

# Dynamical Laws of the Coupled Gross-Pitaevskii Equations for Spin-1 Bose-Einstein Condensates<sup>1</sup>

Weizhu Bao

Department of Mathematics and Center for Computational Science and Engineering  
National University of Singapore, Singapore 117543

Yanzhi Zhang

Department of Scientific Computing  
Florida State University, Tallahassee, FL 32306-4120

## Abstract

In this paper, we derive analytically the dynamical laws of the coupled Gross-Pitaevskii equations (CGPEs) without/with an angular momentum rotation term and an external magnetic field for modelling nonrotating/rotating spin-1 Bose-Einstein condensates. We prove the conservation of the angular momentum expectation when the external trapping potential is radially symmetric in two dimensions and cylindrically symmetric in three dimensions; obtain a system of first order ordinary differential equations (ODEs) governing the dynamics of the density of each component and solve the ODEs analytically in a few cases; derive a second order ODE for the dynamics of the condensate width and show that it is a periodic function without/with a perturbation; construct the analytical solution of the CGPEs when the initial data is chosen as a stationary state with its center-of-mass shifted away from the external trap center. Finally, these dynamical laws are confirmed by the direct numerical simulation results of the CGPEs.

**Keywords:** rotating spin-1 Bose-Einstein condensate; coupled Gross-Pitaevskii equations; angular momentum rotation; condensate width; angular momentum expectation.

---

<sup>1</sup>The authors acknowledge support from Ministry of Education of Singapore grant No. R-158-000-002-112. Y. Z. acknowledges the supports from the US Department of Energy under grant number DE-FG02-05ER25698.