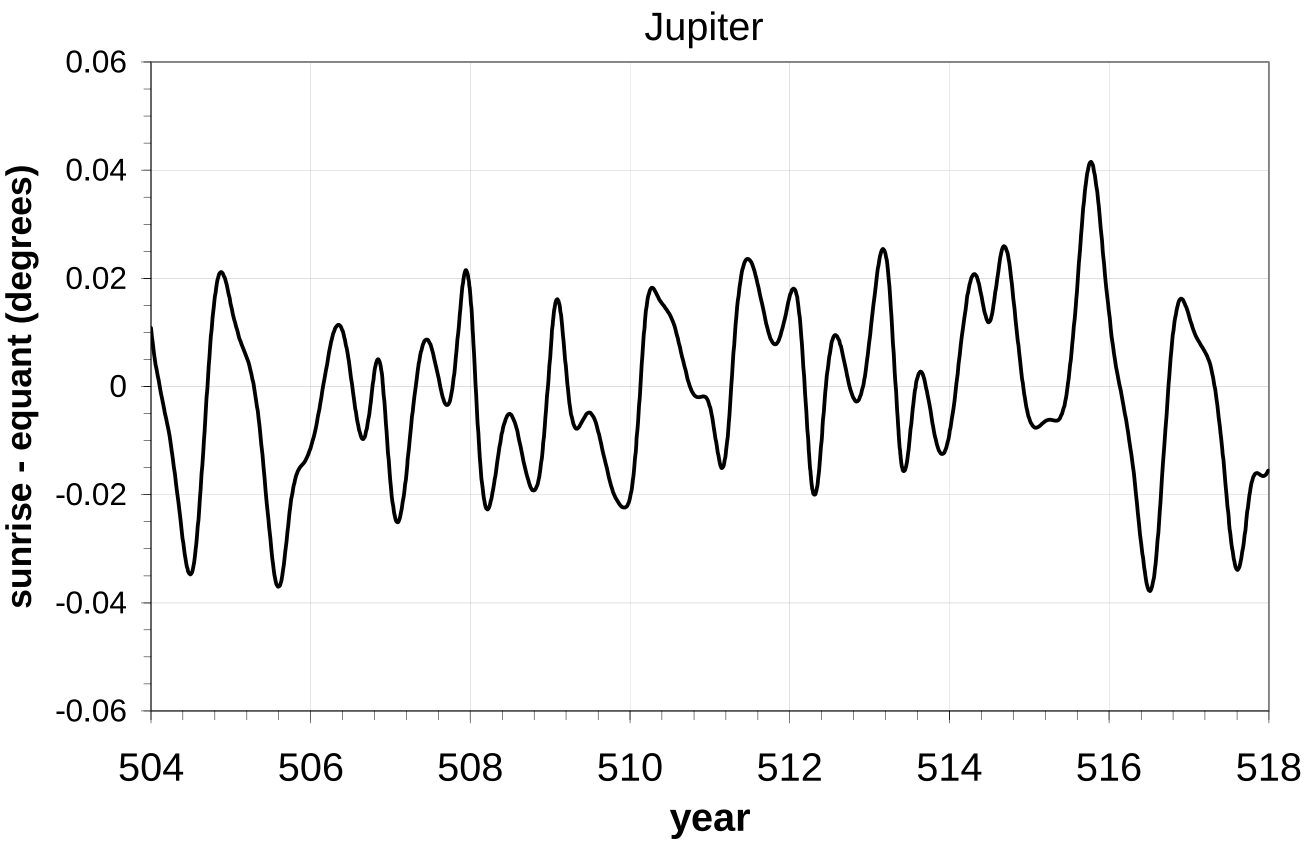

Jupiter
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
sunrise - equant (degrees)
504
506
508
510
512
514
516
518
year

Fig 3

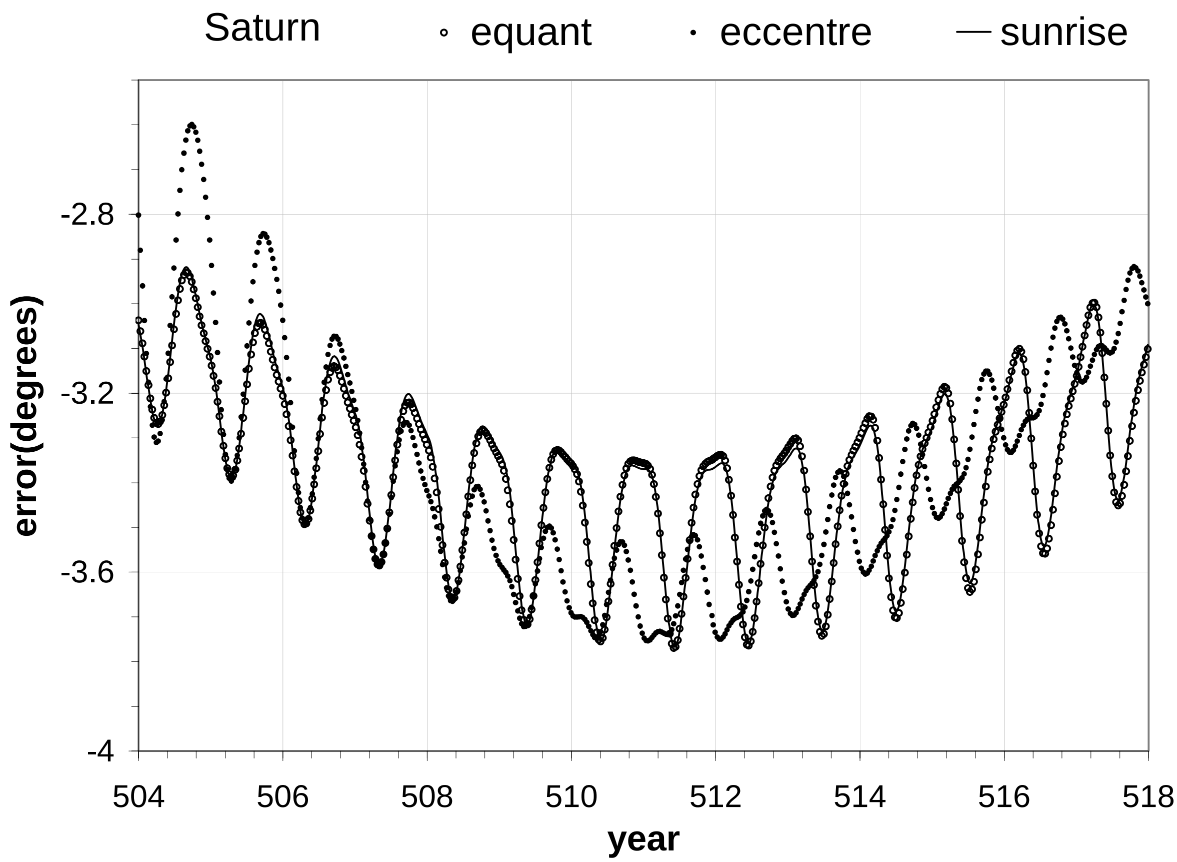
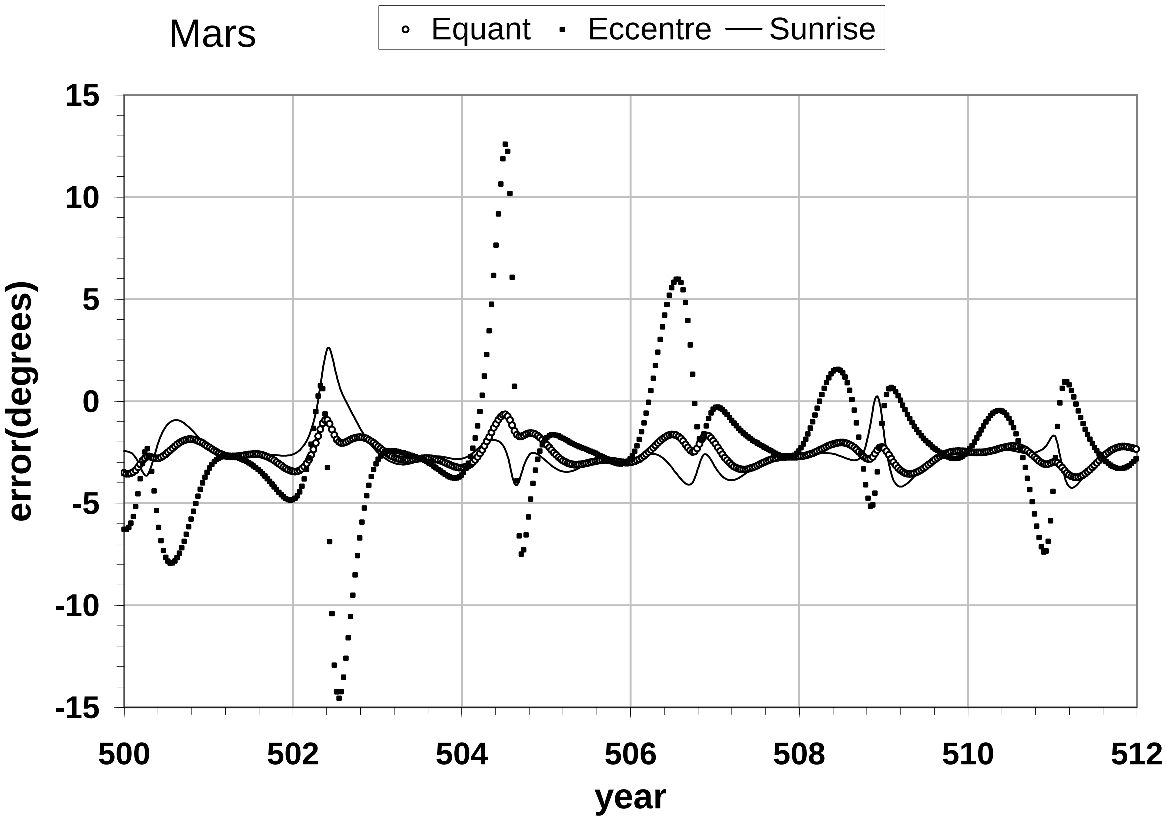

Saturn
equant
eccentre
sunrise
error(degrees)
-2.8
-3.2
-3.6
-4
504
506
508
510
512
514
516
518
year

Fig 4



Mars
Equant
Eccentre
Sunrise
15
10
5
0
-5
-10
-15
error(degrees)
500
502
504
506
508
510
512
year

Fig 5

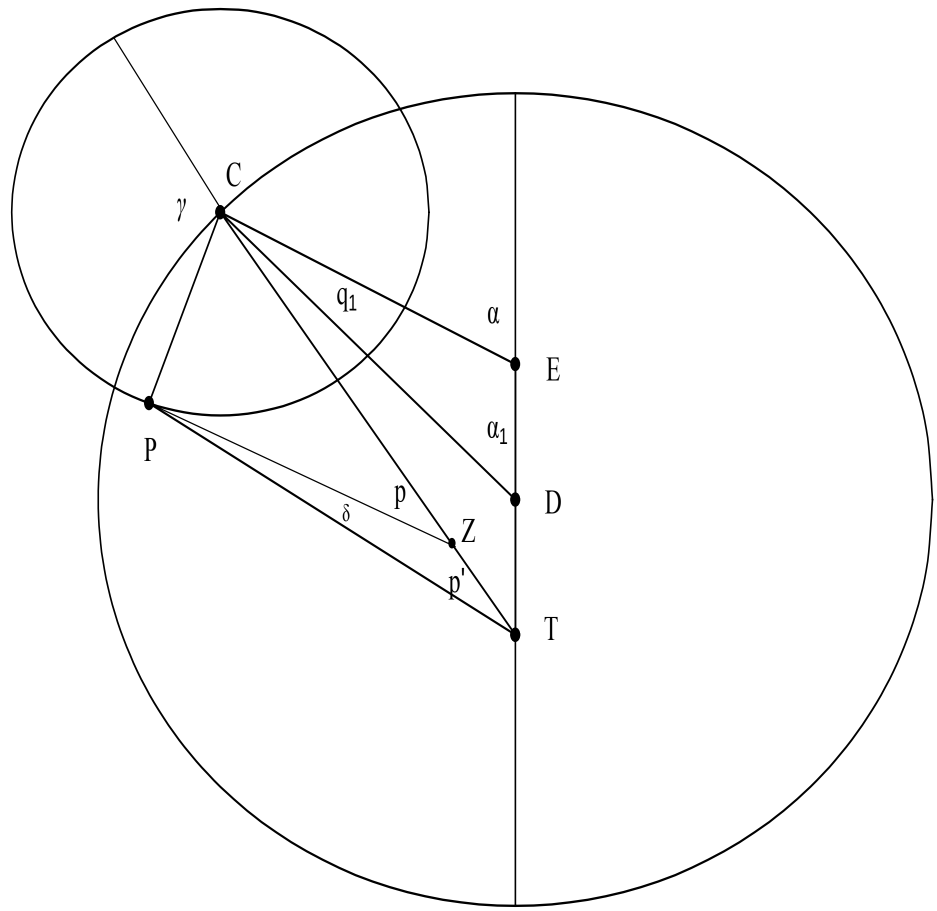


Fig 6

REFERENCES

[1] D. Pingree, “History of Mathematical astronomy in India”, *Dictionary of Scientific Biography*, 15 (1978), 533-633.
[2] K. S. Shukla, *Aryabhatiya of Aryabhata* (1976); K. S. Shukla, *Mahabhaskariya of Bhaskara I* (1960).
[3] O. Neugebauer and D. Pingree, *The Pancasiddhantika of Varahamihira* (2 vols, Copenhagen, 1970-71); B. Chatterjee, *The Khandakhadyaka of Brahmagupta* (1972).
[4] Thus for both outer and inner planets, the mean motion given is the *heliocentric* mean motion of the planet. There is no textual evidence that the Indians knew anything about this, and there is an overwhelming amount of textual evidence confirming their geocentric point of view. Some commentators, most notably van der Waerden, have however argued in favor of an underlying ancient Greek heliocentric basis, of which the Indians were unaware. See, e.g. B. L. van der Waerden, “The heliocentric system in greek, persian, and indian astronomy”, in *From deferent to equant: a volume of studies in the history of science in the ancient and medieval near east in honor of E. S. Kennedy*, Annals of the new york academy of sciences, 500 (1987), 525-546. More recently this idea is developed in about as much detail as the scant evidence allows in L. Russo, *The Forgotten Revolution* (2004).
[5] D. Pingree, “The *Paitamahasiddhanta* of the *Visnudharmottapurana*”, *Brahmavidya*, xxxi-xxxii (1967-68), 472-510.
[6] E. Burgess and W. D. Whitney, “Translation of the Surya Siddhanta”, *Journal of the American Oriental Society*, (1858) 141-498.
[7] O. Neugebauer, “The Transmission of planetary theories in ancient and medieval astronomy”, *Scripta mathematica*, 22 (1956) , 165-192. Essentially the same argument was presented again in O. Neugebauer and D. Pingree, *ibid.* (ref. 4).
[8] B. L. van der Waerden, “Ausgleichspunkt, ’methode der perser’, und indische planetenrechnung”, *Archive for history of exact sciences*, 1 (1961), 107-121.
[9] see, e.g. appendix 7 of B. Chatterjee, *ibid.* (ref.1); S. N. Sen, “Epicyclic Eccentric Planetary Theories in Ancient and Medieval Indian astronomy”, *Indian Journal of History of Science*, 9 (1974), 107-121; S. N. Sen, “Survey of studies in European languages”, *Indian Journal of History of Science*, 20 (1985), 49-121; D. A. Somayaji, “The yuga system and the computation of mean and true planetary longitudes”, *Indian Journal of History of Science*, 20 (1985), 145-187.
[10] see, e.g., D. Pingree, “On the Greek Origin of the Indian Planetary Model Employing a Double Epicycle”, *Journal for the history of astronomy*, ii (1971), 80-85; D. Pingree, “The Recovery of Early Greek astronomy from India”, *Journal for the history of astronomy*, vii (1976), 109-123; D. Pingree, *ibid*. (ref. 1).
[11] R. Mercier, ‘Astronomical tables in the Twelfth century’, *Adelard of Bath, an English Scientist and Arabist of the early Twelfth century*, ed. Charles Burnett, (London:Warburg Institute, 1987), 87-118, reproduced in: *Studies on the Transmission of Medieval Mathematical Astronomy*, Raymond Mercier. Variorum Collected Studies Series CS787, Ashgate: Aldershot, Hants, 2004.
[12] D. Rawlins, “Ancient heliocentrists, Ptolemy, and the equant”, *American journal of physics*, 55 (1987) 235-9.

[13] H. Thurston, "Greek and Indian Planetary Longitudes", *Archive for History of Exact Sciences*, 44 (1992) 191-195.
[14] In this and the succeeding plots, the *y*-axis gives the difference in degrees of the planet's position computed from modern models and the position computed from an ancient model. Hence the curve is, for all practical purposes, the error in the prediction of the ancient model.
[15] For example, for Mars I use $e = 12$ for the eccentricity of the eccentre and $2e = 12$ for the total eccentricity of the bisected equant. Using the values that Ptolemy finds at the first iteration of his equant solutions for the eccentricity of the eccentre, *e.g.* $e = 13;07$ for Mars, has no significant effect on this discussion.
[16] The reader might also wonder about the pattern of errors of the sunrise and equant models with respect to the true longitude of the planets. In the case of Jupiter the errors using the *Almagest* parameters can be significantly reduced by (1) adjusting the mean longitude to remove the bias in Ptolemy's parameters due to the well-known fact that his tropical year is too long, and (2) making a small adjustment to the period in longitude. These changes reduce the mean error to essentially zero and reduce the r.m.s error by about a factor of 3, the residual error being due to the fact that the equant combines the eccentricities of the Sun and Jupiter. For Mars the *Almagest* parameters are already much nearer optimal, but when Mars is near the Earth the errors resulting from combining the eccentricities are greatly magnified: see, e.g. J. R. Voelkel and O. Gingerich, "Giovanni Antonio Magini's "Keplerian" tables of 1614 and their implications for the reception of Keplerian astronomy in the seventeenth century', *Journal for the history of astronomy*, 32 (2001), 237-262. The fact that the equant and the sunrise theories agree poorly for Mars is, of course, simple a result of the fact that $r / R \simeq 0.66$, and the approximations used to derive the sunrise model are not very accurate.
[17] G. J. Toomer, *Ptolemy's Almagest* (1984), 420-3. B. L. van der Waerden, *ibid.*, ref (4), suggests that these remarks of Ptolemy are referring explicitly to presumed Greek precursors of the Indian schemes, but the argument seems tenuous, at best.
[18] O. Neugebauer, *A history of ancient mathematical astronomy*, (1975), 781-833.
[19] Even in the case of the *Almagest*, we know now that few if any of Ptolemy's parameters were derived as he describes. See J. P. Britton, *Models and Precision: the quality of Ptolemy's observations and parameters*, (Princeton, 1992), derived from his unpublished 1966 Yale PhD thesis; C. Wilson, "The Inner Planets and the Keplerian revolution", *Centaurus*, 17 (1972) 205-248; R. R. Newton, *The crime of Claudius Ptolemy*, (Baltimore, 1977); N. M. Swerdlow, "Ptolemy's Theory of the Inferior Planets", *Journal for the history of astronomy*, 20 (1989) 29-60; D. Duke, "Ptolemy's Treatment of the Outer Planets", *Archive for History of Exact Sciences*, (to be published 2004).
[20] The chief proponents of the western view are D. Pingree. *ibid.* ref. (10), and B. L. van der Waerden, *ibid.* ref. (4) and ref. (8), although they differ in opinion on some of the details. The view of many Indian scholars is that the Indian astronomy of the siddhantic period, including the planetary schemes, was largely developed in India and was the culmination of many centuries of pre-siddhantic astronomical investigation. For a survey of this view, see, e.g. S. Kak, "The development of astronomy from Vedanga Jyotisa to Aryabhata", in *Science and Civilization in India*, Vol. 1, Part 2: Life, Thought and Culture in India, edited by G.C. Pande, (2001), 866-885, and references therein.

[21] B. L. van der Waerden, *ibid.* (ref. 8).