Early Mathematical Astronomy: Stories from the Lost Years

Dennis Duke
Florida State University
Neugebauer Conference 2010

Recent developments in Greek kinematic astronomy during the years between Hipparchus and Ptolemy

Papyrus Fouad 267A
Antikythera Mechanism
Keskinto Inscription
India: trigonometry, planets, moon

Standard Solar Model
(according to the *Almagest)*

tropical year $Y_t = 365^d + \frac{1}{4} - \frac{1}{300}$

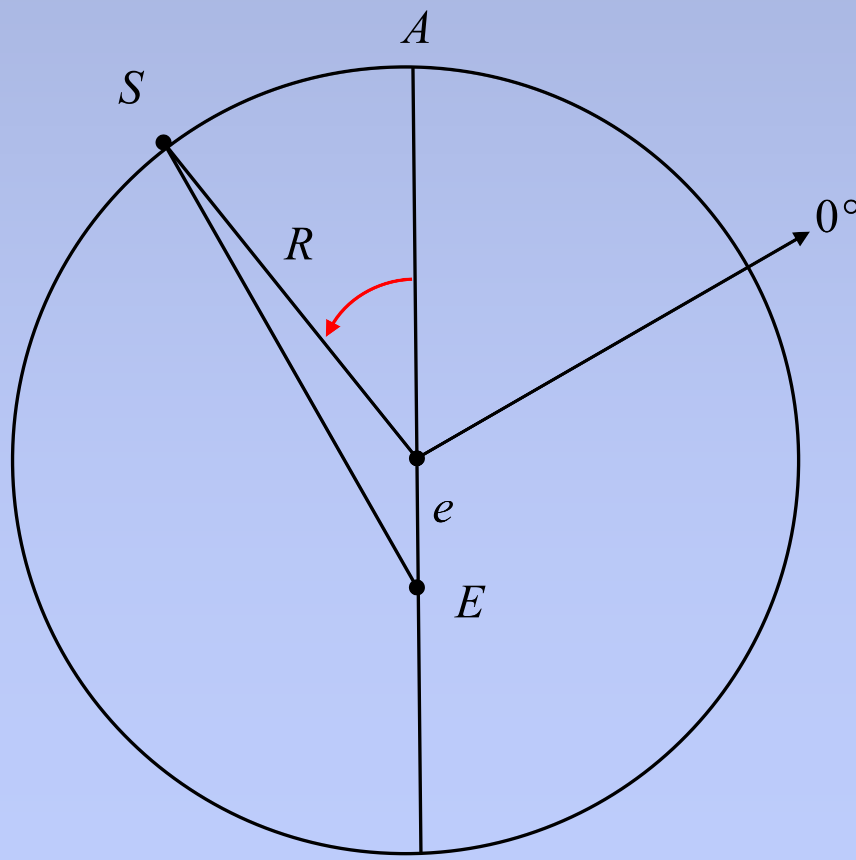
sidereal year $Y_s = 365^d + \frac{1}{4} + \frac{1}{147}$

precession $\pi_t = \frac{1°}{100\ ty}$

single eccentric anomaly: $e = 2;30$

$R = 60$

tropically fixed apogee: $A = 65;30°$



P. Fouad 267 A

Anne Tihon, *Ptolemy in Perspective* (2010)
supplemented by Jones, *PiP,* and Britton (unpub.)

- For a horoscope, calculates the mean and tropical longitude of the Sun at +130 Nov 9 3:20 AM (AMT) (9^{th} seasonal hour of the night)
- Three year lengths ‘conforming to the observations of Hipparchus’:

$$Y_s = 365^d + \tfrac{1}{4} + \tfrac{1}{102}$$

$$Y_j = 365^d + \tfrac{1}{4} \qquad \pi_j = \frac{6°}{625\ ey}$$

$$Y_t = 365^d + \tfrac{1}{4} - \tfrac{1}{309} \qquad \pi_t = \frac{8°}{625\ ey} \simeq \frac{1°}{78y}$$

- A summer solstice at -157 June 26 9 pm (AMT) associated with Hipparchus
- Mean motions from ‘...the table of the *Syntaxis*...’ with slightly adjusted year lengths: 102 → 102 2/3 and 309 → 307 1/6
- an epoch at -37,244 Thoth 1 era Philip (-323 Nov 12), and a secondary epoch 37,500 *ey* later (-158 Oct 2) $\left[\, 37500 = 2^2 \cdot 3 \cdot 5^5 = 60 \cdot 625 \,\right]$

Reconstruction of *P Fouad* 267 A

t_0 — $\Delta t_{01} = 37,500\ ey$ — -158/10/2 t_1 — -157/6/26 t_{SS} — 130/11/9 t_2

$L_s = A_s = \lambda_s = 73;55,18°$

A sidereally fixed

$\lambda_t = 90°$

$\mathrm{L}_t = 90;55,36°$

$\Rightarrow a, g(e,a)$

$\Rightarrow \mathrm{e} = 2;29,58$

$A_t = 67;9°$

$L_s = 228;30°$

$a = L_s - A_s = 154;34,43°$

$\mathrm{e} = 2;30$

$\mathrm{g} = \text{-}1;3,53°$

$\lambda_t = 224;21°$

$\mathrm{L}_t = \lambda_t - g = 225;24,53°$

Neugebauer, *HAMA,* p297-8: $126007^d 1^h = 345^r - 7\frac{1}{2}°$

acc. +1/101.4

Thus one finds by simple division

$$1 \text{ sid. rot.} = 365;15,35,29,28,\ldots^d \approx 365\ 1/4\ 1/100 \qquad (3)$$

for the length of the sidereal year.

The corresponding difference between sidereal and tropical year is therefore

$$\Delta t = 365;15,35,29 - 365;14,48 = 0;0,47,29^d$$

requiring a solar motion of

$$0;0,47,29 \cdot 0;59,8 = 0;0,46,47,51°.$$

Hence (2) implies

$$\text{precession per year: } 0;0,46,48° \text{ or } 1° \text{ precession in 77 Eg. y.} \qquad (4)$$

It seems hardly possible to assume that Hipparchus in his investigations of the differences between sidereal and tropical years could have overlooked such a direct consequence of some of his basic parameters. Hence one must conclude

$$Y_s = 365^d + \tfrac{1}{4} + \tfrac{1}{102}$$

$$\pi_t = \frac{8°}{625\ ey} \simeq \frac{1°}{78y}$$

Hipparchus	accurate
-161/9/27 6 pm	(9/27 2 am)
-158/9/27 6 am	(9/26 8 pm)
-157/6/26 9 pm	(6/26 6 pm) the *only* Hipparchan solstice or equinox *not* at 6^h or 12^h
-157/9/27 noon	(9/27 2 am)
-146/9/27 midnight	(9/26 6 pm)
etc....down to	
-127/3/23 6 pm (20 in all)	

Antikythera Mechanism
(discovered in a ~100 BC shipwreck in 1901)

Price (1970s)
Bromley(1990s)
Wright(1990s-present)

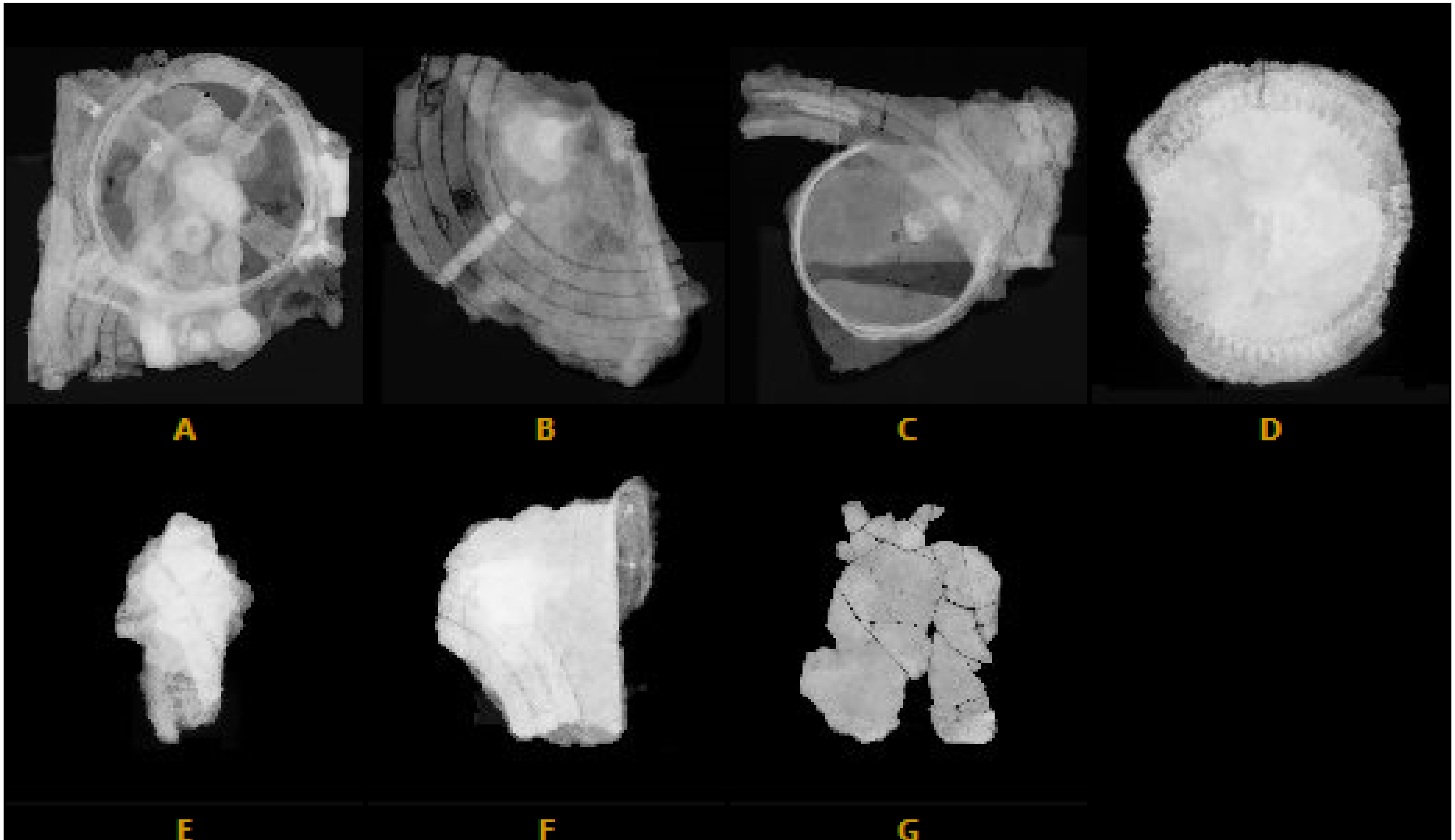
Nature (2006)
Freeth et al. (AMRP)

Nature (2008)
Freeth, Jones, Steele, Bitsakis

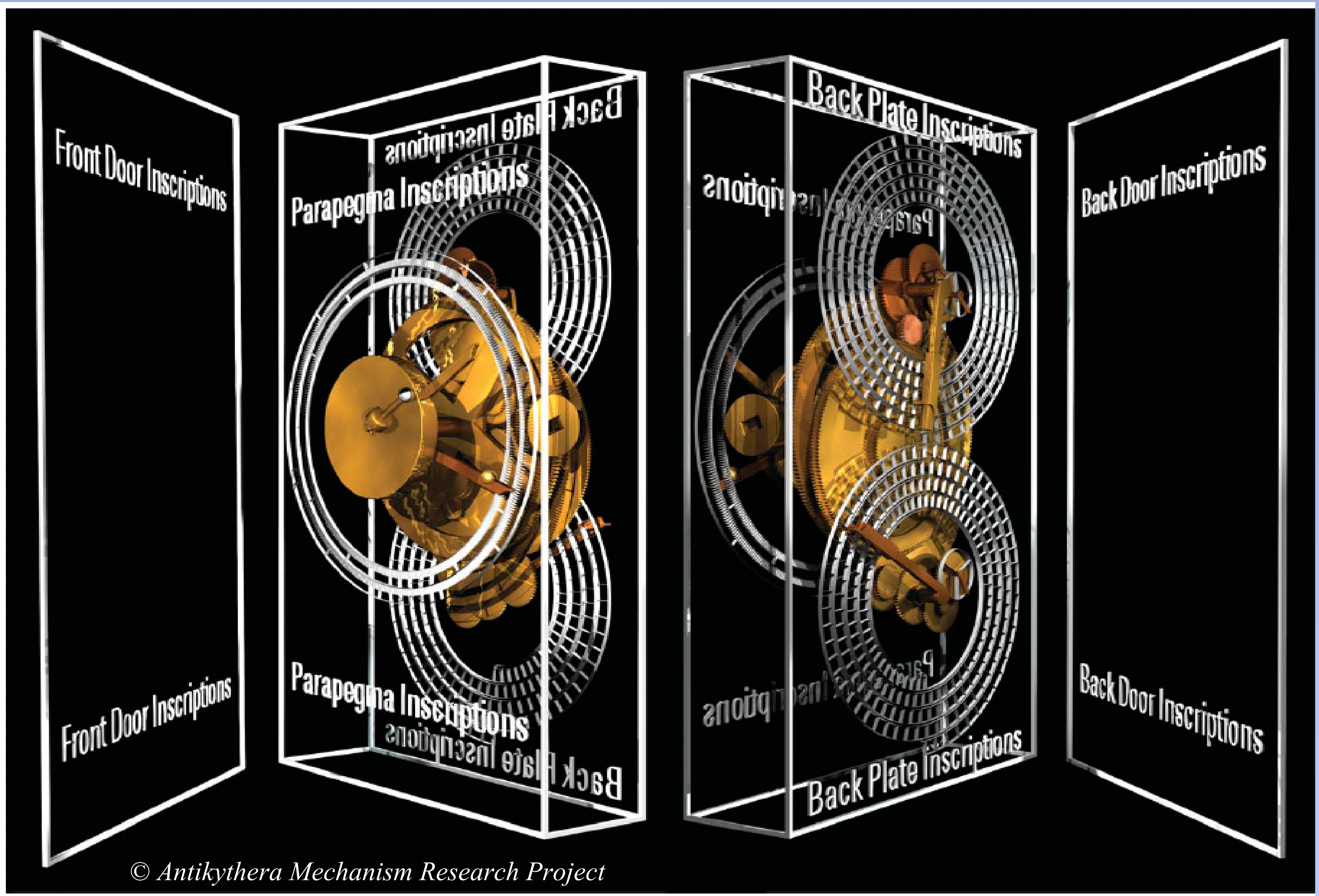
JHA (2010)
Evans, Carman, Thorndike

Many working models,
Youtube videos, etc.

Radiographs of 7 major fragments

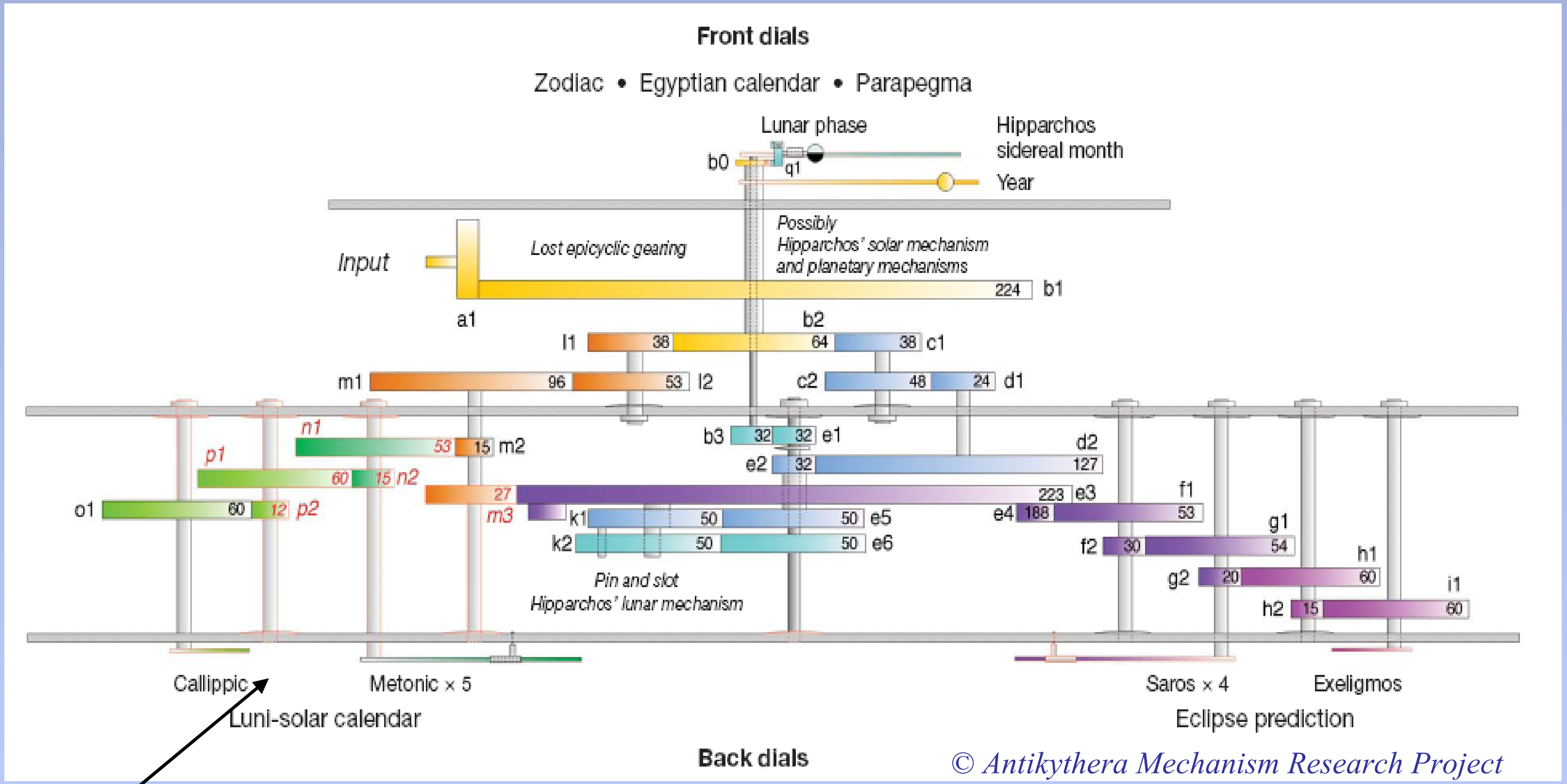


Antikythera Mechanism



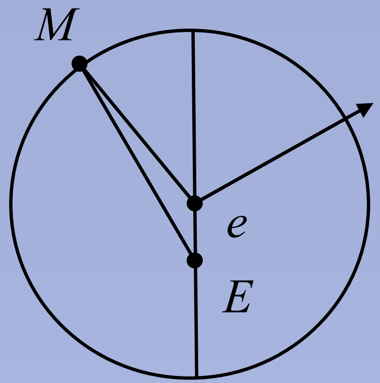
about 12.4" × 7.5" × 4"

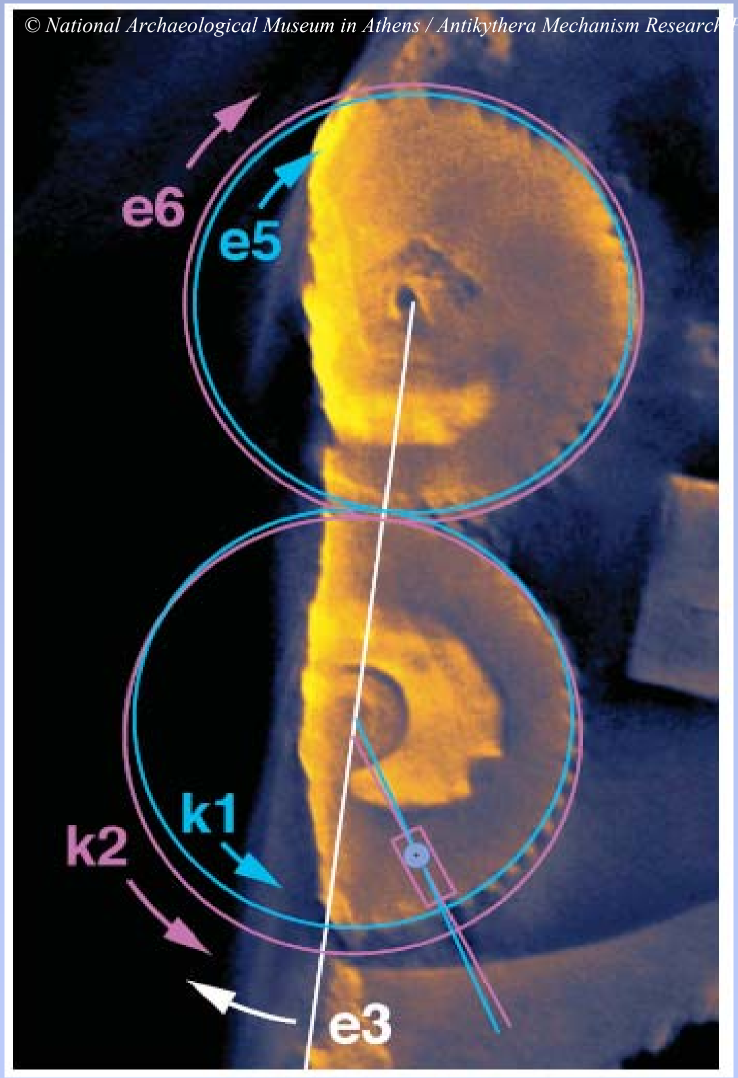
Antikythera gears



civil use of an astronomical calendar (Freeth, Jones, Steele, Bitsakis)

Pin-and-slot mechanism for lunar anomaly

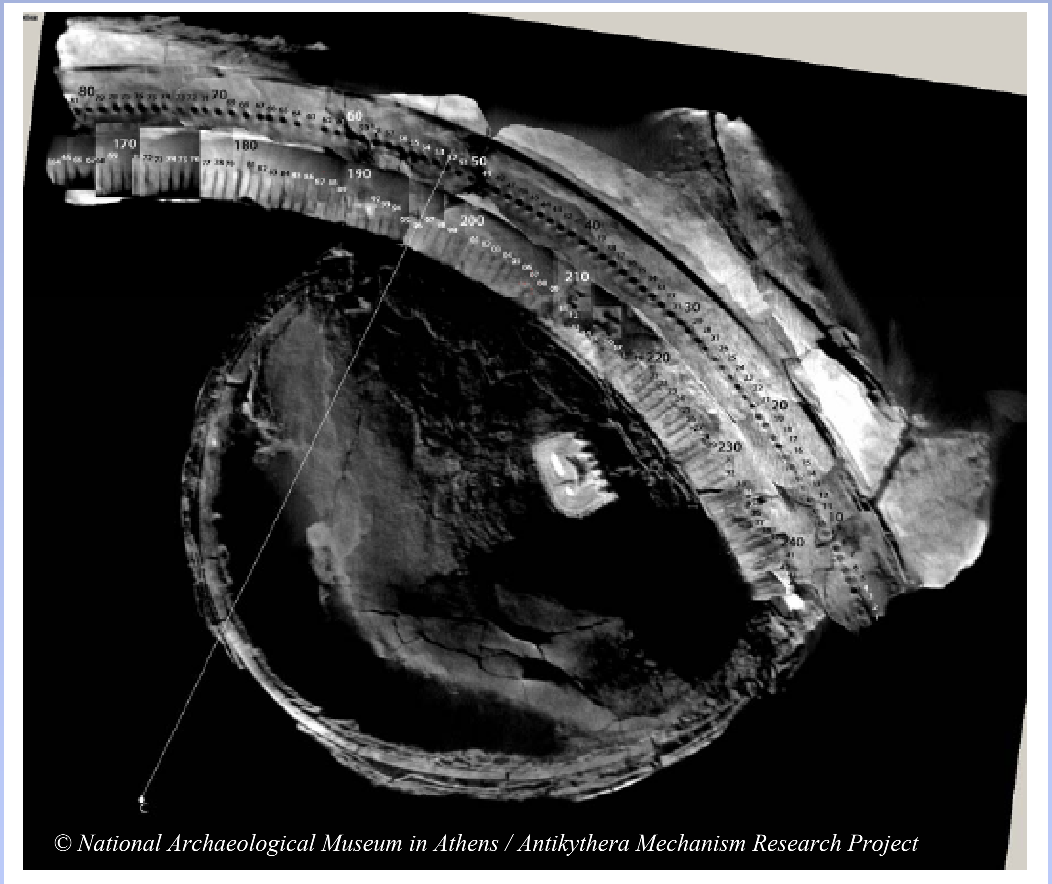


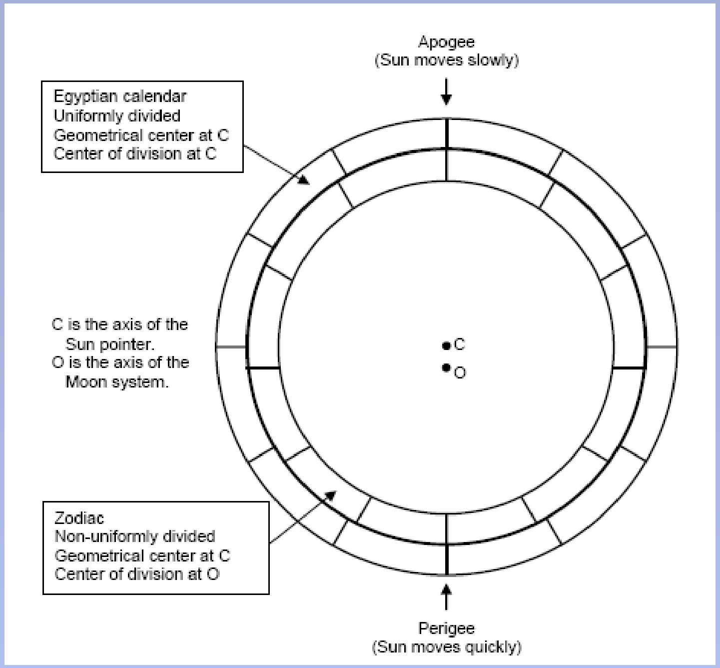


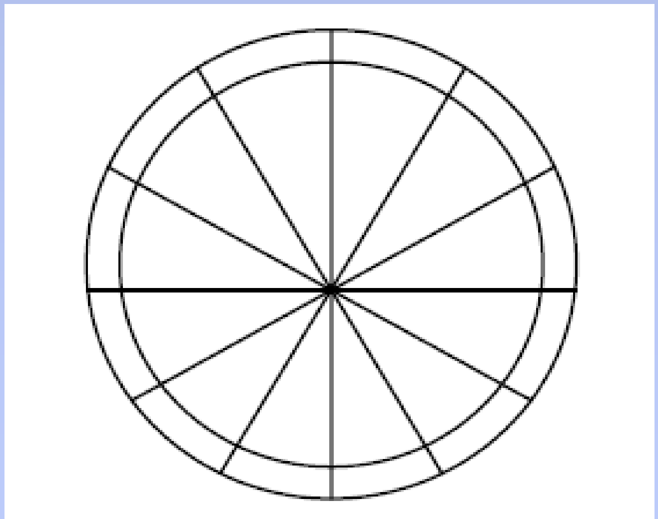
Solar inequality on the Antikythera Mechanism

Evans, Carman, Thorndike, *JHA* (2010)

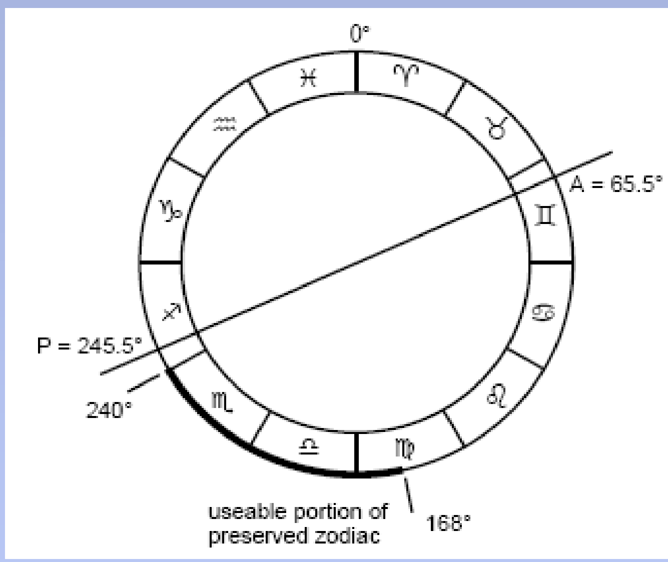
Photoshop composite of multiple CT scans

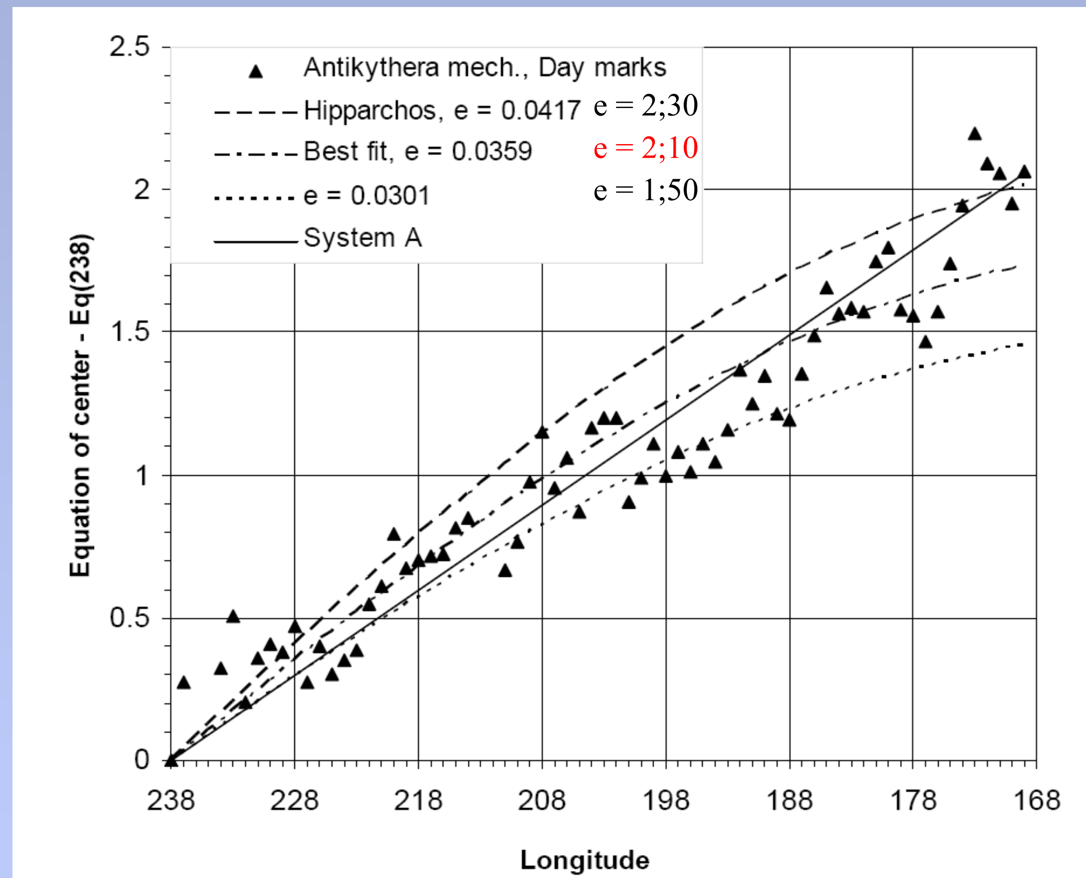






the implied solar anomaly

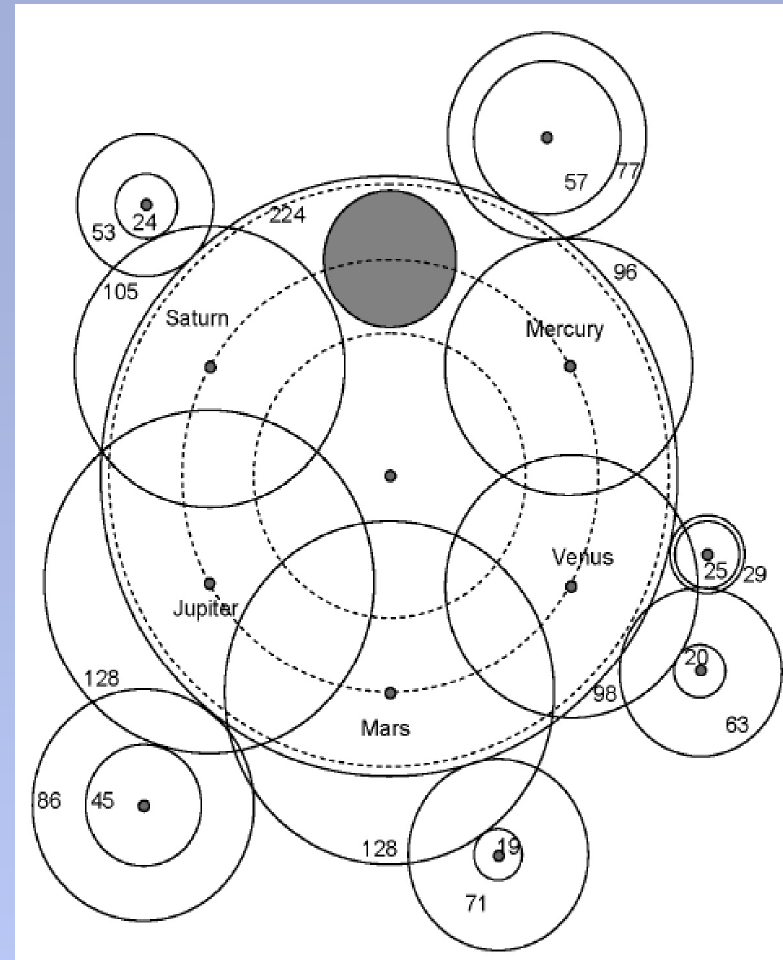




Null hypothesis (no solar anomaly), a horizontal line, is clearly ruled out

Planetary mean motions

	revs	yrs
Venus	720	1151
Mars	133	284
Saturn	256	265
Jupiter	315	344
Mercury	1223	388
	- 684	- 217
	= 539	171



Keskinto Inscription

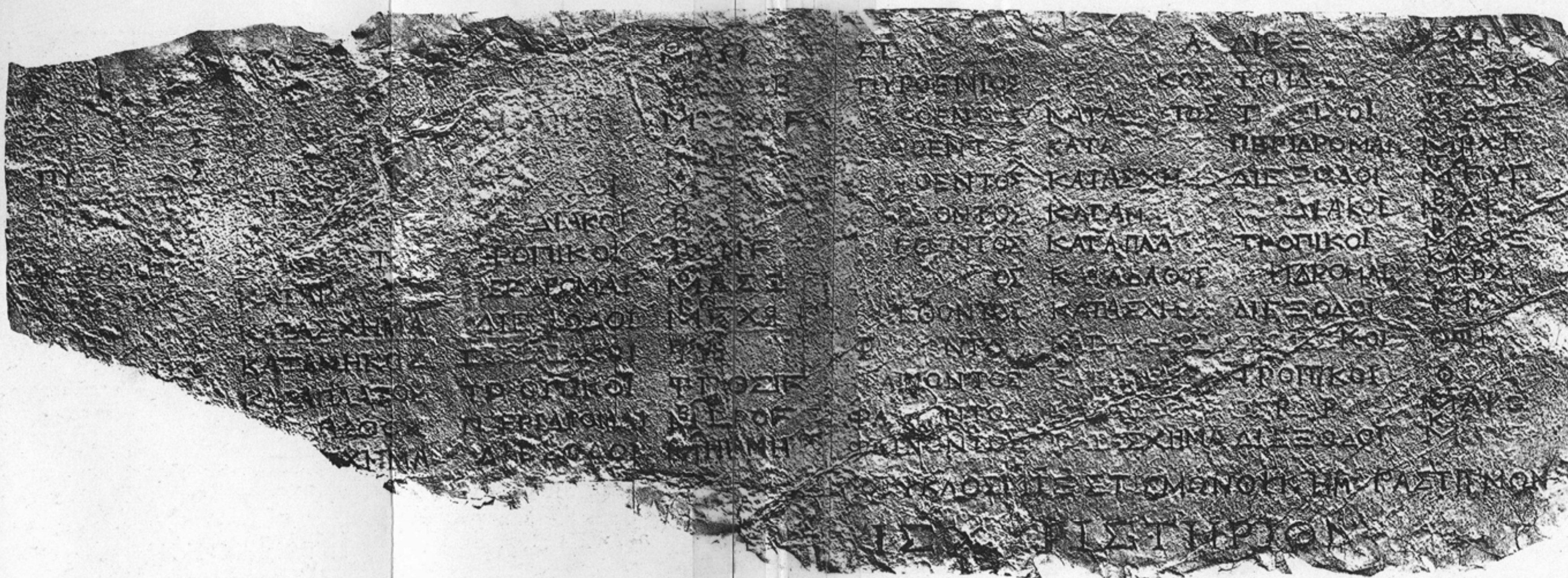
Fig. 1. The Keskintos Astronomical Inscription (*IG* 12.1, 913, = SK 14472)
Staatliche Museen zu Berlin, Preußischer Kulturbesitz, Antikensammlung, Photo Johannes Laurentius.

Tannery 1895

Neugebauer 1975 (*HAMA* 698-705)

Jones 2006

Tannery’s rubbing



KESKINTO

(Rhodes)

Reproduction d'un document dont les caractères ont été précisés au crayon

PAR PAUL TANNERY

Alexander Jones' text...

i	ii	iii	iv	v	vi	vii	viii
Σ[τίλβοντος]	[κατὰ σχῆμα]	[διέξοδοι]	. . . .	Στίλβ[οντος]	[κατὰ] σχῆμα	διέξοδο[ι]	$\overset{\text{[ϘΑ]}}{\text{M}}$ [ʽ]ΗΥ[ ]
Πυ[ρόεντ]ος	κατὰ μῆκ[ο]ς	[ζωι]διακοὶ	$\overset{\text{A}}{\text{M}}$ ʽΕΥϘΒ —	Πυρόεντος	κατὰ μῆκος	ζωιδια[κοὶ]	$\overset{\text{ΙΕ}}{\text{M}}$ ʽΔϡΚ
Πυρόεντος	κατὰ πλάτ[ος]	[τρο]πικοὶ	$\overset{\text{A}}{\text{M}}$ ʽΕΥΛϚ [—]	Πυρόεντος	κατὰ πλάτος	τρ[ο]πικοὶ	$\overset{\text{ΙΕ}}{\text{M}}$ ʽΔΤΞ
Πυρόεντος	κατὰ βά[θος]	[περι]δρομαὶ	$\overset{\text{Δ}}{\text{M}}$ϡΞ . —	Πυρόεντος	κατὰ βάθος	περιδρομαὶ	$\overset{\text{M}}{\text{M}}$ ʽΑΧΝ
Πυρόεντος	κατὰ σχ[ῆμα]	[διέ]ξοδοι	$\overset{\text{A}}{\text{M}}$ ʽΓ[ΧΜ]Η —	Πυρόεντος	κατὰ σχῆμα	διέξοδοι	$\overset{\text{ΙΓ}}{\text{M}}$ ʽϚΥΠ
Φαέθοντος	κατὰ [μῆ]κος	[ζ]ωιδιακοὶ	[ʽ]ΒΥΝ —	Φαέθοντος	κατὰ μῆκος	ζωιδιακοὶ	$\overset{\text{B}}{\text{M}}$ ʽΔΦ
Φαέθοντος	κατ[ὰ πλ]άτος	τροπικοὶ	ʽΒΥΝϚ —	Φαέθοντος	κατὰ πλάτος	τροπικοὶ	$\overset{\text{B}}{\text{M}}$ ʽΔΦΞ
[Φαέ]θοντος	κατὰ βάθος	περιδρομαὶ	$\overset{\text{B}}{\text{M}}$ ʽΔΣΞ —	Φαέθοντος	κατὰ βάθος	περιδρομαὶ	$\overset{\text{ΚΔ}}{\text{M}}$ ʽΒΧ
[Φαέθ]οντος	κατὰ σχῆμα	διέξοδοι	$\overset{\text{B}}{\text{M}}$ ʽϚΧϘ —	Φαέθοντος	κατὰ σχῆμα	διέξοδοι	$\overset{\text{ΚϚ}}{\text{M}}$ [ʽϚϡ]
[Φαίνο]ντος	κατὰ μῆκος	ζωιδιακοὶ	ϡϘΒ —	Φαίνοντος	κατὰ μῆκος	ζωιδιακοὶ	[ʽ]ΘϡΚ
[Φαίνοντος]	κατὰ πλάτος	τροπικοὶ	ϡΠΘ ΣΙϚ	Φαίνοντος	κατὰ πλάτ[ος]	τροπικοὶ	ʽΘΩϘϚ
[Φαίνοντος]	κατὰ βάθος	περιδρομαὶ	$\overset{\text{B}}{\text{M}}$ ʽΖΡΟϚ —	Φα[ίνο]ντος	κα[τ]ὰ βάθος	περιδρομαὶ	$\overset{\text{ΚΖ}}{\text{M}}$ ʽΑΨΞ
[Φαίνοντος]	[κατὰ] σχῆμα	διέξοδοι	$\overset{\text{B}}{\text{M}}$ ʽΗΡΜΗ —	Φαίνοντος	κατὰ σχῆμα	διέξοδοι	$\overset{\text{ΚΗ}}{\text{M}}$ ʽΑΥΠ

] . [. . . .] . . . ὁ κύκλος μο(ιρῶν) $\overline{\text{ΤΞ}}$, στιγμῶν $\overline{\text{[ ]ΘΨΚ}}$. ἡ μοῖρα στιγμῶν $\overline{\text{Κ[Ζ.]}}$

] . . αις χαριστήριον

...and translation

i	ii	iii	iv	v	vi	vii	viii
Mercury	[In relative position]	[passages]	xxxx	Mercury	[In] relative position	passages	[91]84xx
Mars	In longitude	zodiacals	15492	Mars	In longitude	zodiacals	154920
Mars	In latitude	tropicals	15436	Mars	In latitude	tropicals	154360
Mars	In depth	revolutions	4096x	Mars	In depth	revolutions	401650
Mars	In relative position	passages	13648	Mars	In relative position	passages	136480
Jupiter	In longitude	zodiacals	2450	Jupiter	In longitude	zodiacals	24500
Jupiter	In latitude	tropicals	2456	Jupiter	In latitude	tropicals	24560
Jupiter	In depth	revolutions	24260	Jupiter	In depth	revolutions	242600
Jupiter	In relative position	passages	26690	Jupiter	In relative position	passages	266900
Saturn	In longitude	zodiacals	992	Saturn	In longitude	zodiacals	9920
[Saturn]	In latitude	tropicals	989 216	Saturn	In latitude	tropicals	9896
[Saturn]	In depth	revolutions	27176	Saturn	In depth	revolutions	271760
[Saturn]	[In] relative position	passages	28148	Saturn	In relative position	passages	281480

]… A circle comprises 360 degrees or 9720 *stigmai*. A degree comprises 2[7] points.

] to … a thank-offering.

also, $29160 = 3 \cdot 9720 = 81 \cdot 360 = 162 \cdot 180 = 2^3 3^6 5$ (*recall* $37500 = 2^2 3\ 5^5$)

Sometimes, in Hindu texts, an *arcminute* comprises 27 *yohanas*.

27 ‘points’ to a degree

Canobic Inscription:

“..at the mean distance of the Sun and Moon at syzygies, the diameter of either luminary subtends at the sight $\frac{1}{162}$ of a right angle...”

implying that each body subtends 15 ‘points’.

Similarly, in the Hindu text *Pancasiddhantika* of Varahamihira (Neugebauer and Pingree 1971), likely derived from Greek sources, divides the Moon’s disk into 15 parts, and

Hipparchus, Ptolemy, and Hindu texts specify the sizes of planets and stars as fractions of the Sun’s diameter.

PAITĀMAHASIDDHĀNTA 479

III. 8. The measure of the [diameter of the] disc of the Sun is 6,500 [*yojana*-*s*]; that of the Moon 480; that of Mars 15; that of Mercury 60; that of Jupiter 120; that of Venus *240*; and that of Saturn 30.

THE TEN GĪTI STANZAS 15

5. A *yojana* consists of 8,000 times a *nṛ* [the height of a man]. The diameter of the Earth is 1,050 *yojanas*. The diameter of the Sun is 4,410 *yojanas*. The diameter of the Moon is 315 *yojanas*. Meru is one *yojana*. The diameters of Venus, Jupiter, Mercury, Saturn, and Mars are one-fifth, one-tenth, one-fifteenth, one-twentieth, and one-twenty-fifth of the diameter of the Moon. The years of a *yuga* are equal to the number of revolutions of the Sun in a *yuga*.

Great Year
(in Egyptian years)

$$29160^{ey} \cdot 365^{d/ey} = 29140^{r} \cdot \left(365 + \tfrac{1}{4} + \tfrac{1}{1942\frac{2}{3}}\right)^{d/r}$$

$$\simeq 29140^{r} \cdot 365\tfrac{1}{4}^{\,d/r}$$

~ 45 s

All planets:

$$L + A = 29140^{r} \text{ (solar revolutions)}$$

slowly moving apogee

Saturn $12^{\circ/y}$
or 1^{r} in 30^{y}

$$L = 992^{r} = \frac{1}{360^{\circ/r}}\left(12^{\circ/ey} + \left(\tfrac{20}{81}\right)^{\circ/ey}\right) \cdot 29160^{ey} = 972^{r} + 20^{r}$$

$$B = 989\tfrac{3}{5}^{\,r} = \frac{1}{360^{\circ/r}}\left(12^{\circ/ey}\left(1 - \tfrac{1}{405}\right) + \left(\tfrac{20}{81}\right)^{\circ/ey}\right) \cdot 29160^{ey}$$

$$A = 28148^{r} = 29140^{r} - L$$

$$G = 27176^{r} = A - (L - 20^{r})$$

very slowly moving node

The minus signs indicate a planet moving *clockwise* (the ‘wrong’ way)

Jupiter $30^{\circ/y}$
or 1^{r} in 12^{y}

$$L = 2450^{r} = \frac{1}{360^{\circ/r}}\left(30^{\circ/ey} + \left(\tfrac{20}{81}\right)^{\circ/ey}\right) \cdot 29160^{ey} = 2430^{r} + 20^{r}$$

$$B = 2456^{r} = \frac{1}{360^{\circ/r}}\left(30^{\circ/ey}\left(1 + \tfrac{1}{405}\right) + \left(\tfrac{20}{81}\right)^{\circ/ey}\right) \cdot 29160^{ey}$$

$$A = 26690^{r} = 29140^{r} - L$$

$$G = 24260^{r} = A - (L - 20^{r})$$

Saturn: 2^{r} in 60^{y} + apogee ~ 2^{r} in 59^{y}
Jupiter: 6^{r} in 72^{y} + apogee ~ 6^{r} in 71^{y}

Apparent underlying model

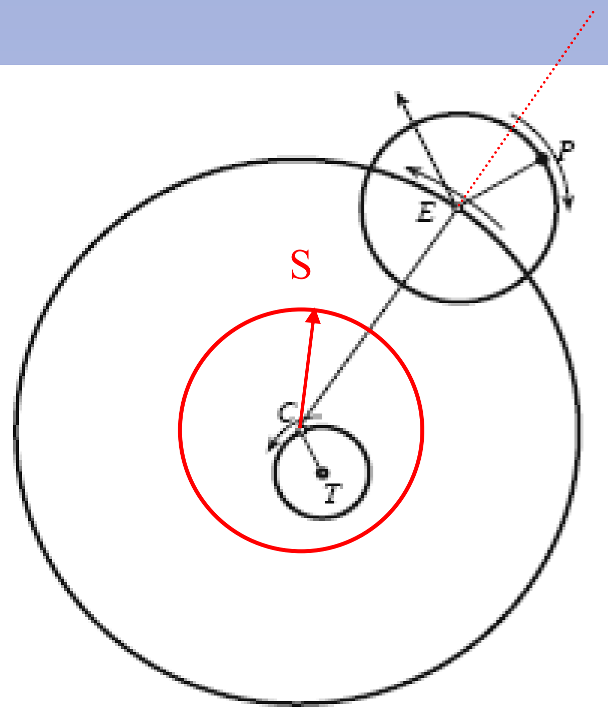


Fig. 2. Possible epicyclic model for Jupiter or Saturn in the Keskintos Inscription.

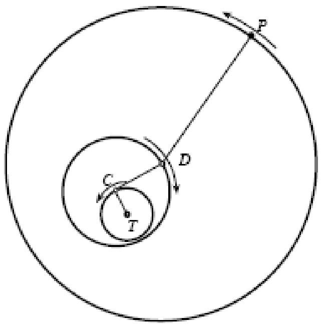


Fig. 3. Possible model for Jupiter or Saturn with revolving eccentre.

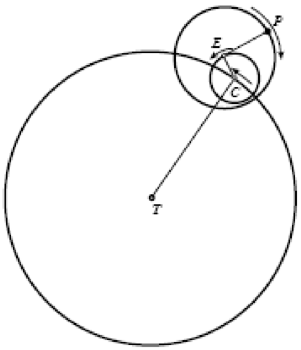


Fig. 4. "Eccentric epicycle" model for Jupiter or Saturn.

slowly moving apogee

$$L_{Saturn} = 992^r \quad = \frac{1}{360^{\circ/r}}\left(12^{\circ/ey} + \left(\tfrac{20}{81}\right)^{\circ/ey}\right)\cdot 29160^{ey} = 972^r + 20^r$$

$$L_{Jupiter} = 2450^r \quad = \frac{1}{360^{\circ/r}}\left(30^{\circ/ey} + \left(\tfrac{20}{81}\right)^{\circ/ey}\right)\cdot 29160^{ey} = 2430^r + 20^r$$

Theon of Smyrna mentions, quite routinely, a solar model with periods of 365¼ days in longitude, 365½ days in anomaly (hence a solar apogee moving ¼° per year), and 365 days in latitude.

Two papyrus fragments, *P. Oxy LXI.4174a* and PSI inv. 515, give kinematic solar motion tables that are consistent with the model parameters mentioned by Theon, and so remove all doubt that the models mentioned by Theon were actively used.

Mars 191°/y

$$L = 15492^r = \frac{1}{360^{\circ/r}}\left(191^{\circ/ey} + \left(\tfrac{21}{81}\right)^{\circ/ey}\right)\cdot 29160^{ey} = 15471^r + 21^r$$

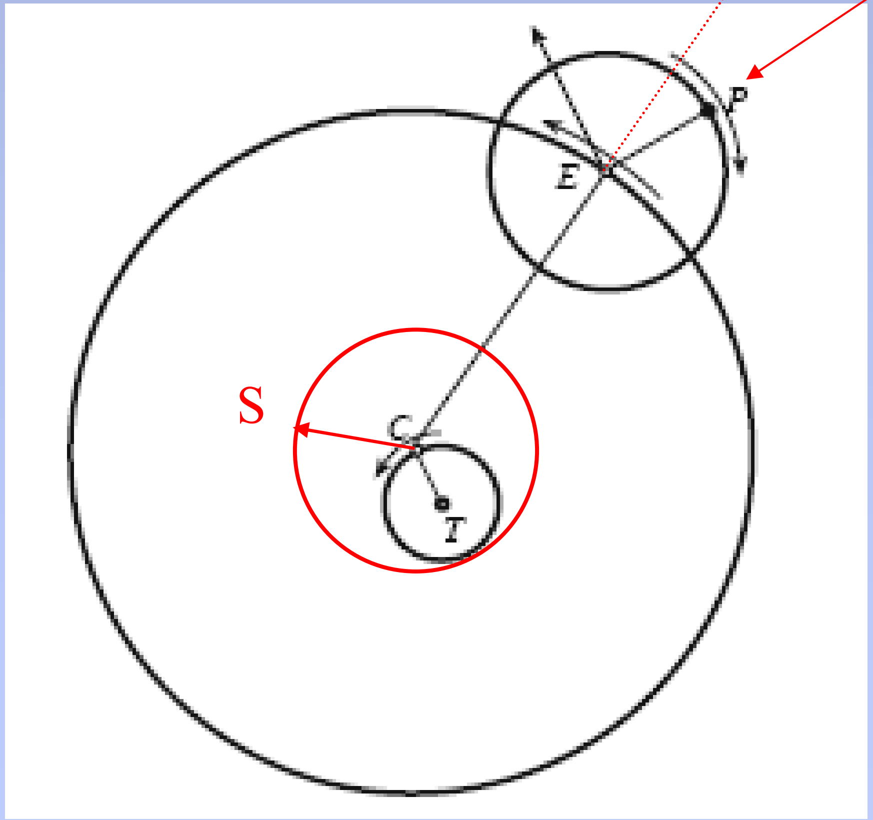
$$B = 2456^r = \frac{1}{360^{\circ/r}}\left(191^{\circ/ey}\left(1-\frac{280/191}{405}\right) + \left(\tfrac{21}{81}\right)^{\circ/ey}\right)\cdot 29160^{ey}$$

$$A = 13648^r = 29140^r - L$$

$$G = 40965^r = 3A + 21^r$$ (probably, but the reading is uncertain)

Mars: $191^{\circ/ey}$ + apogee ~ 42^r in 79^y (almost exactly)

3x, but this is all rather uncertain



Comparing Greek and Hindu Astronomy

There are many similarities between Greek and Hindu astronomy, but in general the level of development in the Hindu versions is lower than what we find in the *Almagest:*

- *The equation of time.*
- *Obliquity of the ecliptic.*
- *Parallax.*
- *Trigonometry scales.*
- *Retrograde motion.*
- *Model of Mercury.*
- *Determination of orbit elements.*
- *Values of orbit elements.*
- *Star catalog.*
- *Zodiacal signs.*
- *The second lunar anomaly.*

Thus the essentially universally accepted view that **the astronomy we find in the Indian texts is pre-Ptolemaic**. Summarizing the prevailing opinion, Neugebauer wrote in 1956:

"Ptolemy's modification of the lunar theory is of importance for the problem of transmission of Greek astronomy to India. The essentially Greek origin of the *Surya-Siddhanta* and related works cannot be doubted – terminology, use of units and computational methods, epicyclic models as well as local tradition – all indicate Greek origin.

But it was realized at an early date in the investigation of Hindu astronomy that the Indian theories show no influence of the Ptolemaic refinements of the lunar theory [2nd lunar anomaly].

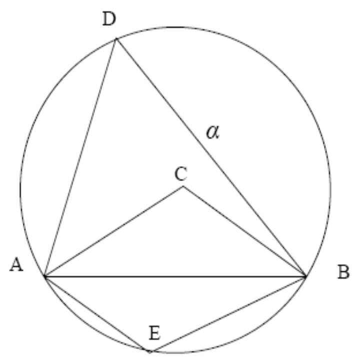
This is confirmed by the planetary theory, which also lacks a characteristic Ptolemaic construction, namely, the "*punctum aequans* [equant] ".

So if we are interested in what happened in Greek astronomy during 130 BC – 120 AD, Hindu texts may be a good place to look.

Let's look at some examples.

Greek-Hindu Trigonometry

Neugebauer (*PAPS* 1972)



$$\mathrm{crd}\,\alpha = \frac{AB}{R'} = 2\sin\frac{\alpha}{2}$$

$$R' = \frac{360^\circ \cdot 60'/^\circ}{2\pi} = \frac{21,600}{2\pi} \simeq 3438$$

or $$D' = \frac{21600}{\pi} \simeq 6875$$

Table of chords

Angle(degrees)	Chord
0	0
7 ½	450
15	897
22 ½	1341
30	1780
37 ½	2210
45	2631
52 ½	3041
60	3438
67 ½	3820
75	4186
82 ½	4533
90	4862
97 ½	5169
105	5455
112 ½	5717
120	5954
127 ½	6166
135	6352
142 ½	6511
150	6641
157 ½	6743
165	6817
172 ½	6861
180	6875

ĀRYABHAṬĪYA

10. The (twenty-four) sines reckoned in minutes of arc are 225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22, 7.

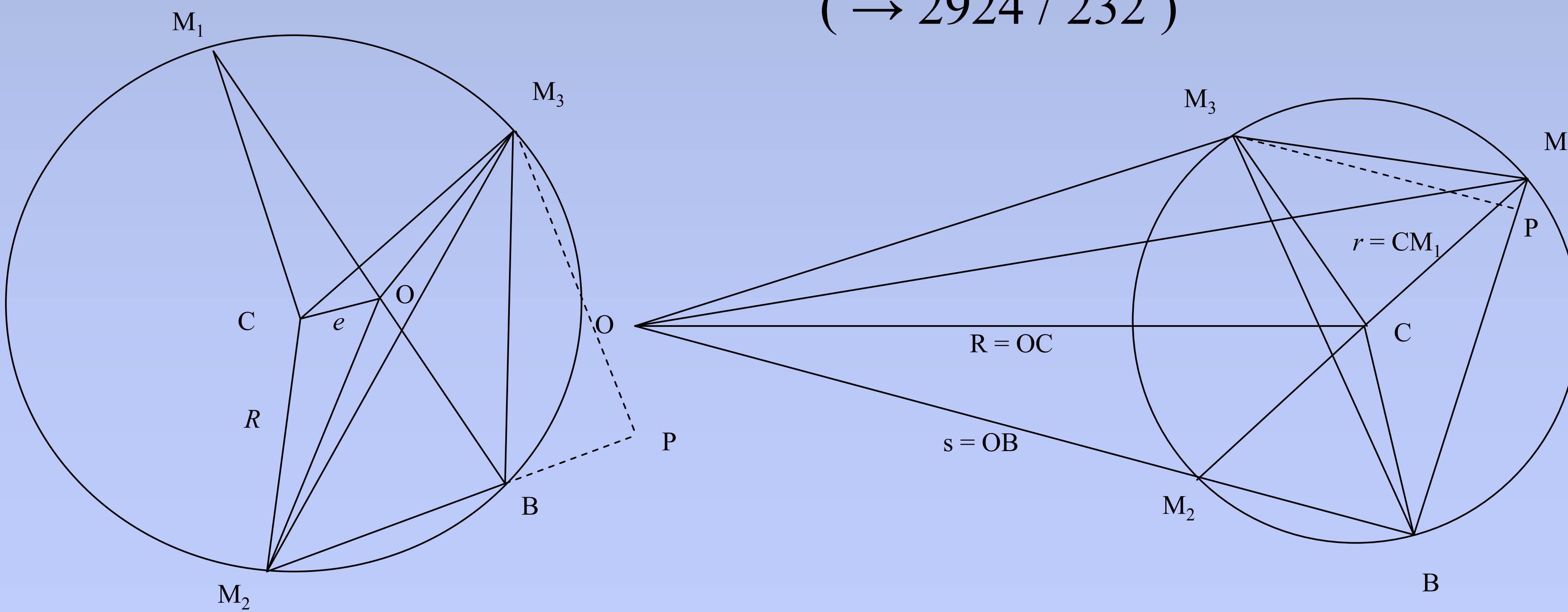
In Indian mathematics the "half-chord" takes the place of our "sine." The sines are given in minutes (of which the radius contains 3,438) at intervals of 225 minutes. The numbers given here are in reality not the values of the sines themselves but the differences between the sines.

Toomer (*Centaurus*, 1973)

from *Almagest* 4.11 and Hipparchus

$R / e = 3144 / 327\frac{2}{3}$ $\quad$ $R / r = 3122\frac{1}{2} / 247\frac{1}{2}$

$(\rightarrow 2924 / 232)$



$$M_1P = M_1B - \frac{M_3B \,\mathrm{Crd}\, 2\left(\frac{180^\circ - \alpha_3}{2}\right)}{2R'} = M_1B - \frac{M_3B \,\mathrm{Crd}\, 128;27^\circ}{2R'}$$

$$= s\left(\frac{1000}{6669\frac{1}{3}} - \frac{1112\frac{1}{2}\cdot 6189\frac{1}{2}}{6750\frac{1}{2}\cdot 2\cdot 3438}\right) = s\left(\frac{515\frac{1}{2} - 510\frac{1}{6}}{3438}\right) = s\cdot\frac{5\frac{1}{3}}{3438}.$$

$$M_1M_3 = \sqrt{M_3P^2 + M_1P^2} = s\cdot\frac{246\frac{1}{3}}{3438}. \qquad (1)$$

$$r = \frac{M_1M_3\cdot R'}{\mathrm{Crd}\,\alpha_3} = s\cdot\frac{246\frac{1}{3}}{3438}\cdot\frac{3438}{2989} = s\cdot\frac{246\frac{1}{3}}{2989}.$$

=3162 x 231 3/4 / 2960 2/5

$$\mathrm{Crd}\,\widehat{M_3CB} = \frac{M_3B\cdot R'}{r} = \frac{s\cdot 1112\frac{1}{2}\cdot 2989\cdot 3438}{s\cdot 6750\frac{1}{2}\cdot 246\frac{1}{3}} = 6875.$$

=247 1/2

$$\pi \simeq \sqrt{10} \simeq 3.1622...$$

This suggests that Hipparchus was using a circle of circumference 20,000 (*i.e.* two Greek myriads), and hence a radius of

$$s = \frac{20,000}{2\pi} = \frac{10,000}{\sqrt{10}} = 1,000\sqrt{10} \simeq 3162$$

of heaven by [the number of] its revolutions [in a Kalpa]; that [planet] which makes many revolutions has an inferior orbit, that which makes few a superior orbit. The diameter of the orbit equals the circumference divided by the square-root of 10; the planet is situated above the earth by [half] the height of this. The circumference equals the diameter multiplied by the square-root of 10. Thus are computed the circumferences and diameters of all circles.

Hindu Mean Motions

$37,500 = 2^2 3\ 5^5$

$29,160 = 2^3 3^6 5$

$4,320,000 = 2^8 3^3 5^4$

t_0 — $\Delta t_{01} = 3,240,000\ y$ — t_1 (-3101/2/18 6 am) — $\Delta t_{12} = 3,600\ y$ — t_2 (499/3/21 noon) — $t_0 + 4,320,000$ y

$L_p = 0°$ (for all planets) $\qquad L_p = 0° \qquad L_p = 0°$

Similar Great-Year structure to *P Fouad* 267A and the *Keskinto Inscription*

Cycles of Time: An Extraordinary New View of the Universe Roger Penrose *Bodley Head*: 2010. 320 pp.

"It is possible that our early universe is the late universe of a previous era. This is Penrose's big idea: deliciously absurd, but just possibly true."

....the essentially Greek origin of the *Surya-Siddhanta* and related works cannot be doubted – terminology, use of units and computational methods, epicyclic models as well as local tradition – all indicate Greek origin (Neugebauer, 1956).

e.g. for Jupiter:

$$\omega = \frac{364,224^r}{4,320,000^y} = 0;5,3,31,12^{r/y}$$

$$\frac{R_L}{Y} = \frac{83-76}{83} = 0;5,3,36,52...$$

$$L_{Jup} = 187;12°$$

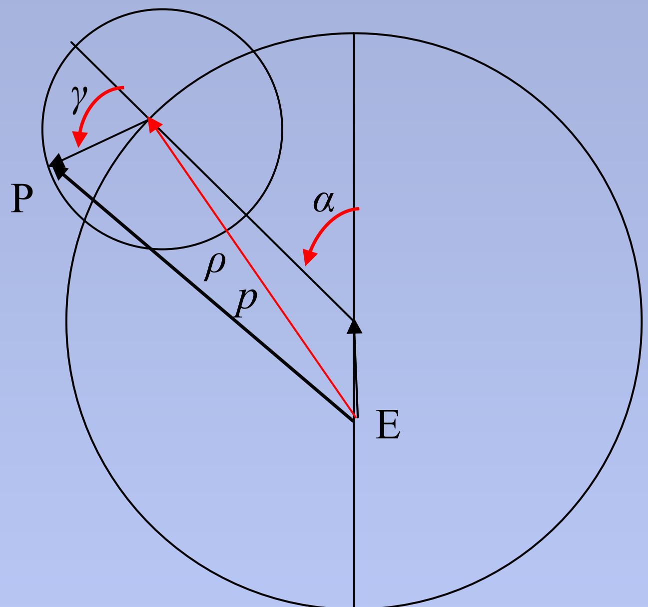
$$= 187;12° \cdot \frac{1^r}{360°}$$

$$= 0;31,12^r$$

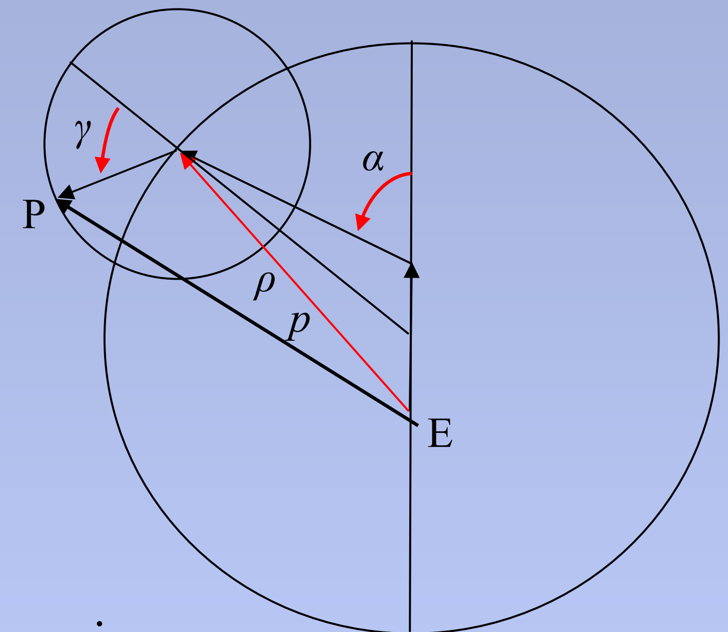
There is a *constraint:*
L must be a multiple of 1;12°

Greek-Hindu Planetary Models

eccentric plus epicycle model



equant plus epicycle model

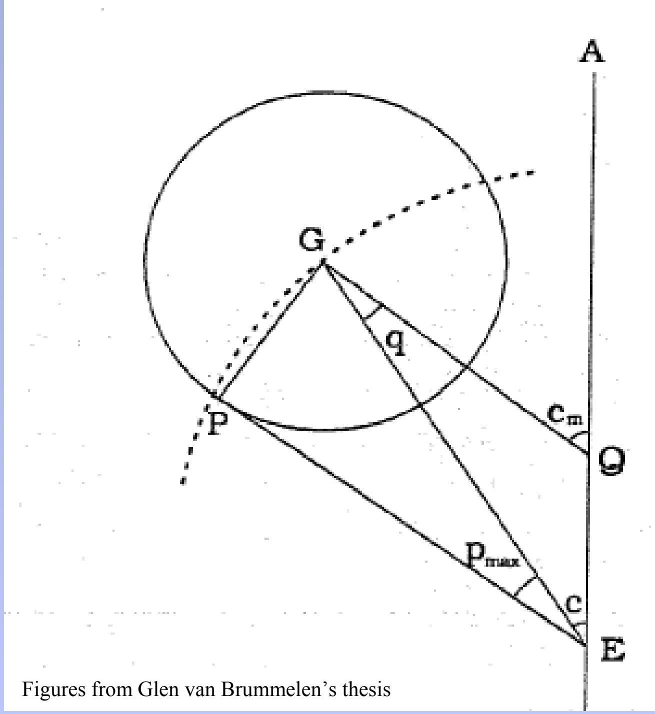


$$\tan p(\alpha,\gamma) = \frac{-r\sin\gamma}{\rho(\alpha) + r\cos\gamma}$$

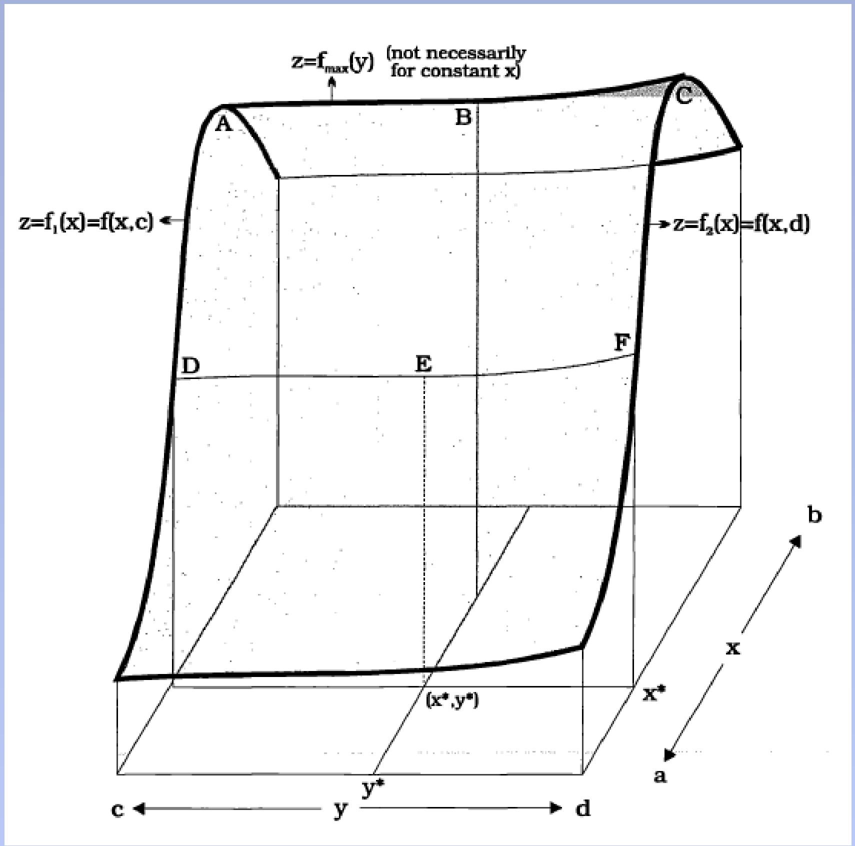
The ***final longitude is a function of two variables***, so the computation is probably too complicated for the primary customers (astrologers) and ***a simplification that uses only single variable functions is needed***.

We know of two schemes.

1. The *Almagest* Solution:
A sophisticated interpolation scheme



Figures from Glen van Brummelen's thesis



$$f(x,y) = f_1(x) + \frac{H(y) - H(c)}{H(d) - H(c)} \cdot [f_2(x) - f_1(x)]$$

$$H(y) = f_{\max}(y) = \max\{f(x,y) : x \in [a,b]\}$$

2. The Hindu Solution: factorization by iteration

eccentric orbits (*manda*) for the zodiacal anomaly

$$\sin q(\alpha) = -e \sin \alpha$$

epicycles (*sighra*) for the solar anomaly

$$\tan p(\gamma) = \frac{r \sin \gamma}{1 + r \cos \gamma} \neq \frac{r \sin \gamma}{\rho + r \cos \gamma}$$

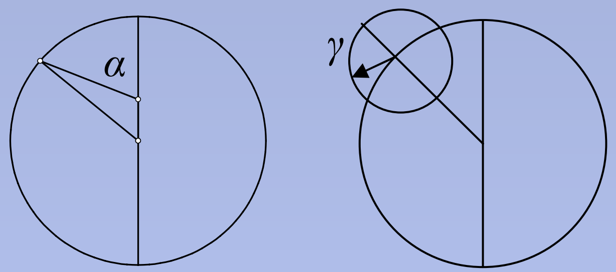
(1) $\alpha = \bar{\lambda} - \lambda_A \qquad \nu_1 = \bar{\lambda} + \tfrac{1}{2} q(\alpha)$

(2) $\gamma = \bar{\lambda}_S - \nu_1 \qquad \nu_2 = \nu_1 + \tfrac{1}{2} p(\gamma)$

(3) $\alpha = \nu_2 - \lambda_A \qquad \nu_3 = \bar{\lambda} + q(\alpha)$

(4) $\gamma = \bar{\lambda}_S - \nu_3 \qquad \lambda = \nu_3 + p(\gamma)$

...but what is the Hindu solution an approximation *to*?

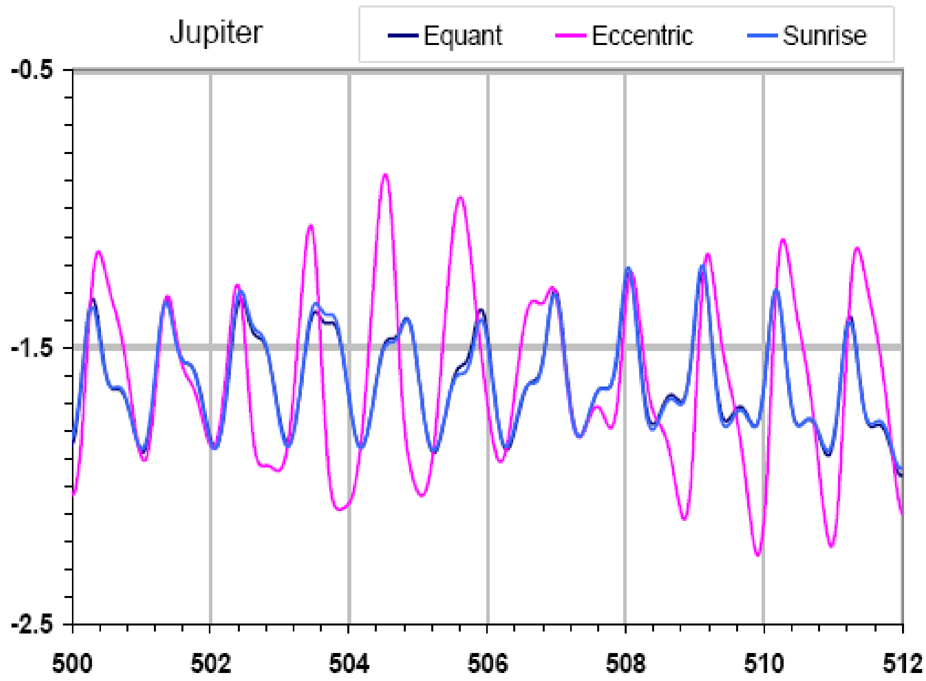


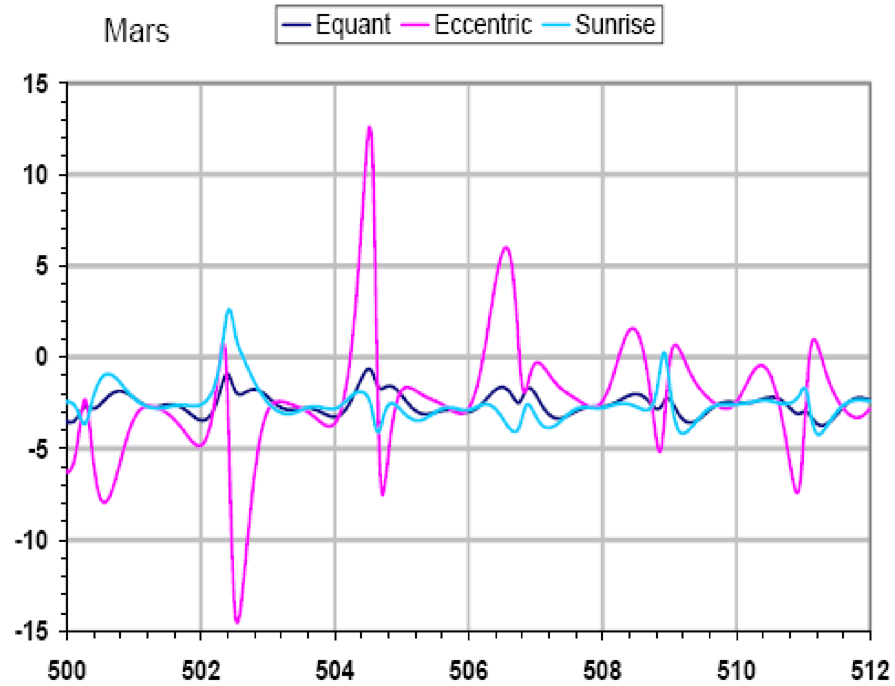
		$q(\alpha)$ Mars equation of center		$p(\gamma)$ equation of anomaly	
6	354	1	12	2	25
12	348	2	23	4	49
18	342	3	33	7	13
24	336	4	40	9	36
30	330	5	45	11	59
36	324	6	46	14	20
42	318	7	42	16	40
48	312	8	33	18	59
54	306	9	19	21	16
60	300	9	59	23	30
66	294	10	32	25	42
72	288	10	58	27	50
78	282	11	17	29	55

etc...

we can see by using *identical* orbit elements in both models

Below is plotted the discrepancy between modern theory and three ancient models.



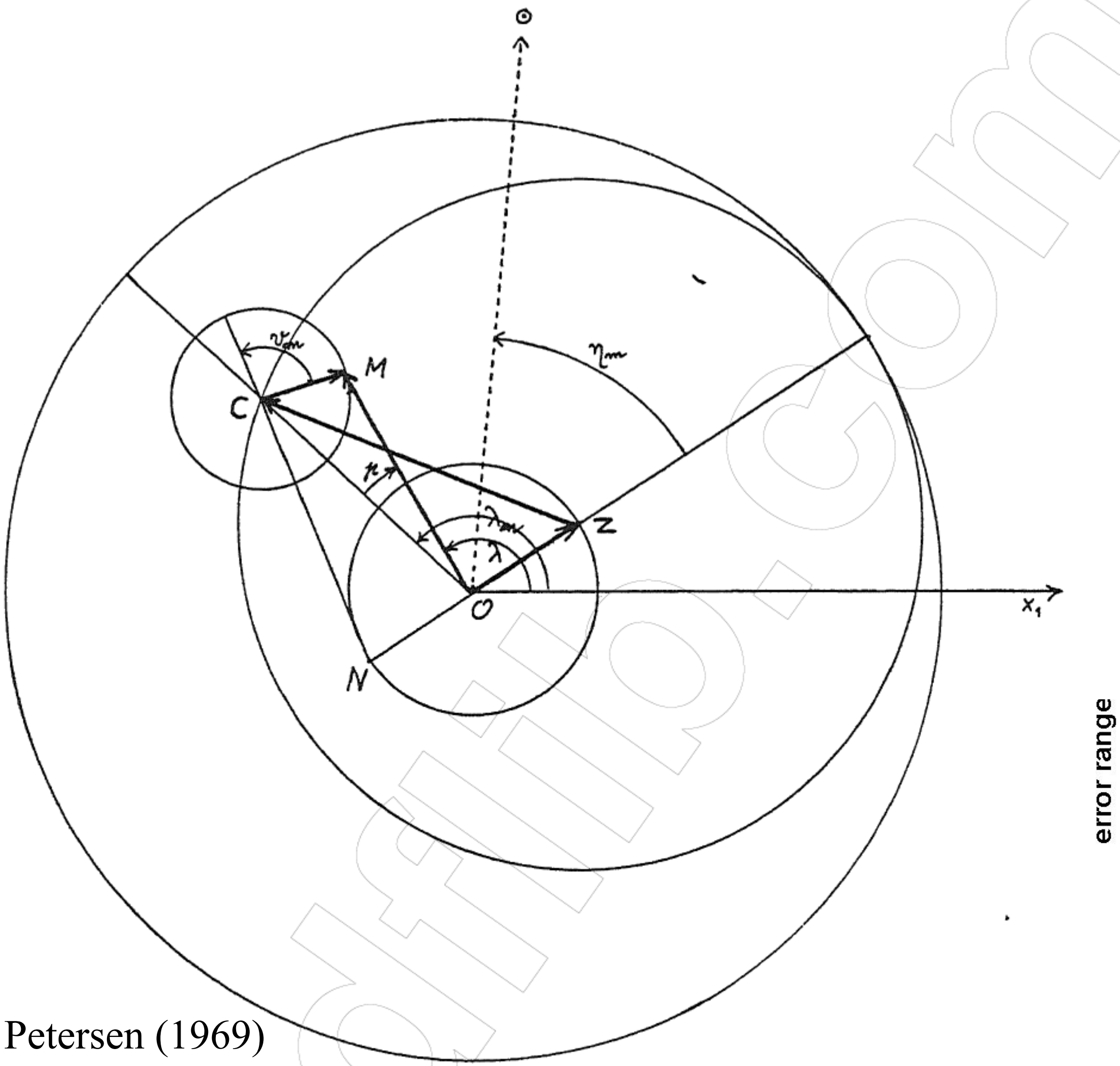


The Hindu scheme is clearly approximating the *equant*, not the eccentric.

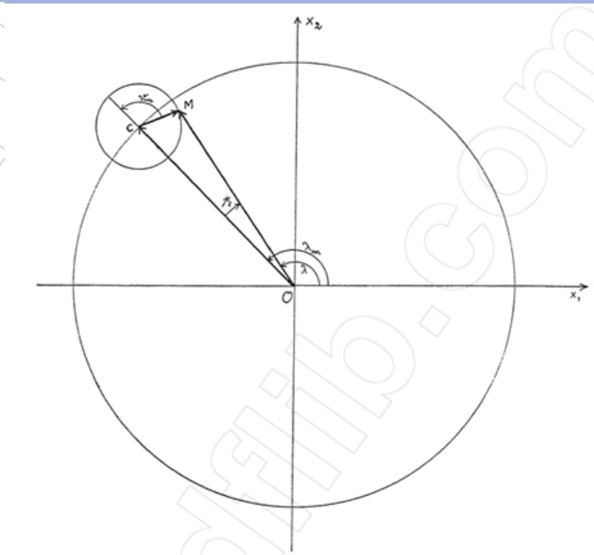
The approximation is excellent for Jupiter (and Saturn), and pretty good for Mars, but definitely not as good as the *Almagest* interpolation scheme.

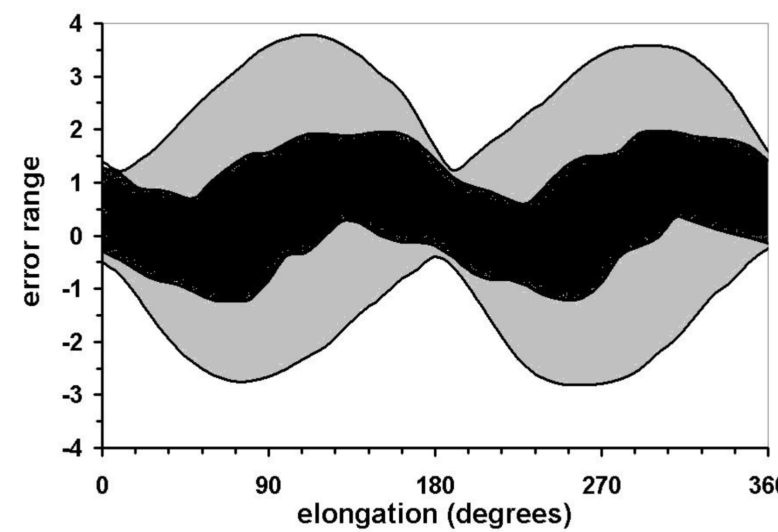
Greek-Hindu Lunar Models

Almagest:
At syzygy, r = 5;15, epicycle; otherwise, a crank mechanism and adjusted epicycle apogee



Petersen (1969)





Greek-Hindu Lunar Models

Hindu models:

At syzygy, r = 5;15, concentric equant;

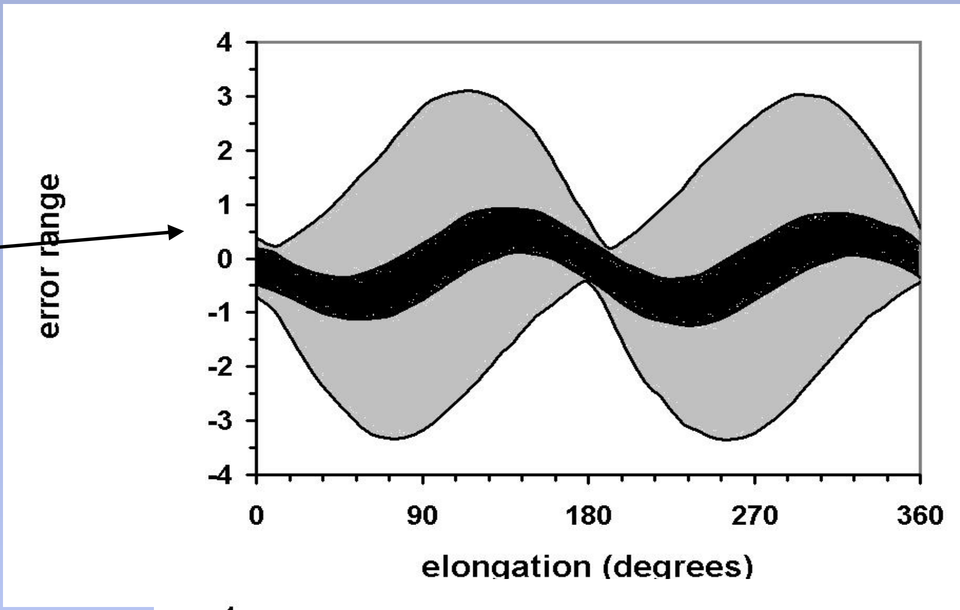
Otherwise

$$q = \lambda - L$$

$$= -5;01^\circ \sin\alpha - 2;29^\circ \cos\psi \sin\eta$$

Aryabhata *et al.* (*ca.* 500 AD)

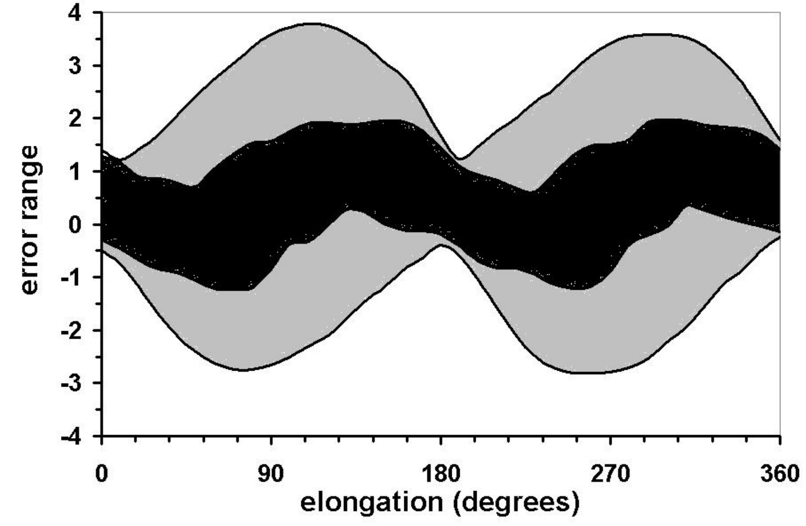
Munjala, Vatesvara(?) (*ca.* 902-930 AD)



Almagest (*ca.* 150 AD):

At syzygy, r = 5;15, epicycle;

otherwise, a crank mechanism and adjusted epicycle apogee



Modern lunar theory

$\alpha = L - A =$ mean lunar anomaly

$\eta = L - L_S =$ mean lunar elongation from Sun

$\psi = L_S - A$

$\alpha = \eta + \psi$

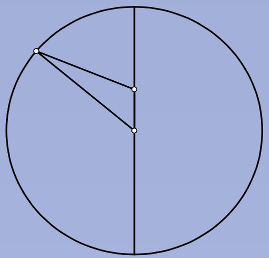
The first two terms in modern theory:

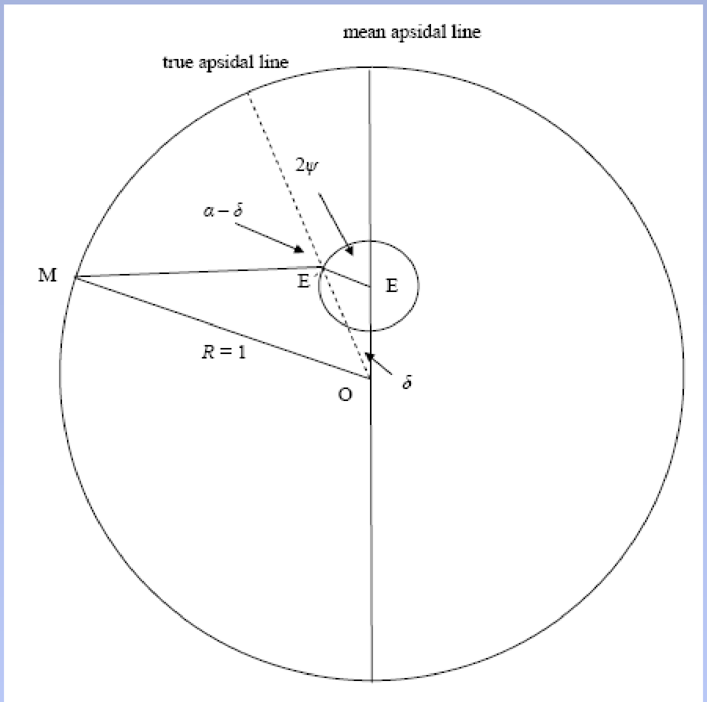
$$
\begin{aligned}
q(\alpha,\eta) &= -2e\sin\alpha - \varepsilon\sin(2\eta-\alpha) + O(e^2,\varepsilon^2) \\
&= -2e\sin\alpha + \varepsilon\sin\alpha - 2\varepsilon\cos\psi\sin\eta \\
&= -(2e-\varepsilon)\sin\alpha - 2\varepsilon\cos\psi\sin\eta \\
&= -r\sin\alpha - r'\cos\psi\sin\eta \\
&= -5;01^\circ\sin\alpha - 2;33^\circ\cos\psi\sin\eta
\end{aligned}
$$

(Munjala gives 2;29°)

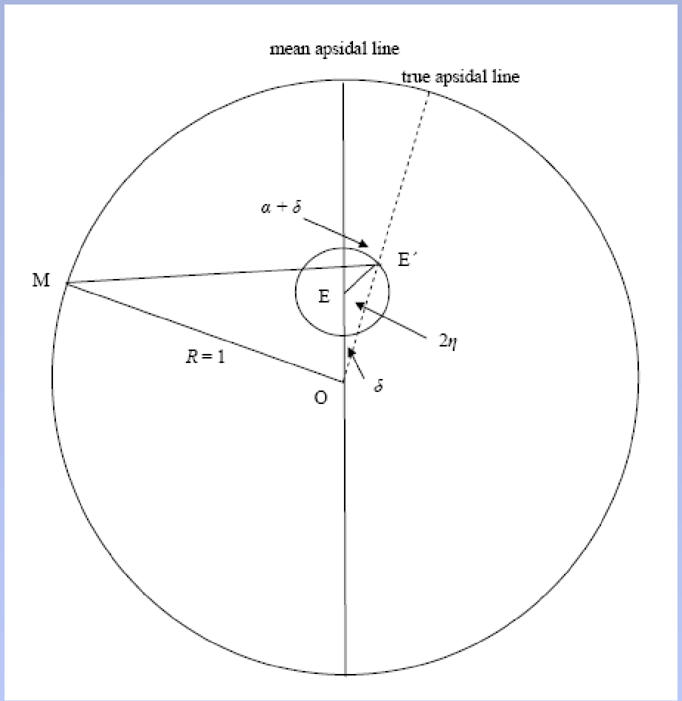
so how do you get $-5;01^\circ\sin\alpha - 2;29^\circ\cos\psi\sin\eta$ from geometry?

In fact, there are two ways to modify the concentric equant, and they are equivalent:





2ψ period 7 months



2η period 15 days

Munjala, and later Kepler, Horrocks, and probably Newton used the 2ψ version, while Ibn ash-Shatir, and later Copernicus and Lansbergen used the 2η version (everyone after Munjala using an eccentric version).

Thus the essentially universally accepted view that **the astronomy we find in the Indian texts is pre-Ptolemaic**. Summarizing the prevailing opinion, Neugebauer wrote in 1956:

"Ptolemy's modification of the lunar theory is of importance for the problem of transmission of Greek astronomy to India. The essentially Greek origin of the *Surya-Siddhanta* and related works cannot be doubted – terminology, use of units and computational methods, epicyclic models as well as local tradition – all indicate Greek origin.

But it was realized at an early date in the investigation of Hindu astronomy that the Indian theories show no influence of the Ptolemaic refinements of the lunar theory [2^{nd} lunar anomaly].

This is confirmed by the planetary theory, which also lacks a characteristic Ptolemaic construction, namely, the "*punctum aequans* [equant] ".

So things might be a little more involved that once thought...

The safest general characterization of the European ~~philosophical~~ history of ancient mathematical astronomy tradition is that it consists of a series of footnotes to ~~Plato~~ Neugebauer. I do not mean the systematic scheme of thought which scholars have doubtfully extracted from his writings. I allude to the wealth of general ideas scattered through them. His personal endowments, his wide opportunities for experience at a great period of civilization, his inheritance of an intellectual tradition not yet stiffened by excessive systemization, have made his writings an inexhaustible mine of suggestion.

Alfred North Whitehead,
Process and Reality (1929), p.39

Stories from the Lost Years

P. Fouad 267A

Antikythera Mechanism

Keskinto Inscription

India: trigonometry, planets, moon

Directly or indirectly, Neugebauer had a role in all of these, often fundamental.