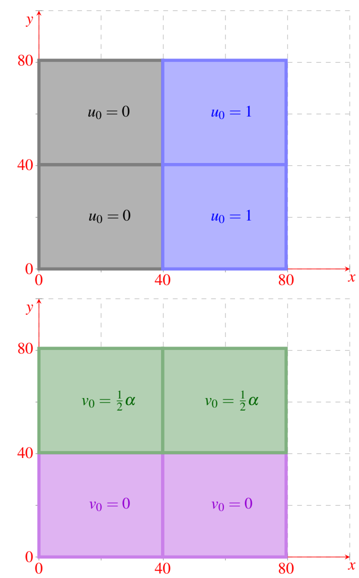
EXAMPLE 3.7.

The following is a simple reaction-diffusion model with solution - a spiral rotating around the center of the spatial domain (see e.g., [2, page 301] and the reference therein).



$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta u + \dfrac{1}{\varepsilon} u(1-u)\big(u - \dfrac{v+\beta}{\alpha}\big), \\ \dfrac{\partial v}{\partial t} = \delta \Delta v + u - v, \end{cases} \qquad (x,y) \in \Omega = (0,80)^2,\ t > 0,$$

$$\begin{cases} u_0(x,y,0) = \begin{cases} 0, & x < 40, \\ 1, & x \geq 0 \end{cases} \\ v_0(x,y,0) = \begin{cases} 0, & y < 40, \\ \frac{1}{2}\alpha, & y \geq 0 \end{cases} \end{cases} \qquad \text{(I.C.)}$$

$$\frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0, \qquad \text{(homogeneous Neumann B.C.)}$$

where the parameters are:

$$\delta = 0, \quad \varepsilon = 0.002, \quad \alpha = 0.25, \quad \beta = 0.001.$$

On a fixed spatial grid of 400×400 *(*$h = 0.2$*), plot the solutions at* $t = 10$ *obtained with a finite difference 5-point Laplace discretization, and respectively with the 9-point Laplacian. Observe that the 5-point Laplacian exhibits a spiral of a 'square' form, aligned to the grid, while the 9-point Laplacian solution gives a 'round' spiral. With these parameters, the PDE is not well resolved. When finer grids are used, the differene between the results becomes smaller, both solutions converging to the exact solution.*

REFERENCES

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[2] W. HUNDSDORFER AND J. VERWER, *Numerical solution of time-dependent advection-diffusion-reaction equations*, vol. 33 of Springer Series in Computational Mathematics, Springer-Verlag, Berlin, 2003.

[3] R. E. LYNCH, *Fundamental solutions of nine-point discrete Laplacians*, vol. 10, 1992, pp. 325–334. A Festschrift to honor Professor Garrett Birkhoff on his eightieth birthday.