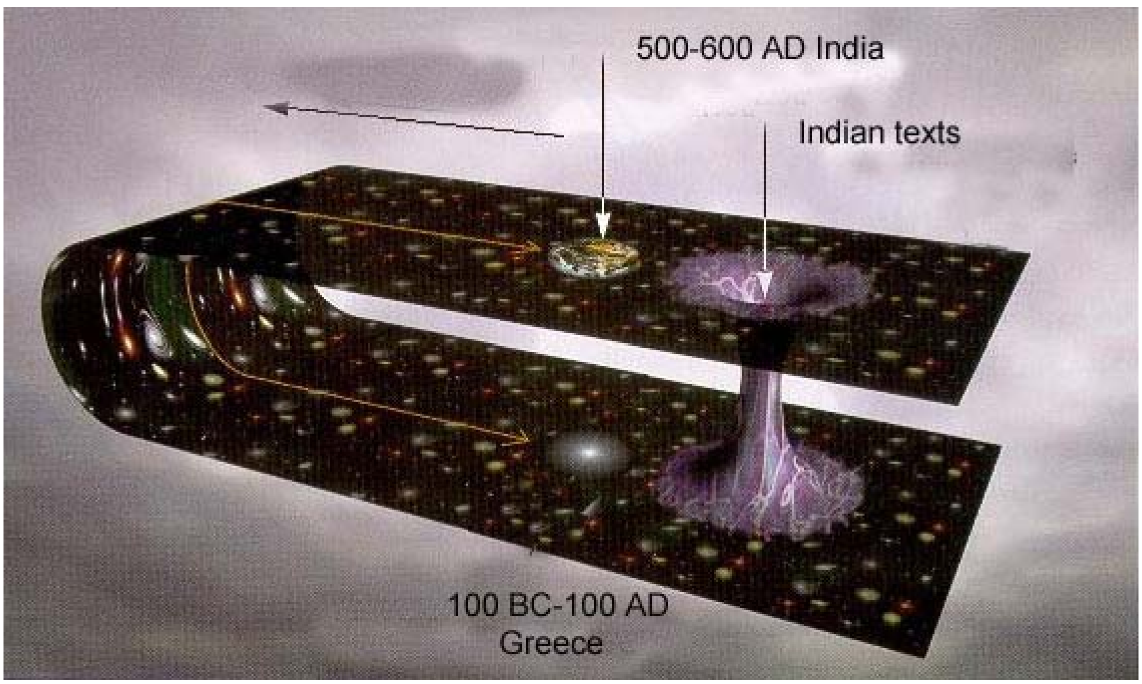
Indian Planetary Theories and Greek Astronomy

Dennis Duke, Florida State University

1. the Pingree – van der Waerden Hypothesis
2. the equant
    a. the equant in India
    b. pulsating epicycles
    c. the concentric equant
3. second lunar anomaly
4. planetary latitudes

1. The Pingree – van der Waerden Hypothesis (not exactly their words…)

The texts of ancient Indian astronomy give us a sort of wormhole back through space-time into an otherwise largely inaccessible era of Greco-Roman development of astronomy.



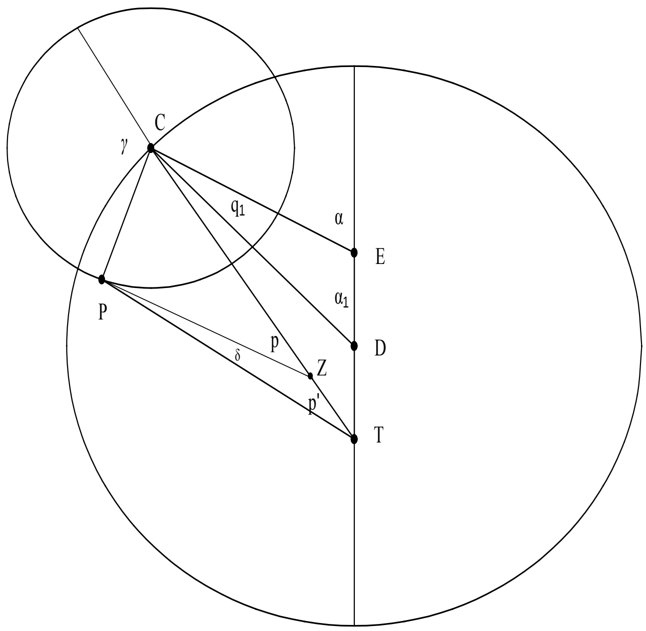
The idea dates from the 1800's. It is more or less universally accepted in the West (and equally rejected by many Indian scholars). Pingree and van der Waerden are/were the most prominent modern champions (while disagreeing on many of the details).

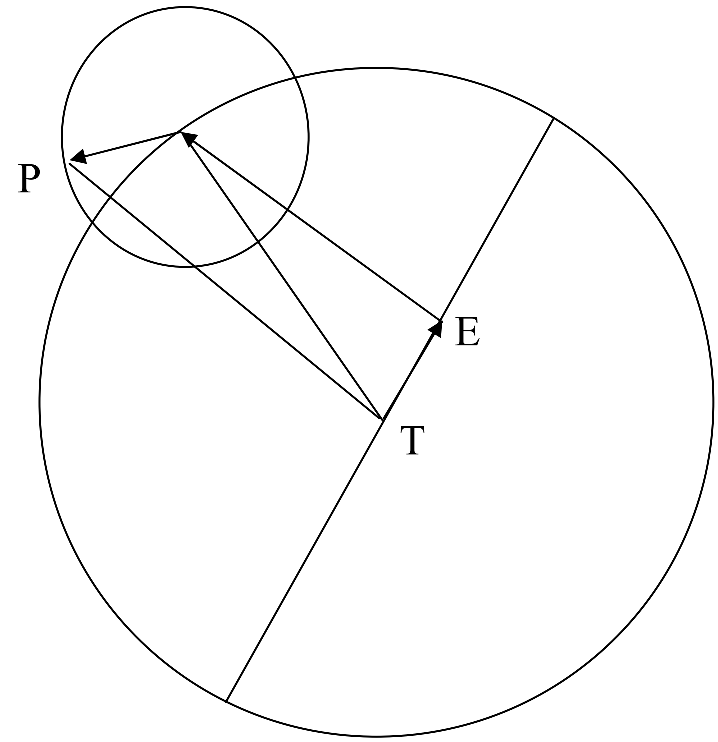
Some examples related to Ptolemy:

- *The equation of time*. Throughout the first millennium the Indians used an abbreviated version which includes only the effect of the zodiacal anomaly of the Sun, and neglects the effect of the obliquity of the ecliptic.
- *Obliquity of the ecliptic*. When used in spherical trigonometry, the Indians use either 24° or 23;40°, both associated with Hipparchus, but never the Eratosthenes/*Almagest* value 23;51,20°.
- *The second lunar anomaly*. The Indians did not discuss evection until the beginning of the second millennium, and then in a form different from that used by Ptolemy.
- *Accurate discussions of parallax*. The Indians were aware of parallax and used it for computing eclipses, but always used various approximations.
- *Decoupling the anomalies*. The scheme used by the Indians works well for moderate eccentricities and epicycle sizes, such as those appropriate to Jupiter, but as we have noticed it breaks down for larger values, such as those appropriate to Mars. Presumably they would have used Ptolemy's more accurate tabular interpolation scheme if they had known about it.
- *Trigonometry scales*. The Indians used a variety of values for the radius of the reference circle, and mostly the value R = 3438 in the earliest texts. This is a value used by Hipparchus but apparently abandoned by the time of Ptolemy.
- *Retrograde motion*. When mentioned at all in connection with the multi-step models we are discussing, the Indians quoted specific values of the sighra anomaly that correspond to first and second station. There is no mention of the variation in the size of retrograde arcs with zodiacal position.
- *Model of Mercury*. Unlike Ptolemy, who used a complicated crank mechanism to generate a pair of perigees for Mercury, the Indians used the same model for Mercury and Venus, which is also often the same or closely related to the model used for the outer planets. The basis of all of these models is the equant.
- *Determination of orbit elements*. While the bulk of the *Almagest* is devoted to explaining how to determine orbit elements from empirical data, it is not at all obvious that any comparable derivation is even possible in the context of the Indian approximation schemes.
- *Values of orbit elements*. The values used in the Indian schemes for e, r, and A are generally different from the values found in the *Almagest*. Except for Mercury, the resulting Indian model predictions for true longitudes are generally inferior to those in the *Almagest*.
- *Star catalog*. The Indian coordinates for star positions are generally inaccurate, and bear no relation to those found in the *Almagest* star catalog.
- *Zodiacal signs*. The Indian texts routinely divide circles such as epicycles into 30° segments and refer to them in terms of the zodiacal signs. The only other known use of this practice is in Hipparchus' similar description of circles of constant latitude in the *Commentary to Aratus*.

Many other examples in Babylonian/Greco-Roman astronomy.

2(a). The equant in India





eccentric orbits (*manda*) for the zodiacal anomaly

$$\sin q(\alpha) = -e \sin \alpha$$

epicycles (*sighra*) for the solar anomaly

$$\tan p(\gamma) = \frac{r \sin \gamma}{1 + r \cos \gamma} \neq \frac{r \sin \gamma}{\rho + r \cos \gamma}$$

(1) $\alpha = \bar{\lambda} - \lambda_A \qquad \nu_1 = \bar{\lambda} + \tfrac{1}{2} q(\alpha)$

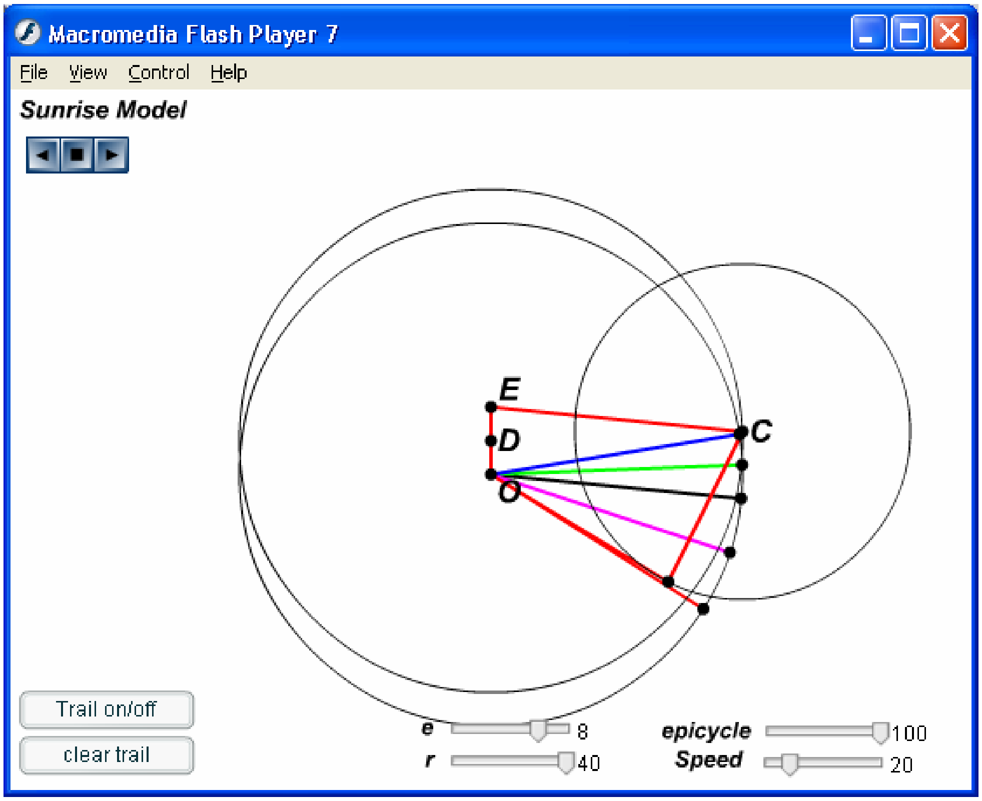
(2) $\gamma = \bar{\lambda}_S - \nu_1 \qquad \nu_2 = \nu_1 + \tfrac{1}{2} p(\gamma)$

(3) $\alpha = \nu_2 - \lambda_A \qquad \nu_3 = \bar{\lambda} + q(\alpha)$

(4) $\gamma = \bar{\lambda}_S - \nu_3 \qquad \lambda = \nu_3 + p(\gamma)$

		Mars			
		equation of center		equation of anomaly	
6	354	1	12	2	25
12	348	2	23	4	49
18	342	3	33	7	13
24	336	4	40	9	36
30	330	5	45	11	59
36	324	6	46	14	20
42	318	7	42	16	40
48	312	8	33	18	59
54	306	9	19	21	16
60	300	9	59	23	30
66	294	10	32	25	42
72	288	10	58	27	50
78	282	11	17	29	55
84	276	11	29	31	55
90	270	11	33	33	50
93	267	11	32	34	45
96	264	11	29	35	38
99	261	11	24	36	29
102	258	11	17	37	18
105	255	11	9	38	4
108	252	10	58	38	48
111	249	10	46	39	28
114	246	10	32	40	5
117	243	10	16	40	38
120	240	9	59	41	7
123	237	9	40	41	31
126	234	9	19	41	49
129	231	8	57	41	60
132	228	8	33	42	5
135	225	8	8	41	60
138	222	7	42	41	46
141	219	7	14	41	21
144	216	6	46	40	42
147	213	6	16	39	48
150	210	5	45	38	36
153	207	5	13	37	3
156	204	4	40	35	6
159	201	4	7	32	40
162	198	3	33	29	43
165	195	2	59	26	11
168	192	2	23	22	1
171	189	1	48	17	13
174	186	1	12	11	52
177	183	0	36	6	3
180	180	0	0	0	0

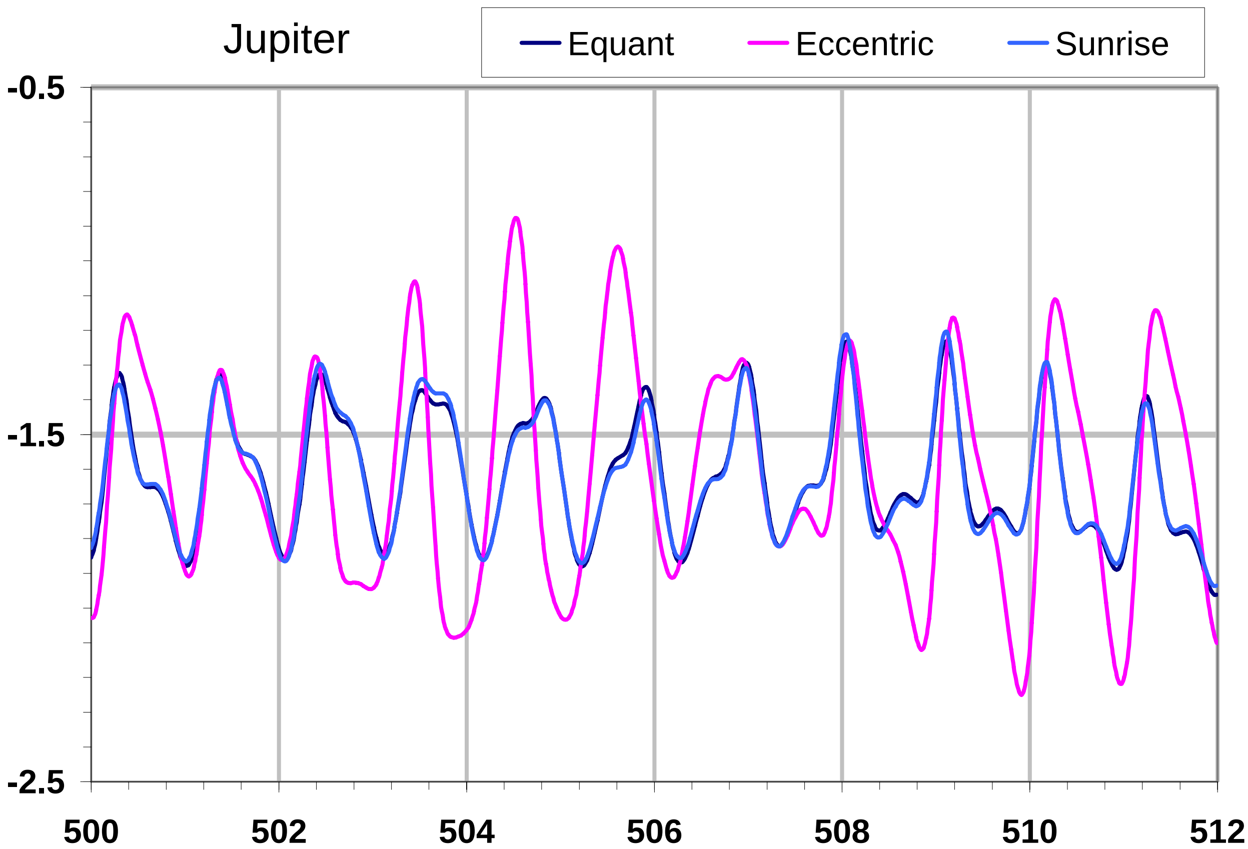
Can we visualize the algorithm geometrically?

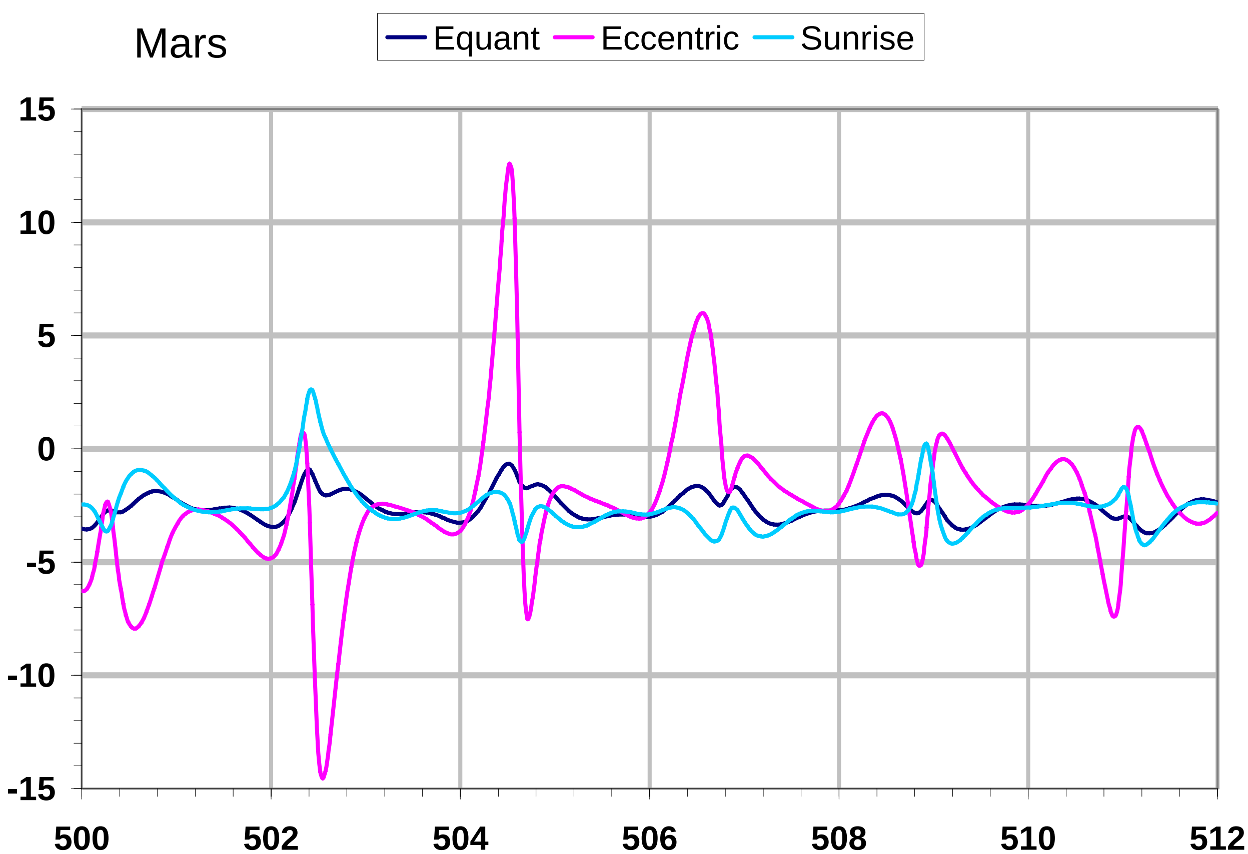


Pingree (*JHA*, 1971):

The Double Epicycle Indian Planetary Model 81

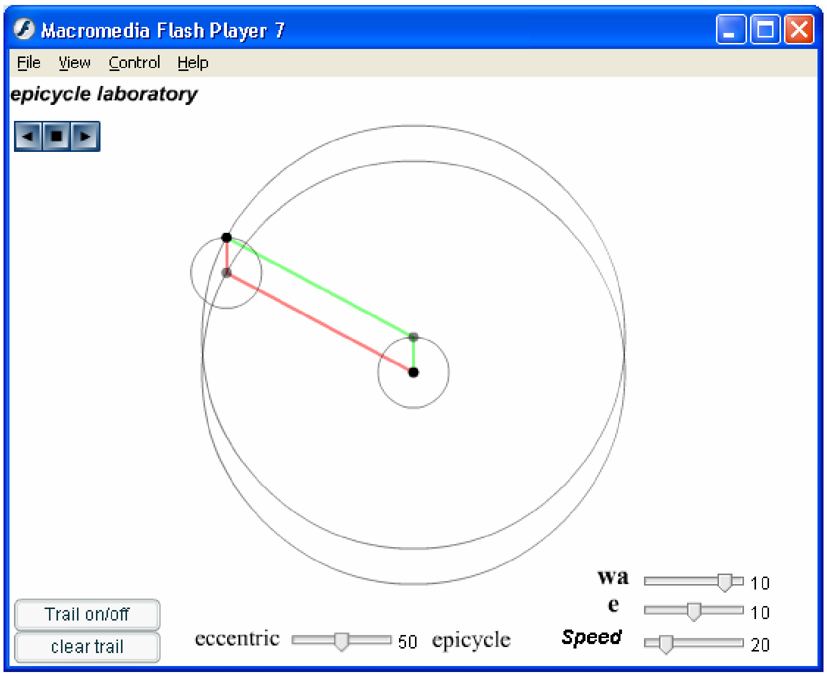
to another, or even from astronomer to astronomer within a *pakṣa*. But the fundamental concept remains clear: the planet is always situated on the circumference of a deferent circle concentric with the centre of the Earth, while two epicycles (one each for the Sun and Moon) revolve about it. As the planet progresses with its mean velocity about this deferent circle, at each instant it is pulled by the two epicycles away from its mean to its true longitude. These instantaneous true longitudes are subject to computation, but a true course of the planet over a period of time can only be conceived of as a series of such instantaneous true longitudes.


Jupiter
Equant
Eccentric
Sunrise
-0.5
-1.5
-2.5
500
502
504
506
508
510
512

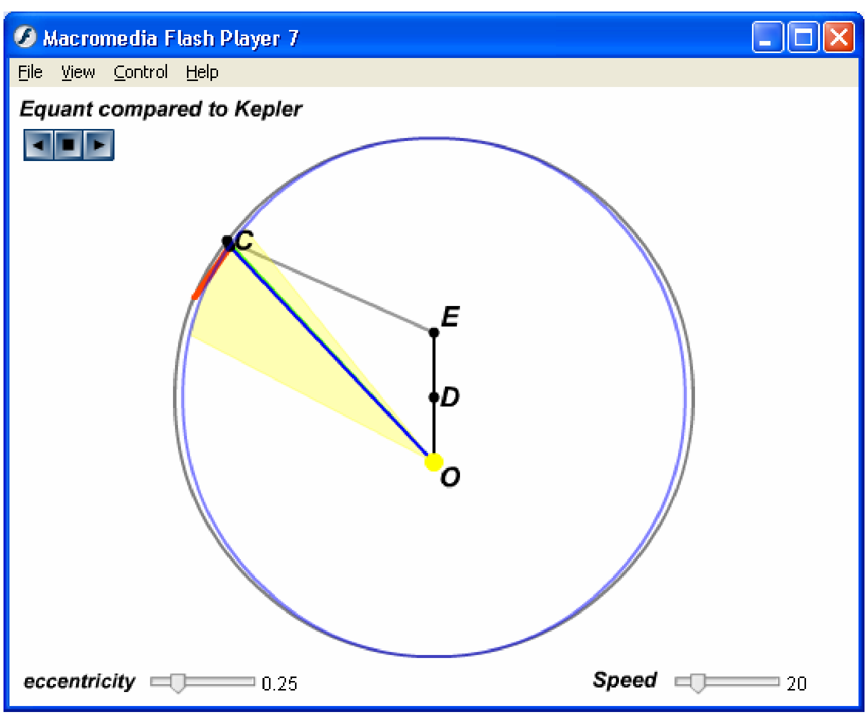

Mars
Equant
Eccentric
Sunrise
15
10
5
0
-5
-10
-15
500
502
504
506
508
510
512

2(b). Pulsating Epicycles

normal eccentric and epicycle models are equivalent:



But what is the epicycle equivalent of the equant?



Consider:

(1) an eccentric model with eccentricity $2e$, and
(2) an equant model with total eccentricity $2e$.

For the eccentric the equation of center is given by

$$\tan q_1 = \frac{2e\sin\alpha}{1+2e\cos\alpha},$$

For the equant the equation of center is given by

$$\tan q_2 = \frac{2e\sin\alpha}{\sqrt{1-e^2\sin^2\alpha}+e\cos\alpha}$$

Ptolemy tabulates q_1 in column 3 of his anomaly tables, and $q_2 - q_1$ in column 4.

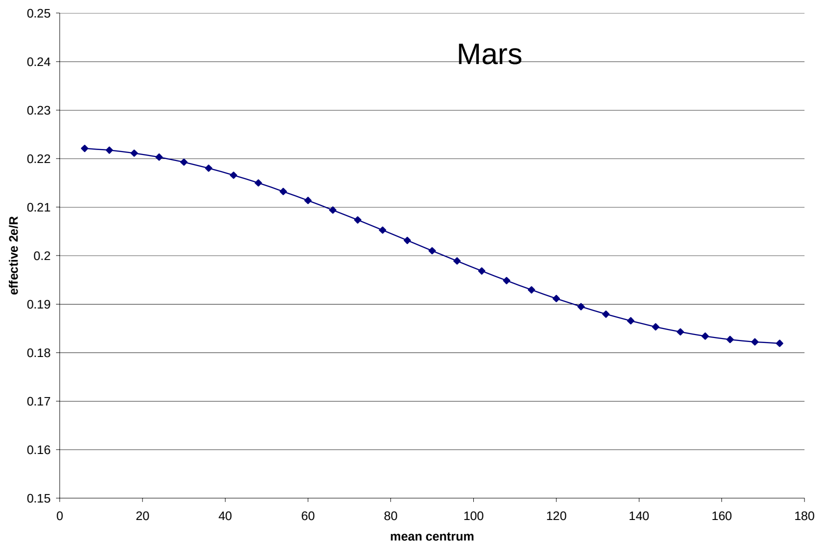
Suppose we want to stick with a simple eccentric model, but we are willing to tolerate a *variable* $e' = e'(\alpha)$. Can we find a function $e'(\alpha)$ such that for a fixed α,

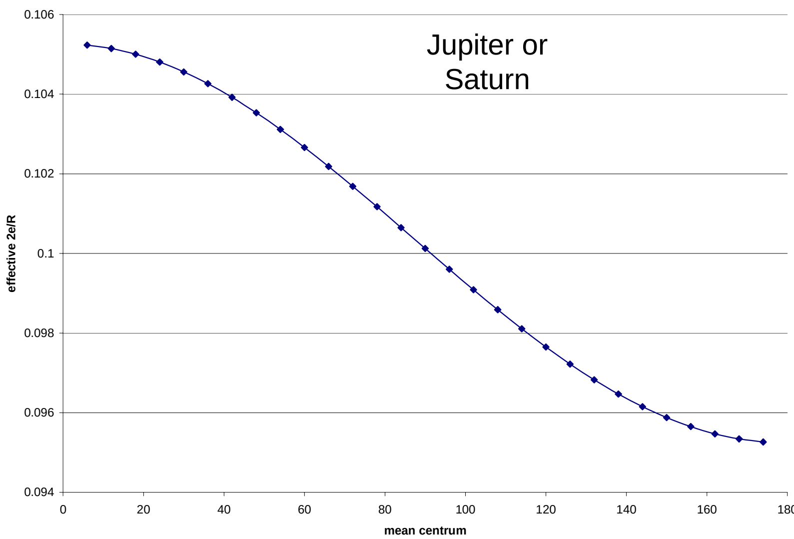
$$q_1(e'(\alpha),\alpha) = q_2(e,\alpha).$$

The required function $e'(\alpha)$ is given by

$$e' = \frac{\tan q_2}{\sin\alpha - \cos\alpha\tan q_2}$$

It is





the function $e'(\alpha)$ is approximately sinusoidal, and of the general form

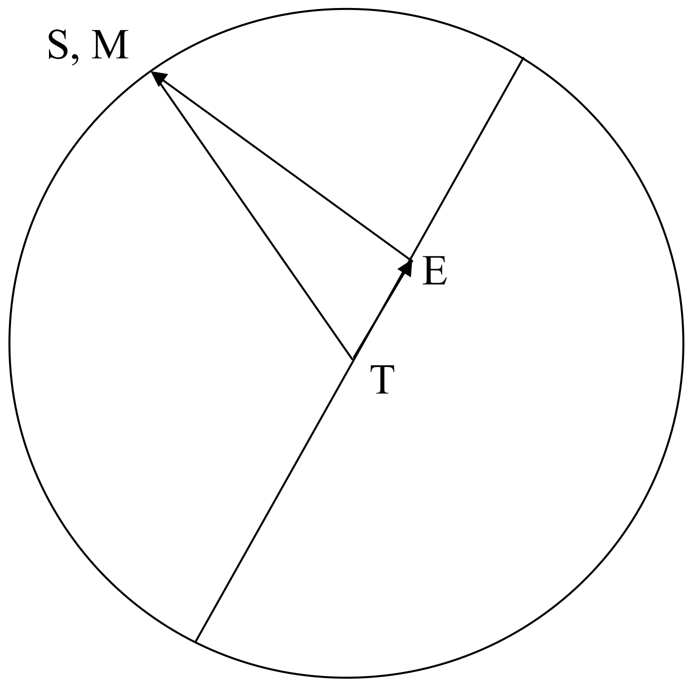
$$e' = e'_{\max} - (e'_{\max} - e'_{\min})\sin\alpha$$

It contrast, the Indian texts use

$$e' = e'_{\max} - (e'_{\max} - e'_{\min})\left|\sin\alpha\right|,$$

This exercise is not entirely academic…

2(c). The concentric equant



used for the Sun and Moon in Indian theories. Uniform motion about E, and Earth is at the center of the deferent.

The texts have Moon's r = 5;15 and Sun's e = 2;10.

Further, it is explicitly attested that the concentric equant is equivalent to an ordinary eccentric or epicycle with a variable e or r.

Concentric with equant 27

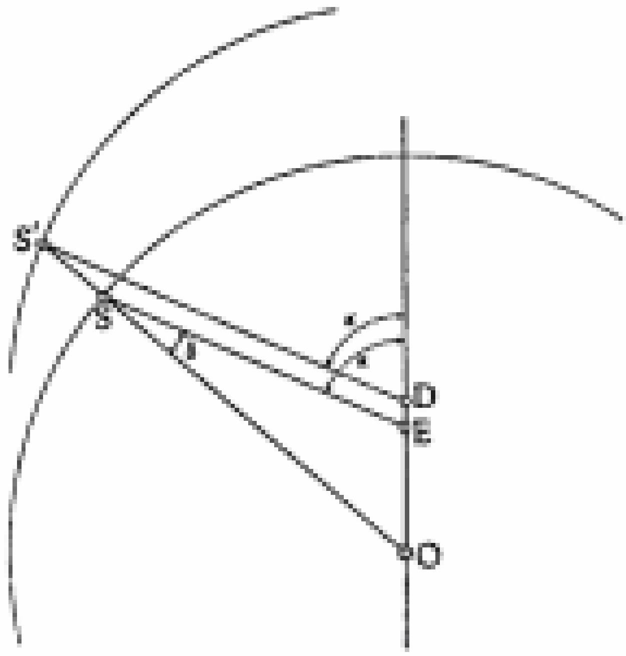


Fig. 1

IV 19. Subtract (the Sine of) the final equation from (the Sine of) the koṭi or again add it, depending on the quadrant; the square-root of the sum of the square of that and the square of the bâhu is the hypotenuse.

IV 20. Multiply (the Sine of) the final equation by the hypotenuse (and) divide (the product) by the Radius; add (the quotient) to or subract it from the previous koṭi (repeatedly) until the hypotenuse is equal (to the hypotenuse obtained in the immediately preceding computation).

IV 21. Multiply the Radius by the Sine of the bâhu (and) divide (the product) by the (final) hypotenuse. Add the are (corresponding to that quotient) to (the longitude of) the apogee according to the quadrant of the argument.

$OS = DS' = 1$.

the mean anomaly κ and $EO = e$ are given.

we want to find OD and $OS' = h$ = ‘the hypotenuse’, and use those to compute the true anomaly (as in verse 21).

First, drop a perpendicular line from S' to a new point F on the apsidal line.

The algorithm amounts to solving

$$h = \sqrt{(eh + \cos\kappa)^2 + \sin^2\kappa}$$

by iteration, beginning with $h = 1$ as the first trial value.

(verse 19) assume $OD = e$

then $OF = OD + \cos(\kappa)$ and $S'F = \sin(\kappa)$

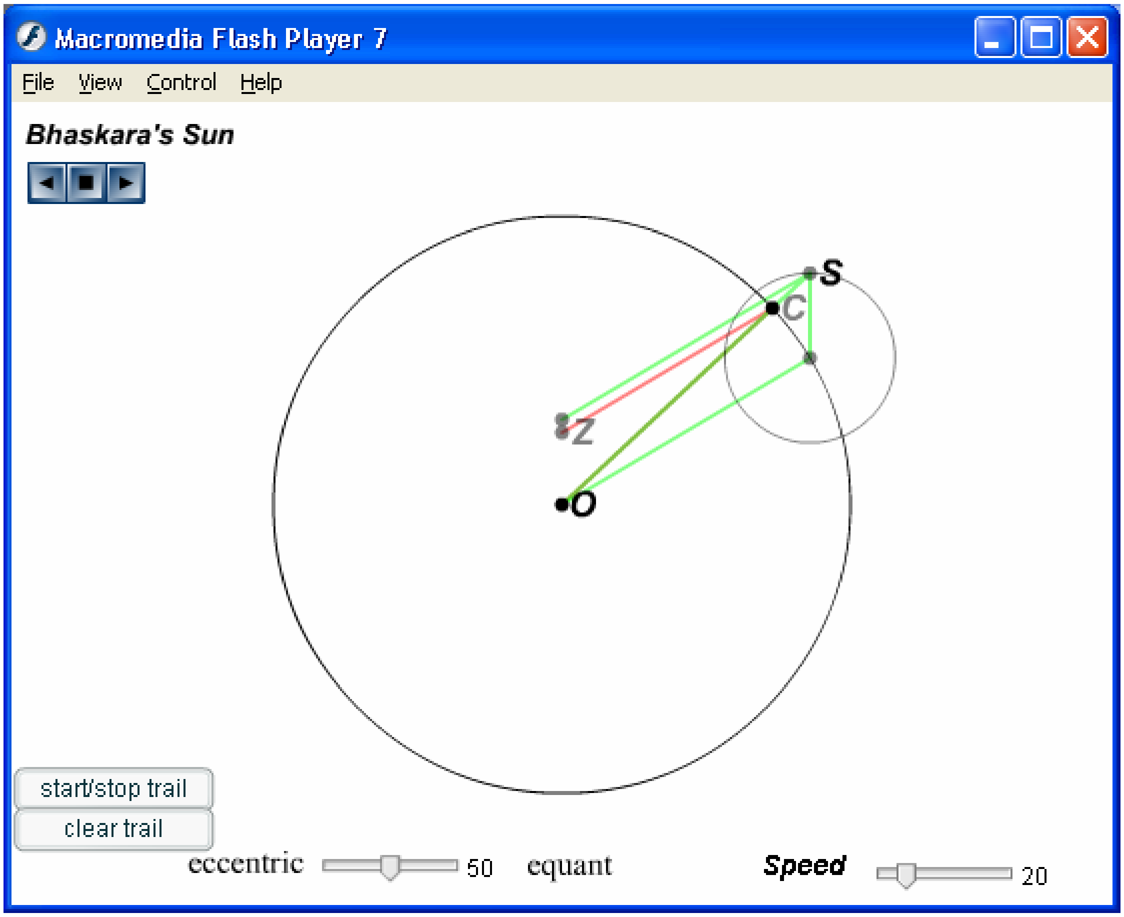
then $OS' = \sqrt{OF^2 + S'F^2} = h$

(verse 20) by similar triangles $\frac{OS'}{DO} = \frac{1}{e}$, so we have a new estimate: $DO = e\,h$.

go to step 2 with the new estimate of DO and recompute H. When H stops changing, go to step 6.

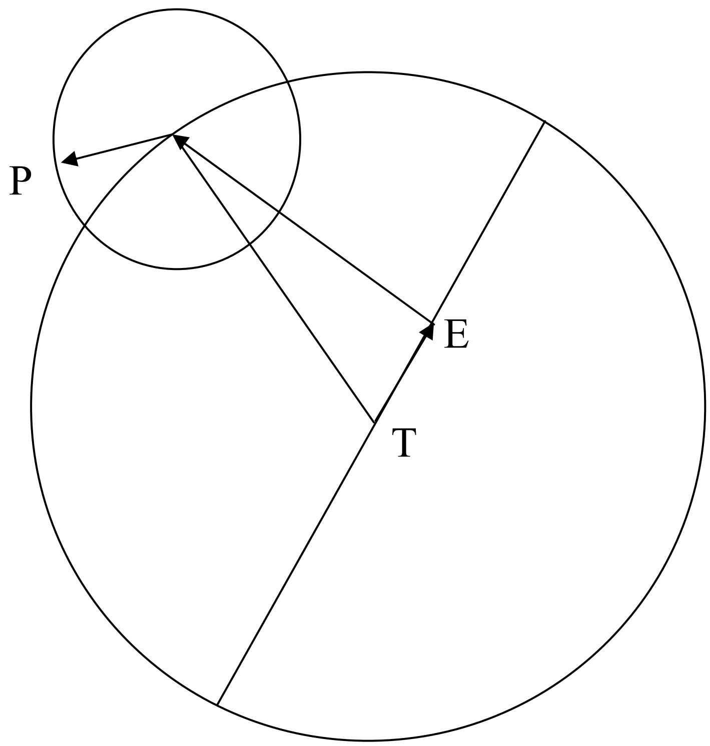
(verse 21) compute angle $DOS' = c = \arcsin(\sin\kappa / h)$. It will be the same value you could have gotten without iteration from angle $EOS = c = \kappa + q = \kappa + \arcsin(-e\sin\kappa)$.

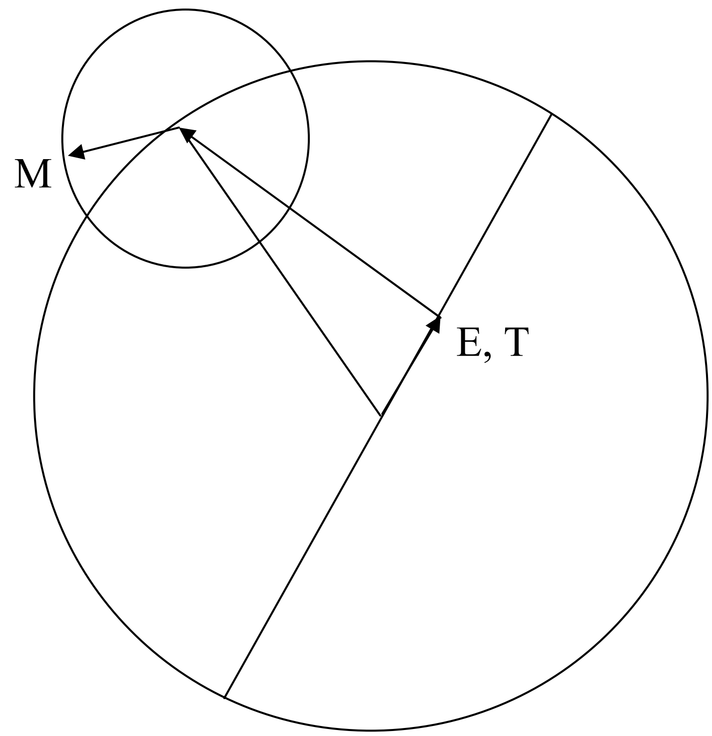
Verses IV 9-12 give an equivalent solution employing an epicycle of varying radius. This is all illustrated in the animation at www.csit.fsu.edu/~dduke/pingree2.html.



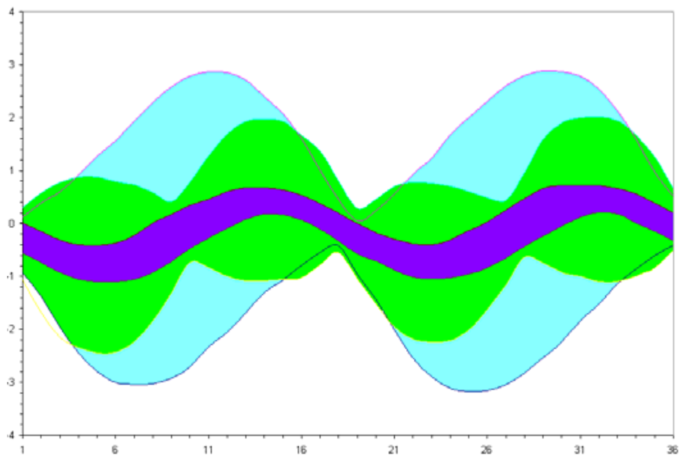
so the Pingree – van der Waerden Hypothesis suggests this was all known to some Greco-Roman astronomers.

A variant of the concentric equant does show up in the lunar model of Ptolemy's *Planetary Hypotheses*:

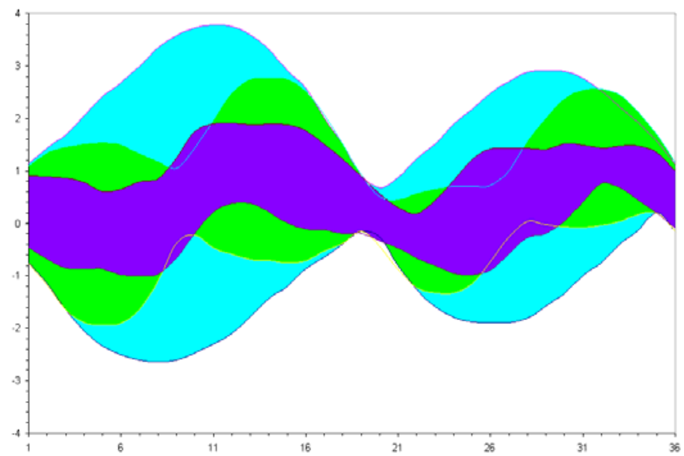




3. The second lunar anomaly



Indian model



Almagest model

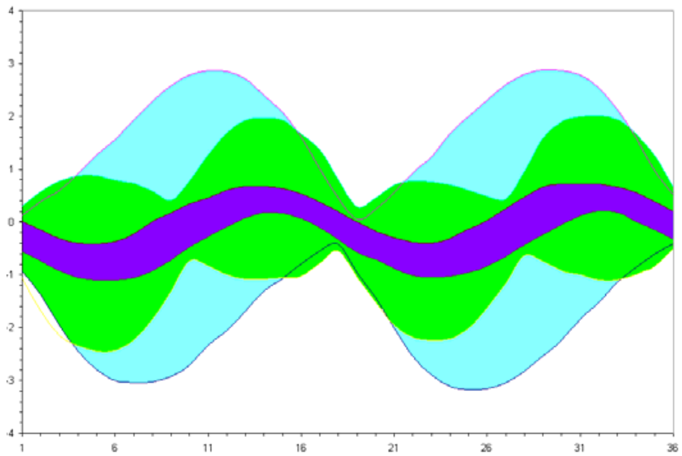
Maximum and minimum discrepancies in Sun – Moon elongation *vs.* elongation.

The models for the outlying blue envelopes are single anomaly lunar models at syzygy.

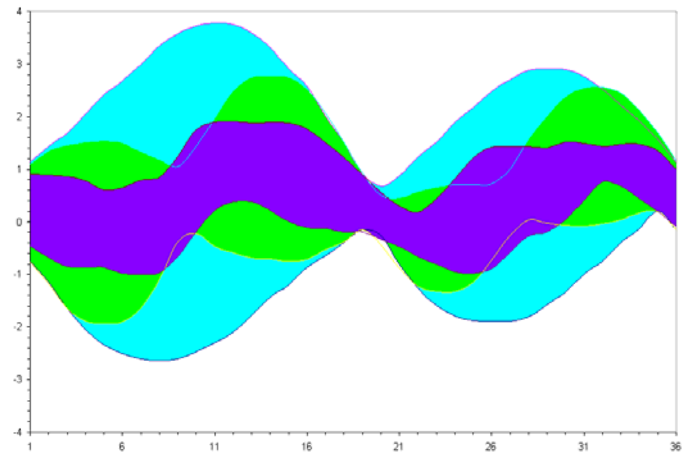
The models for the intermediate green envelopes are
- for the *Almagest*, the so-called second lunar model, i.e. the crank mechanism to move the epicycle closer to the Earth at quadrature, no *prosneusis*.
- for the Indian model, a variable epicycle radius of the form

$$r = r_0 + r_1 \left|\sin \eta\right|$$

where η is the Moon-Sun elongation (This model is not in any known text, but the quality of the description is about as good as the second *Almagest* model, and it avoids the lunar distance problem of the *Almagest*).



Indian model



Almagest model

The models for the central purple envelopes are

(a) for the *Almagest*, the full lunar model including *prosneusis*

(b) for the Indian model, the equation of center for the simple model is supplemented with an explicitly attested evection term proportional to

$$-2\cos(\alpha-\eta)\sin\eta = -\sin\alpha + \sin\left(\alpha-2\eta\right)$$

where α is the mean lunar anomaly, and which, as the r.h.s. shows, is exactly the modern form of evection plus a small correction to the primary contribution to the equation of center.

There is no known geometrical derivation of the evection term (*or anything else!*) in the Indian models. It is remarkable that the Indian term, described in a text from around 900 AD, shows nearly perfect agreement with modern theory for the evection term, and is clearly empirically superior to the *Almagest* model.

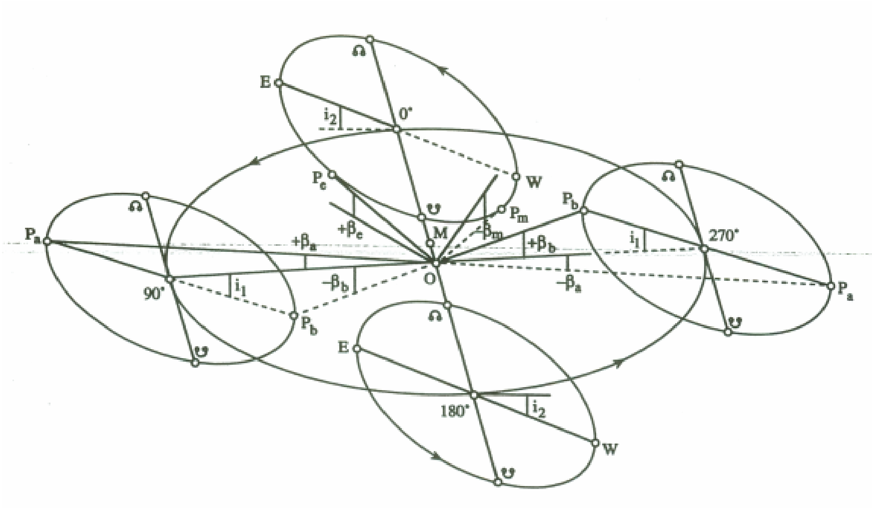
इन्दूच्चोनार्ककोटिघ्ना गत्यंशा विभवा विधोः ।
गुणो व्यर्कन्दुदोः कोट्योः रूपपञ्चाप्तयोः क्रमात् ॥ १ ॥
फले शशाङ्कतद्गत्योः लिप्ताद्ये स्वर्णयोर्वधे ।
ऋणं चन्द्रे धनं भुक्तौ स्वर्णसाम्यवधेऽन्यथा ॥ २ ॥

and

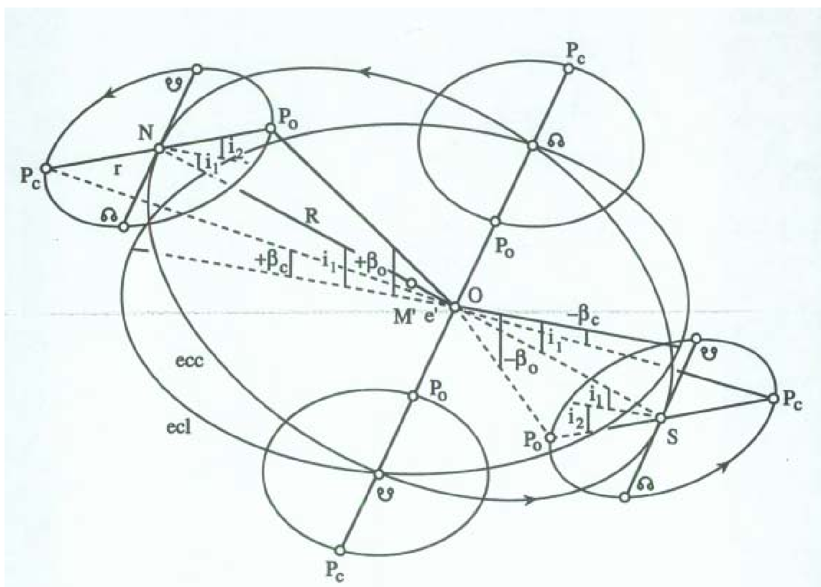
त्रि भविरहितचन्द्रोच्चोन भास्वद्भुजज्या
गगननृप (160) विनिघ्नी भत्रयज्याविभक्ता ।
भवति चरफलाख्यं, तत्पृथक्स्थं शर (5) घ्नं
हृतमुडुपतिकर्णत्रिज्ययोरन्तरेण ॥ २ ॥
परमफलमवाप्तं तद्धनर्णं पृथक्स्थे
तुहिन किरणकर्णे त्रिज्यकोनाऽधिकेऽस्य ।
स्फुटदिनकरहीनादिन्दुतो या भुजज्या
स्फुट परमफलघ्नी भाजिता त्रिज्ययाऽऽप्तम् ॥ ३ ॥
शशिनि चरफलाख्यं सूर्यहीनेन्दुगोलात्
तदृणमुतधनं चेन्दूच्च हीनार्कगोलम् ।
यदि भवति हि साम्यं व्यस्तमेतद्विधेयं
स्फुटगणितदृगैक्यं कर्तुमिच्छद्भिरत्र ॥ ४ ॥

On the other hand, as far as we know all the arguments that conclude that Indian astronomy lacked any empirical observational basis in 500–600 AD are just as valid (or invalid – take your pick) in 900 AD, so the source of the empirical accuracy is a mystery.

4. Planetary latitudes



Almagest latitudes – outer planets



Almagest latitudes – inner planets
(both figures from NMS)

the computation of these is rather complicated.

```
c latitude of Mercury
      if(xav.le.180d0) then
         eta = 180d0-xav
      else
         eta = xav - 180d0
      endif
      pprime = abs(re*cosd(eta)*sind(xi1))
      oprime = sqrt((0.94444d0 - re*cosd(eta)*cosd(xi1))**2 +
     * (re*sind(eta))**2)
      c3 = atan2(pprime,oprime)*dpr
      c6 = abs(atan2(re*sind(xav),1d0+re*cosd(xav))*dpr)
      c4 = (6d0/60d0+48d0/60d0**2)*c6
      xc = crc(xcm + xq)
      xkappa0p = crc(xc + 270d0)
      if(xav.ge.90d0.and.xav.le.270d0) then
        beta1 = cosd(xkappa0p)*c3
      else
        beta1 = -cosd(xkappa0p)*c3
      endif
      xkappa0pp = crc(xc + 180d0)
      if(xav.le.180d0) then
        sgn = 1d0
      else
        sgn = -1d0
      endif
      if(xc.ge.90d0.and.xc.le.270d0) then
        beta2 = cosd(xkappa0pp)*1.1d0*sgn*c4
      else
        beta2 = cosd(xkappa0pp)*0.9d0*sgn*c4
      endif
      beta3 = xi0*cosd(xc)**2
      xlat = beta1 + beta2 + beta3

c latitude of an outer planet
      xrhop = 1d0 + e*cosd(xomegaa)
      xrho3 = sqrt((xrhop+re*cosd(xav))**2 + (re*sind(xav))**2)
      xc3 = ((xi0-xi1)*re*cosd(xav)+xrhop*xi0)/xrho3
      xrhopp = 1d0 - e*cosd(xomegaa)
      xrho4 = sqrt((xrhopp+re*cosd(xav))**2 + (re*sind(xav))**2)
      xc4 = ((xi0-xi1)*re*cosd(xav)+xrhopp*xi0)/xrho4
      xkappa0 = crc(xcm + xq)
      omega = crc(xkappa0 + xomegaa)
      xc5 = abs(cosd(omega))
      if(omega.le.270d0.and.omega.ge.90d0) then
        xlat = -xc5*xc4
      else
        xlat = xc5*xc3
      endif
```

Indian latitudes

$$\rho' = \sqrt{(1 + r\cos\gamma)^2 + (r\sin\gamma)^2}\,,$$

$$\sin\beta = \frac{\sin i \sin\omega}{\rho'}$$

88 *E. S. Kennedy and Walid Ukashah*

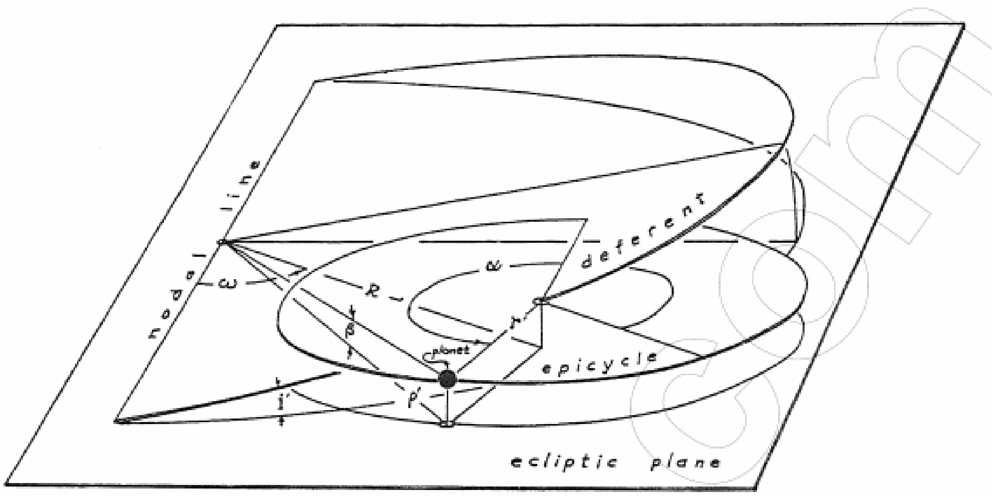


Fig. 2

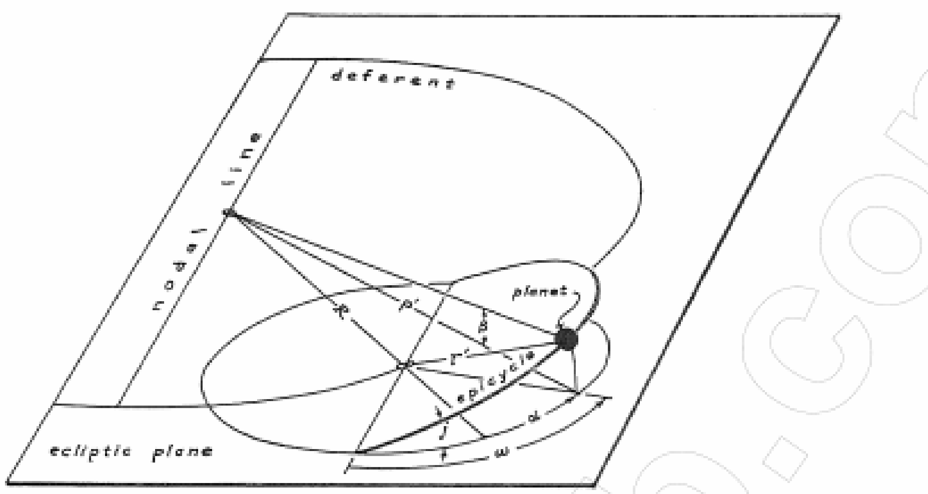


Fig. 3

Identical to Ptolemy's *Planetary Hypotheses*

	a	e	i	node	perigee	apogee	L
J0000	AU		deg	deg	deg	deg	deg
Mercury	0.387	0.206	7.0	24.6	46.3	226.3	210.8
Venus	0.723	0.007	3.4	58.7	103.5	283.5	157.7
Earth	1.000	0.017	0.0	0.0	68.6	248.6	85.1
Mars	1.524	0.093	1.9	34.1	299.2	119.2	201.5
Jupiter	5.203	0.048	1.4	80.1	342.5	162.5	148.3
Saturn	9.537	0.054	2.6	96.2	53.2	233.2	59.7

		Paitamaha	Midnight	Sunrise	BSS	Almagest	PH
Saturn	e	5;00	10;00	9;45	5;00	6;50	6;50
	e'			6;45			
	r	6;40	6;40	6;00	5;48	6;30	6;30
	r'			6;45			
	A	261	240	236	261	233	
	N	103	100	100	103	100	
	i	2;10	2;00	2;00	2;10	2;30/4;30	2;30
Jupiter	e	5;30	5;20	6;00	5;30	5;30	5;30
	e'			5;15			
	r	11;20	12;00	11;15	11;20	11;30	11;30
	r'			12;00			
	A	173	160	180	173	161	
	N	82	80	80	82	90	
	i	1;16	1;00	1;00	1;16	1;30/2;30	1;30
Mars	e	11;40	11;40	13;30	11;40	12;00	12;00
	e'			10;30			
	r	40;30	39;00	39;45	40;36	39;30	39;30
	r'			38;15			
	A	128	110	118	128	115;30	
	N	22	40	40	32	29;30	
	i	1;50	1;30	1;30	1;50	1;00/2;15	1;50
Venus	e	1;50	2;20	3;00	1;48	2;30	2;30
	e'			1;30	1;30		
	r	43;00	43;20	44;15	43;48	43;10	43;10
	r'			42;45	43		
	A	81	80	90	81	55	
	N	60	60	60	60	325	
	i	2;16	2;00	2;00	2;16	2;30/3;30	3;30
Mercury	e	6;20	4;40	5;15	6;20	3;00/3;00	3;00/2;30
	e'			3;45			
	r	22;00	22;00	23;15	22;00	22;30	22;15
	r'			21;45			
	A	225	220	210	225	190	
	N	21	20	20	21	100	
	i	2;32	2;00	2;00	2;32	6;15/7;00	6;30
Sun	e	2;16,40	2;20	2;15		2;30	2;30
	A	78	80	78		65;30	65;30
Moon	e	5;16	5;10	5;15		5;15	5;15
	i	4;30		4;30		5;00	5;00

The Relative Chronology of the *Planetary Hypotheses* and the *Almagest*?

Provisional translation of Ptolemy's *Planetary Hypotheses*, Book 1 part 1

We have worked out the models of the celestial motions, Syrus, in the books of the Mathematical Syntaxis, demonstrating by arguments in each instance both how it is plausible and how it is everywhere in agreement with the phenomena, with a view to the exhibiting of uniform circular motion, which necessarily applies to things that share in eternal and regular motion and that are not liable in any manner to undergoing increase or decrease. **Here we have taken on the task of setting out the facts themselves succinctly and in such a way that they can be much more handily comprehended both by ourselves and by people who choose to exhibit them in a mechanical construction**, whether they do this in a more "naked" manner with each of the motions restored to its proper positions by hand, or they accommodate them to each other and to the motion of the whole by mechanical methods. [I definitely do] not [mean] the customary manner of "spheremaking," since this sort of thing, besides failing to represent the models, yields the appearance only and not the underlying reality, so that an exhibition is made not of the models but of the craftsmanship; but rather [the manner] according to which the arrangement together with the variety of the motions is in our view along with the anomaly that is apparent to observers and that is caused by the uniform circular courses, even if it is not possible to weave them all together in a way that is worthy of the aforesaid purpose, but only to display each one as it is separately.

We shall make the setting out, so far as the general assumptions are concerned, in agreement with the things that are delineated in the Syntaxis, but **so far as the details are concerned, following the corrections that we have made in many places on the basis of more prolonged comparisons of observations, [corrections] either in the models themselves or in their spatial ratios or in their periodic restitutions.** [Our presentation will] also adhere to the demonstrations of the models themselves, that is, for the uniform motions, splitting apart or for that matter joining together wherever necessary the [motions] given in [the Syntaxis] in order that their definitions should be relative to the parts of the zodiac and the starting points, since this is convenient for calculations, in such a way that the individual character of each course should here be manifest, even if several [motions] are carried out in the same direction. **In the case of the positions and arrangements of the circles that cause the anomaly we use the simpler ones among the [available] methods for the sake of easy execution in instrument construction, even if some small discrepancy follows**, and moreover for the time being we fit the motions to the circles themselves as if they are freed from the spheres that contain them, so that we can gaze upon the impact of the models stripped and as it were laid bare….

But on the other hand…..

the mean motions for the planets in sections 10-14 of the *PH* are in essentially the same form we find in the Indian texts: the number of heliocentric rotations in some number of years, for both outer and inner planets.

the latitude theory in the *Almagest* and the *Handy Tables* is quite complicated. The version in the *PH* is much simpler, and is identical to that found in Indian texts.

the full lunar theory in the *Almagest* has an extra complication – the inclination of the epicycle radius – beyond that found in the *PH.*

a passage in *Ptolemy's First Commentator* seems to imply that the inclination might be Ptolemy's *only* original contribution to the lunar theory.

Ptolemy says in *Almagest* 4.9 that he has revised various values for the Moon, Mercury, and Saturn because "we had got our hands subsequently on more indisputable observations".

For Mercury, the *Canobic Inscription* and the *PH* agree with each other, but not with the *Almagest*.

So…. *Almagest*, 1st Ed…*PH*…*Almagest*, 2nd Ed.?

Unresolved Issues

1. is there a plausible scenario for a *post–Almagest* transmission of the equant to India?

2. do we need to worry that Ptolemy's *third* latitude theory might be *pre–Almagest*?

3. do we need to worry that Ptolemy's *final* lunar theory might be *pre–Almagest*?

4. could the publishing history of the *Almagest* be more complicated than normally thought?

5. do we really understand the Pingree – van der Waerden Hypothesis?