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Quantized Vortex Stability and Interaction in the Nonlinear Wave Equation[1]

Weizhu Bao
Department of Computational Science and Center for Computational Science and Engineering
National University of Singapore, Singapore 117543

Rong Zeng
Department of Electrical Engineering
Tsinghua University, Beijing, 100084, China

Yanzhi Zhang
Department of Mathematics
National University of Singapore, Singapore 117543

Abstract

The stability and interaction of quantized vortices in the nonlinear wave equation (NLWE) are investigated both numerically and analytically. A review of the reduced dynamic law governing the motion of vortex centers in the NLWE is provided. The second order nonlinear ordinary differential equations for the reduced dynamic law are solved analytically for some special initial data. Using 2D polar coordinates, the transversely highly oscillating far field conditions can be efficiently resolved in the phase space, thus giving rise to an efficient and accurate numerical method for the NLWE with non-zero far field conditions. By applying this numerical method to the NLWE, we study the stability of quantized vortices and find numerically that the quantized vortices with winding number $m = \pm 1$ are dynamically stable, and resp. $|m| > 1$ are dynamically unstable, in the dynamics of NLWE. We then compare numerically quantized vortex interaction patterns of the NLWE with those from the reduced dynamic law qualitatively and quantitatively. Some conclusive findings are obtained, and discussions on numerical and theoretical results are made to provide further understanding of vortex stability and interactions in the NLWE. Finally, the vortex motion under an inhomogeneous potential in the NLWE is also studied.

Keywords: Nonlinear wave equation; quantized vortex; reduced dynamic law; stability; vortex interaction.

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