What did Ptolemy set out to do in the *Almagest*? Considering the historical importance of the book, and the intensity with which it has been studied, it is surprisingly difficult to find an entirely satisfying statement of its purpose. Typically it is described as an encyclopedic work, setting out comprehensively an entire field of study. Olaf Pedersen, for example, speaks of the *Almagest* as "this brilliant exposition of everything achieved by Ptolemy himself and by the most remarkable of his predecessors among the Greek astronomers," while Gerald Toomer describes it as "a complete exposition of mathematical astronomy as the Greeks understood the term.\(^1\)

Ptolemy himself seems to say something along the same lines at the end of the first chapter (1.1), where he writes that

> We shall try to write up *[hypomnematisasthai]* everything that we believe has come to light for us at present, as succinctly as possible and so that those who have already progressed to some degree may be able to follow. For the sake of completeness in the treatise we shall set out in the appropriate arrangement everything useful for the theory of the things in the heavens.\(^2\)

But this does not mean that the *Almagest* was a capacious sack into which Ptolemy thrust anything that seemed relevant. His subject is a special one, the "theory concerning divine and heavenly things," that is, the ethereal and eternally unchanging bodies in the heavens. These are bodies that are, at least in part, perceptible to our senses but that are regular

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1 Pedersen 1974, 11; Toomer 1984, 1.
2 Heiberg 1898-1903 v. 1, 8. (References to Heiberg's edition also may be used to locate passages in Toomer's translation, which gives Heiberg's page numbers in the margins.) All translations are my own.
and rational in their motions, so that the study of them too is rational, indeed a part of mathematics. This implies that, to the extent that the celestial bodies share some kind of unity or form a system, the proper study of them should have an analogous unity.

As we shall see, a closer study of the workings of Ptolemy's book bears this out. The *Almagest* is not composed as an assemblage of reasonings but as a single reasoning, the biggest deductive argument in all ancient natural science. This is exceptional in the history of astronomical writing, even (so far as we can tell) in antiquity. Other writers recognize that the solutions of some problems depend on others, but they generally assume that some degree of isolation between problems is possible. But for Ptolemy, mathematical astronomy is like a puzzle that can only be taken apart piece by piece in a particular order starting with a particular key piece.

It is important to grasp that the *Almagest* is a work of proof, not heuristics. Although on a few occasions Ptolemy makes statements about how an element in a model was arrived at by himself or his predecessors, the deductions in the book do not depend for their persuasiveness on any assumption that they reflect the original routes of discovery. Ptolemy makes much use of reports of observations made by himself between A.D. 127 and 141; and since some of the very latest of these are used in the solar theory upon which the entire edifice of reasoning rests, anyone who imagined that Ptolemy worked out his celestial system in the way that it is presented in the *Almagest* would have to conclude that Ptolemy had to wait many years before he could make a start on the project. But in fact close study of the numerical details of the computations in the *Almagest* has repeatedly shown that the results that Ptolemy ostensibly derives from analysis of specific observational data cited in the text were actually prior to the demonstrations in the *Almagest*, which he manipulated in a variety of ways to force out the results he wants.

3 In the present paper I prefer to take Ptolemy's philosophy of science on its own terms, rather than discuss it in relation to Greek philosophical traditions. At the risk of oversimplifying, it may be said that Ptolemy's conception of what mathematics is and its place within the full spectrum of intellectual endeavours ("philosophy") clearly owes something to Aristotle, as does his broad cosmology, while his epistemology and perhaps some of his physical notions are influenced by Stoicism. His outlook is too coherent (and personal) to be characterized as "eclectic."

4 A selection of recent discussions might include Jones 2005 (solar theory), Newton 1975, 115-130 (lunar theory—a more thorough treatment is due), Swerdlow 1989 (inferior planets), and Duke 2005 (superior planets). The opinion I have expressed here is held,
To take an extreme possibility, Ptolemy might have worked out a proto-*Almagest* already in the 130s, with dummy observations standing in for the ones he claims to have made later, provided of course that he already knew what model structures and parameters he was heading for. I do not say that this is how he composed the *Almagest*—on the contrary, I think it is improbable—but the fact that it is conceivable shows how free Ptolemy was to shape his material according to what he saw as logical rather than historical constraints.

Why mathematics?

Our name for Ptolemy’s book, *Almagest*, is of course a medieval nickname simply labelling it as "the Greatest," presumably in relation to other works on astronomy. Ptolemy’s own title for the book was much more indicative of what he thought he was doing: he called it the *Mathematical Composition*. But this word "mathematical" will mislead us if we do not grasp that Ptolemy was a mathematical realist, in the Aristotelian rather than the Platonist sense. In the opening chapter of the *Almagest* he defines mathematics as the study of shapes and spatial movements in all kinds of bodies, whether eternal and ethereal or perpetually changing and composed of the four elements. Mathematics offers "sure and unshakeable knowledge," and when concerned with the ethereal heavens, this knowledge is as eternal as its objects. In other words, the conviction that the heavens are composed of ethereal bodies, which are by their composition both eternal and subject to no kind of change except circular revolution, guarantees the legitimacy and truth of the kind of reasoning that the *Almagest* embodies. It is noteworthy that, while practically every other theoretical hypothesis in the *Almagest* is justified by some empirical argument, the hypothesis of the ethereal nature of the heavens is given axiomatically at the beginning.

The models in the *Almagest* are presented and discussed in terms of circles and lines, which can easily seem to our eyes as disembodied and abstract, but for Ptolemy they are the knowable aspect of the ethereal bodies that must exist out there. He mentions these bodies only in a few passages, where he uses the term "spheres." The most telling of these passages is part of Ptolemy’s first introduction of the basic eccentric and epicyclic model types (3.3), which I translate with some minor paraphrasing of technical terminology:

perhaps to varying degrees, by almost all scholars currently working at a technical level on the *Almagest*, regardless of their opinions on the extent to which Ptolemy was responsible for the discovery of his models and parameters.
The assumption must first be made that the eastward shiftings of the planets are uniform, just like the westward [daily] movement of the whole [heavens], and they are circular by nature, that is, the straight lines that are imagined as leading the heavenly bodies or their circles in their revolutions sweep out in all cases equal angles in equal times with respect to the centres of each one's revolutions, while the apparent anomalies pertaining to them are produced by the positions and arrangements of the circles on their spheres, by means of which they make their motions, and nothing in nature really occurs that is foreign to their eternity in connection with the imagined irregularity of the phenomena.

There can be no mistaking Ptolemy's grounding of his bare circles in corporeal realities, even if he chooses not to speculate in the Almagest about the specifics of these spheres. In his later book called the Planetary Hypotheses Ptolemy did indulge in such speculation, and among other things that he has to say about the ethereal bodies composing the heavens, he maintains that their motions are caused and regulated by intelligent souls embodied in the visible sun, moon, and planets, just as the motions of our limbs are caused and regulated by our souls. And in the Harmonics, which I believe to have been one of Ptolemy's earliest writings, he writes that mathematical regularity exists in our cosmos only because rational souls impose it on matter. Thus when we make a musical instrument, our souls are imposing mathematical order on matter that would otherwise produce mere noise, and one part of mathematics, our science of harmonics, gives us knowledge of the structures and causes of that order. And analogously, the celestial souls, by setting their ethereal spheres into specific patterns of uniform circular motion, impose mathematical order on the spheres, and we gain knowledge of it through another part of mathematics.

* A three-stage epistemological strategy.

The Almagest is the most complex of three treatises in which Ptolemy attempted to demonstrate a system of models for a set of related physical phenomena, the others being the Harmonics and the Optics. Besides the passage already mentioned, the Harmonics has
much else to say about scientific epistemology that turns out to be applicable to the
Almagest. Starting with the recognition that sensation and reason are both criteria of
scientific knowledge, but in distinct ways, Ptolemy proposes a strategy for attaining
knowledge through reasoning based on the analysis of observations that are progressively
more controlled by theoretical presuppositions. He introduces this strategy, interestingly,
in the course of an analogy between harmonic science and astronomy according to which
the modelling assumptions underlying the kanon harmonikos or theorist's monochord play a
comparable role in harmonics to the astronomer's models:

It is the part of an astronomer to preserve [diasosai] the models [hypotheseis] of the
heavenly motions in agreement with the observed courses, where these models
themselves were assumed on the basis of the manifest and rougher phenomena
and [subsequently] got their details accurately, so far as is possible, by means of
reason.\footnote{\textit{Düring} 1930, 5. I consider "models" to be clearly preferable to "hypotheses" as a
rendering of Ptolemy's \textit{hypotheseis} in astronomical contexts (and probably elsewhere as
well). Except in dialectic contexts, when he is discussing and refuting the \textit{hypotheseis} of
others, he means something that definitely exists though it cannot be perceived by our
senses.}

According to this statement, the astronomer starts by hypothesizing a vague model, or as
I prefer to call it, a model structure, on the basis of general empirical considerations that
require few technical presuppositions. An example might be the decision to use an
epicycle or eccentric model for a heavenly body's motion. Next, the astronomer uses
reason to work out the details, which might be the numerical parameters of the model
such as radii and rates of revolution. Of course this cannot mean unassisted reason, but
rather reason guiding the choice and handling of further observations that would be
useless except with reference to a model.

Lastly, the astronomer must "preserve the models... in agreement with the
observed courses." At first glance this stage may seem otiose: one would expect a model
competently derived from the phenomena to reproduce the phenomena. Ptolemy's
intention becomes clearer if we supplement the \textit{Harmonics} passage with another from the

Almagest, where Ptolemy is explaining the difficulties that Hipparchus would have faced in constructing a satisfactory planetary theory accounting for the two periodic effects or "anomalies" apparent in the motions of each planet:

Anyone who intends to convince both himself and future readers would have to demonstrate [apodeixai] the magnitude and periods of both anomalies by means of manifest and agreed-upon phenomena, and moreover combining both, he would have to demonstrate the situation and order (ten te thesin kai ten taxin) of the circles by which they take place, and discover the manner of their motion, and finally fit [epharmosai] practically all the phenomena to the specific character of the circles in the model. 6

There again are three stages, the first two of which correspond more or less to the two first stages in the Harmonics passage, though in this case Ptolemy apparently puts the demonstration of the model structure after the quantitative working out, which seems odd. 7 But the point of the third stage is clearer here, since Ptolemy puts emphasis on a distinction between the "manifest and agreed-upon phenomena" that are used in the construction of the model and the "practically all the phenomena" that must finally be brought into agreement with the model. Certain phenomena may not be used, or indeed may be unusable, for deducing the model but are nevertheless deducible from it. We are speaking here of model verification.

So much for methodological pronouncements. When we turn to Ptolemy's practice in the Almagest, we can easily recognize this three-stage strategy in action. Practically every element in Ptolemy's modelling has some explicit, ostensibly empirical justification. Most of the fundamental, nonquantitative elements are dealt with by first-stage nontechnical arguments. The opening chapters of Book 1, which establish the

6 Heiberg 1898-1903 v. 2, 211. So far as Hipparchus is concerned, Ptolemy's remarks are counterfactual, since so far as he knew Hipparchus neither attempted to model the planets nor explained why he did not.
7 Ptolemy is an exceedingly careful writer, and I am reluctant to suppose that he is speaking loosely here. Perhaps he is making some concession to what he sees as a Hipparchian methodology, by which the parameters of a model are measured independently and, so far as possible, prior to a commitment to a specific model structure.
spherical motion of the heavens, the sphericity and central location of the earth, and the fact that the sun's path, the ecliptic, is an inclined great circle, are of this kind, and brief first-stage arguments back up many modelling decisions later in the work. For example, an epicyclic model is an option for the sun and also for the planets. But which way should we suppose the sun or the planet to revolve about the epicycle? Ptolemy says that since the sun takes longer to go from least apparent speed to mean speed than from mean to greatest speed, its motion on the epicycle must be in the opposite sense to the epicycle's motion around the earth. For the planets, however, the phenomenon and thus the conclusion are the reverse.\(^8\)

In passing, I may remark that some of these first-stage arguments invoke alleged phenomena that, plausible though they sound, are scarcely observable. For example, the difference in time between the intervals from least to mean speed and from mean to greatest speed is only conspicuous for Venus, and frankly undetectable for the sun. Again, the theory is really prior to the argument.\(^9\)

Ptolemy's second-stage arguments are the deductions of numerical parameters such as periodicities and ratios of radii in the models from selected observations. A typical example is *Almagest* 3.4, in which Ptolemy uses the resources of Greek geometry and his table of chords to calculate the direction and distance from the earth to the centre of an eccentric circular orbit for the sun from the dates of two observed equinoxes and one solstice. The second-stage arguments are the parts of the *Almagest* that have received the fullest treatment in modern commentaries.

The third-stage arguments are the demonstrations that the fully worked out models are in agreement with the full range of recognized phenomena. In practice Ptolemy does two things: he provides the means for predicting observable celestial events, such as eclipses, planets' stations, and dates of first and last visibility, and he shows that the models exhibit certain patterns of behaviour that are experienced in nature. An example of the latter kind is his demonstration (13.8) that according to his model Mercury should fail in certain parts of the zodiac to make an expected appearance either as

\(^8\) Heiberg 1898-1903 v. 1, 232; v. 2, 251.

\(^9\) The first-stage arguments dispersed through Books 3-9 have received little discussion; Swerdlow 2004, 249-254 has noticed them as a class practically for the first time, and emphasizes their empirical character.
morning or evening star, a phenomenon well known from Babylonian astronomy. Individual observations of first or last visibility could not have been used to work out the model, because they involve factors that are entirely uncontrollable (variable meteorological conditions, acuity of the observer's vision). 

Occasionally Ptolemy uses an indirect third-stage argument to support a modelling assumption. Thus as part of his contention in 1.6 that the earth is effectively a point relative to the celestial sphere, he notes that sundials and other astronomical instruments work properly on the earth's surface although their design theoretically presumes that they lie at the centre of the cosmos. And again, in 3.1 he asserts that if the sun had a significant second anomaly besides the one assumed in his model, his eclipse theory would result in larger errors in the predicted times of eclipses than in fact occur. But these are presented as supplementary to direct empirical arguments, and in general Ptolemy avoids depending entirely upon the ability of a model to "preserve" the phenomena to validate the model, although in the continuation of the passage quoted above that introduces the planetary theory he concedes that such arguments may sometimes be unavoidable:

We know… that things assumed without demonstration, once they have been grasped [katalambanetai] in agreement with the phenomena, cannot have been found without some method and knowledge, even if the manner of their being grasped is difficult to write down, since in general the cause of first principles is either nothing or inherently difficult to explicate.

An order of deduction dictated by the cosmos itself.

The second-stage arguments in Books 3 through 13 form the backbone of the Almagest; it is the logical interdependence of these that determine the large-scale structure of the work. To work out the model of any heavenly body, Ptolemy requires a selection of observed date-longitude pairs, that is, a precise date and the precise longitude of the

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11 Heiberg 1898-1903 v. 2, 212. Toomer translates the phrase khoris hodou tinos kai epistaseos as "without some careful methodological procedure," which gives the passage a more confident tone than Ptolemy probably intended.
heavenly body on that date. In general, longitudes must be measured relative to other heavenly bodies, so that to get the longitude of body $A$ from an observed elongation with respect to body $B$, one would already need to have a model for $B$.

The historical starting point for developing planetary models in ancient astronomy had been the fixed stars. Taking an arbitrary zodiacal star as the zero point, one could measure longitudes of other stars relative to that star, and finally longitudes of the moon and planets relative to any nearby star.\(^{12}\) For Ptolemy, however, it was unsound methodology to privilege any visible heavenly body over any other, and in any case the presupposition that the stars can furnish a stationary frame of reference had been undermined by Hipparchus' researches into precession, which indicated that the so-called fixed stars have a motion of their own, and perhaps are really very slow planets.\(^{13}\)

Ptolemy instead starts in Book 3 with the only heavenly body that appeared to offer a directly observable frame of reference for its own motion, the sun. If it is true that the sun's path on the celestial sphere is a great circle, then at the moments of solstices and equinoxes the sun must be at points on this circle exactly 90° apart, and one of these, the vernal equinoctial point, can stand as the zero point for all longitudes. Thus on four dates each year, one knows the sun's longitude from its observed altitude, with no reference to other heavenly bodies.

In making a solar theory derived from solstices and equinoxes the foundation of his astronomy, Ptolemy faces two potential obstacles. The first is that there was to guarantee that these dates always correspond to the same stages in the periodic variation in the sun's speed. If they did not, Ptolemy would not find it easy to use them to establish a solar model. Fortunately, Ptolemy was able to satisfy himself that the time intervals between these cardinal dates were constants (\textit{Almagest} 3.1).

The second potential obstacle is that it might prove difficult to progress from a solar model to models for the rest of the heavenly bodies, because most of them are practically never visible while the sun is above the horizon. The sole exception, the moon,

\(^{12}\) This essentially characterizes the treatment of longitudes in Babylonian astronomy, for which the fundamental study is Huber 1958. Most Greek astronomy before Ptolemy, and indeed for some time afterwards, also used a sidereal frame of reference.

\(^{13}\) \textit{Almagest} 3.1, Heiberg 1898-1903 v. 1, 193-194. According to Ptolemy (\textit{Almagest} 7.1, Heiberg 1898-1903 v. 2, 3), Hipparchus considered the hypothesis that the precessional motion is limited to stars in the zodiacal belt.
could provide a deductive bridge between the sun and the stars, but it had long been
recognized that the moon is near enough to the earth so that its apparent position in the
sky is significantly affected by the observer's location on the earth's surface, that is, by
parallax. Again Ptolemy was fortunate in that lunar eclipses were a frequent and well-
recorded phenomenon; at the middle time of a lunar eclipse one knew that the moon was
exactly 180° in longitude from the sun as seen from the earth's centre (Almagest 4.1).
Ptolemy can thus deduce the parameters of an epicyclic model for the moon entirely from
eclipse records.

But nature plays a joke on the astronomer here. It turns out that there is a large
periodic effect in the moon's motion that always assumes the same value when the earth,
moon, and sun are all in line, so that it will not be detected from eclipses. In one of his
rare heuristic passages Ptolemy tells us (Almagest 5.1) that he discovered this effect from
studying observations that Hipparchus made of elongations of the moon from the sun.
Ptolemy uses the same kind of observations to correct his provisional lunar model. There
is a problem here to which he does not draw much attention: these observed elongations
are affected by parallax, and at this stage Ptolemy has not worked out the part of his lunar
theory necessary to correct for parallax. We will return later to this difficulty.

Now that Ptolemy has a model that successfully reproduces the observed
longitudes of the moon, he can assume that it also predicts correctly the relative distances
of the moon from the earth on given dates. Comparison of an observed and calculated
position of the moon gives Ptolemy the absolute lunar distance in terms of earth radii, and
indirectly also the absolute distance of the sun (Almagest 5.13-15). With the solar and lunar
model fully established in three dimensions, Ptolemy now has full control of parallax and,
more impressively, can predict and investigate the behaviour of lunar and solar eclipses
(Book 6). We are now at the half-way mark of the Almagest (Fig. 1).
We can be a bit more brief with the remainder of the work. The next step after the lunisolar theory is Ptolemy's demonstration in Book 7 of how precession works, as a slow uniform revolution of all the stars around the poles of the ecliptic, and of a method for measuring the positions of stars relative to the equinox-based frame of reference. Of course most stars are observed relative to other stars, but some must be referred to the sun. This is done by observing the moon's elongation relative to the sun during the daytime, and the star's elongation from the moon during the nighttime, making appropriate corrections for parallax and for the moon's motion during the intervening time (Fig. 2). Since according to Ptolemy's model for precession the stars all preserve their configurations relative to each other and to the ecliptic, it turns out that for practical purposes they can after all be used as points of reference for observed positions of the other heavenly bodies.
Finally, the models for the planets are established using observations of them relative to stars and to the moon, so that all the preceding parts of the *Almagest* are required. Ptolemy’s theory of the latitudinal deviations of the planets north and south of the ecliptic (*Almagest* 13.1-6) is a more or less independent section, making no use of specific dated observations; Ptolemy needs it for the working out of the conditions of planetary visibility that brings the *Almagest* to a triumphant close (Fig. 3). Ptolemy could plausibly claim that he had done all that mathematical reasoning can do to discover the nature of the heavens, and that all this could not have been properly demonstrated in any other order.
Large-scale structure of the *Almagest* (Books 9-13)

- **Theory of Sun**
- **Final Theory of Moon**
- **Lunar Parallax**
- **Theory of Stars**

**Theory of Longitude of Planets**
- Based on observations of greatest elongations from mean sun, oppositions with mean sun, elongations from stars and moon
- Yielding longitude on given date, retrogradations, and greatest elongations

**Theory of Latitude of Planets**
- Based on generalized observations of latitude
- Yielding latitude on given date

**Theory of Visibility of Planets**
- Based on generalized observations of first and last visibilities
- Yielding criteria of visibility

Fig. 3. Large scale structure of Books 9-13.

*Another tangle of recursions.*

Observed at this large scale, the plan of the *Almagest* appears to be straightforwardly linear, with later steps depending only on earlier ones and not *vice versa*, just as in a Greek geometrical treatise. When we look more closely, however, we find more complex and sophisticated deductive patterns. Take for example Ptolemy's working out of the provisional lunar model in Book 4 (Fig. 4). The heart of this book is 4.6, the chapter in which Ptolemy sets out a method of finding the relative sizes and instantaneous positions of the circles in the model for the date of a lunar eclipse from the observed dates of this eclipse and two neighbouring ones. There are four prerequisites: first, an approximation of the periodicities in the model, good enough to yield accurate intervals of angular motion over one or two years; secondly, a presumed model structure, in this case an epicyclic model with the moon revolving around the epicycle in the opposite sense to the epicycle's revolution around the earth (Fig. 5); thirdly, a complete solar theory, from which we can get the moon's longitude at each mid-eclipse and the exact time
intervals between them; and fourthly, the observation reports themselves. Ptolemy makes the calculations twice, for two trios of eclipses separated by as long an interval of time as possible (in fact more than eight centuries). Then, by comparing the two computed configurations of the model, he corrects the periodicities that were originally assumed from an external authority, Hipparchus. Now a recursion is strictly necessary, since a change in the periodicities should have some effect on all the subsequent numbers. In fact the recursion converges immediately, so Ptolemy saves space and the reader's patience by writing up only the second iteration, starting from Ptolemy's final values for the periodicities.

Fig. 4. Structure of the deduction of the provisional lunar theory (Book 4).
The measurement of periodicities is a problem in Greek mathematical astronomy concerning which we have slightly fuller testimony for the time before Ptolemy than other aspects of modelling, which admittedly is not saying much. So far as we can tell, earlier efforts had sought to isolate the periodicities from the effects of the model by looking for observations separated by integer numbers of all the relevant periodicities.\(^\text{14}\) Ptolemy concedes that this separation is not possible, and his recursive strategy is designed to solve the problem of evaluating mutually dependent quantities.\(^\text{15}\)

\(^{14}\) The relevant texts include Ptolemy's discussion of Hipparchus' establishment of the lunar periodicities in 4.2; a passage on lunar periodicities by Apollinarius (an astronomer of the generation before Ptolemy) quoted in an anonymous commentary on Ptolemy's tables (Jones 1990, 38-45); a fragment of an early second-century A.D. treatise discussing observations of Jupiter in the papyrus P. Oxy. astron. 4133 (Jones 1999 v. 1, 69-80 and v. 2, 2-5); and Ptolemy's own first attempt to correct the lunar periodicity in latitude (\textit{Almagest} 4.9, Heiberg 1898-1903 v. 1, 326-327, on which see Hamilton, Swerdlow, and Toomer 1987).

\(^{15}\) \textit{Almagest} 4.2, Heiberg 1898-1903 v. 1, 277.
Ptolemy does not always work out the recursions that are notionally called for if he can satisfy himself that insignificant corrections will result. There are three instances of such shortcuts in his lunar theory. One has already been mentioned: to analyse the observed lunar elongations on which the final model is based, Ptolemy must correct the data for parallax, and his parallax theory relies on knowledge of the final model. He almost succeeds in evading this problem, since three of the four observations that he uses (Almagest 5.3 and 5.5) catch the moon in a situation where the moon's parallax affects its apparent latitude, not its longitude. But the remaining observation, which was made by Hipparchus, needs a significant longitudinal correction, and here Ptolemy simply uses without justification Hipparchus' correction, which is in fact very nearly the same number that one would get from Ptolemy's own parallax tables.\footnote{Heiberg 1898-1903 v. 1, 369.}

Ptolemy does make sufficient excuses for his other shortcuts. He notes in Almagest 5.10 that it is not strictly true that the second lunar anomaly of the final model vanishes at the moment of mid-eclipse. According to the model, it vanishes when the mean sun and mean moon are 180° apart, but eclipses are oppositions of the true sun and moon. Hence the whole of Book 4 ought to be reworked using the final model. But Ptolemy calculates the maximum discrepancy in time of mid-eclipse between the provisional and final models, and finds it to be smaller than the probable errors of the observed times, so that he dispenses with this correction. A similar argument exculpates his neglect of the inclination of the lunar model to the ecliptic in working out the model (Almagest 6.7).

Ptolemy uses parallel deductive structures when he can, and the ways in which parallelism breaks down generally reflect significant differences in the materials he is working with. Thus it is instructive to compare the plan of the lunar theory in Books 4 and 5 to that of the theory for the three outer planets, Mars, Jupiter, and Saturn, in Books 10 and 11. Each of these planets receives its own treatment in a separate set of four chapters, but the procedure is in each case identical, and Ptolemy's prose is also word-for-word the same except insofar as the numbers and some geometrical situations are different.

The model is rather more complicated than the preliminary lunar model, and one would not expect that there could be much in common in the methods of deriving their
parameters (Fig. 6). The planet again revolves on an epicycle that in turn revolves around the earth, but the centre of the deferent circle on which the epicycle rides is now an eccentre, and the two revolutions have the same sense. Moreover, Ptolemy prescribes that the epicycle's revolution about the earth is uniform not as seen from the centre of the deferent but as seen from an equant point twice as far away from the earth.

![Equant model for a planet](image)

However, Ptolemy shows that when the planet is observed at 180° from the mean sun, the earth, the planet, and the epicycle's centre all lie on one line, so that from observations of this kind one can deduce the parameters of a model for the epicycle's centre, that is, isolate the deferent from the epicycle. The problem becomes very similar to that of *Almagest* 4.6, where three eclipses yielded the parameters of the provisional lunar model; now three observed mean oppositions will yield the parameters of the deferent.\(^{17}\)

Ptolemy starts with the same prerequisites (Fig. 7): provisional periodicities from Hipparchus, a model structure, the solar theory, and the observations. The deduction of

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\(^{17}\) The relevant chapters are 10.7 (Mars), 11.1 (Jupiter), and 11.5 (Saturn).
the eccentricity of the deferent is essentially the same calculation that was used in 4.6 for the moon's epicycle. But this calculation takes no account of the equant, and in fact locates the centre of the deferent roughly where the equant ought to be. Ptolemy therefore makes a complicated calculation of how the initial data need to be corrected if the epicycle's positions at the three oppositions are assumed to lie on a deferent circle with half the eccentricity that he has just found. Since the corrections are significant, Ptolemy has to iterate the entire procedure, achieving convergence after the third iteration. This is the only recursive procedure in the *Almagest* that is worked out stage by stage in the text (though the second and third rounds are merely summarized). It is a formidable piece of mathematics. To prove that the iteration has converged on the astronomically correct result, Ptolemy shows that the model finally arrived at reproduces the time intervals between the observations exactly.

![Diagram](Diagram.jpg)

Fig. 7. Structure of the deduction of the model for a superior planet's deferent.

But this impressive computational engine is only part of the deduction of the planetary model (Fig. 8). Ptolemy still has to measure the radius of the epicycle, and correct the periodicities that were initially assumed from Hipparchus. And of course with
the corrected periodicities one ought to repeat the whole deduction, recursive process and all. Few readers of the *Almagest* will have been sorry that, just as in Book 4, Ptolemy has only written up the second round, in which the final periodicities are plugged in and come out the other end. As if to stifle our curiosity, he even omits to tell us what the original Hipparchian approximations were.\(^\text{18}\)

Fig. 8. Structure of the deduction of the entire longitudinal model for a superior planet.

Ptolemy's treatment of the superior planets turns out to differ structurally from his treatment of the moon, but only to the extent that particular astronomical conditions forced a divergence of method on him. In both cases he knew that the required model had one element of complexity too many to allow him to apply directly the mathematical technique of calculating the model's parameters from three observed positions. In both

\(^{18}\) As he did for the moon, Ptolemy has deduced the longitudinal models for the planets as if they lay in the plane of the ecliptic. This time he does not even mention this shortcut.
cases he solves the problem for a simpler, provisional model and then adjusts the parameters to take account of the final model. But the recursive procedure that should follow this step, which Ptolemy skips for the moon on the plea that it would result in corrections below the threshold of observational precision, turns out to be absolutely indispensable for the planets.

Ptolemy's recursions, whether actual or implied, point to an important difference between his mathematical logic and that of the Greek geometers, who generally avoid recursive methods. Aristarchus and Archimedes shield themselves from the approximations that are unavoidable when geometrical constructions are made to yield numerical measures by operations with inequalities and bounds. This approach is viable when an exact geometrical result is untranslatable into an exact number, for example in taking a square root; but it can run into difficulties when the geometrical result itself has to be approached by approximations, as happens with Ptolemy's final lunar and planetary models. In Ptolemy's celestial mathematics, however, the numerical parameters of models (unlike the model structures themselves) are by their very nature not knowable for human beings to full exactitude; what we must look for is a method by which we can prove that our approximations can be made arbitrarily close to the truth, allowing for the limits of accuracy of our senses. This is a point that he makes explicitly with respect to the measurement of periodicities. Ptolemy's recursive techniques are the logical counterpart of this epistemological stance.

I have tried to show that the deductive plan of the Almagest is a work of great care and deliberation, and that this is not the care of the researching scientist aiming at the most precise results but the care of the mathematician aiming at ineluctable proof. He knew what he was doing when he identified his subject as mathematics, and in fact words related to astrologia and astronomia are wholly absent from the text of the Almagest. It is an

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19 Archimedes, De mensura circuli provides the closest analogue to Ptolemy's procedures, but this work does not invoke the concept of "effective convergence" that is at the heart of Ptolemy's use of recursion. A passage that is instructive on the Greek geometers' avoidance of recursion is Pappus, Collectio 3.1-19 (Hultsch 1875-1878 v. 1, 30-55), a critique of an iterative construction ostensibly solving the classic problem of the two mean proportionals. Interestingly, Pappus suggests using Ptolemy's chord table to evaluate numerically the geometrical magnitudes in this construction, not for the sake of obtaining an approximate result, but in order to refute the claim that the results are exact.

20 Almagest 3.1, Heiberg 1898-1903 v. 1, 202-203.
open question whether the astronomical literature available in Ptolemy's time included works that discussed the derivation of models from observational data in a more heuristic mode; for my part I think that this was probably the case. But the *Almagest* was the only book that survived from antiquity that discussed model construction in relation to specific dated observations at all.

Robert Newton, at the end of his book *The Crime of Claudius Ptolemy*, notoriously concluded that the *Almagest* "has done more damage to astronomy than any other work ever written, and astronomy would be better off if it had never existed." Newton of course believed that all the theories in the *Almagest* were plagiarisms and most of the observations forgeries, and in company with several modern historians (some of whom had a much more positive assessment of Ptolemy's contributions) he believed that the publication of the *Almagest* had actually caused the disappearance of the works of his predecessors. For my own part I am not so sure that the caprices by which some ancient books survived and others perished can be reduced to such simple causal explanations, and on the question of Ptolemy's originality in astronomical research I am frankly an agnostic. But it seems to me that if we are to consider the *Almagest* not merely as a vehicle by which information about astronomy was conveyed from antiquity to the Islamic world, Byzantium, and western Europe, but also as a model that influenced how later people attempted astronomical research and exposition, we may discover that its deductive structures were sometimes as much a hindrance as a stimulus.

*References*


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