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The Dynamics and Interaction of Quantized Vortices in the Ginzburg-Landau-Schrödinger Equation[1]

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Abstract

The dynamic laws of quantized vortex interactions in the Ginzburg-Landau-Schrödinger equation (GLSE) are analytically and numerically studied. A review of the reduced dynamic laws governing the motion of vortex centers in the GLSE is provided. The reduced dynamic laws are solved analytically for some special initial data. By directly simulating the GLSE with an efficient and accurate numerical method proposed recently in [Y. Zhang, W. Bao, and Q. Du, *Numerical simulation of vortex dynamics in Ginzburg-Landau-Schrödinger equation*, European J. Appl. Math., to appear], we can qualitatively and quantitatively compare quantized vortex interaction patterns of the GLSE with those from the reduced dynamic laws. Some conclusive findings are obtained, and discussions on numerical and theoretical results are made to provide further understanding of vortex interactions in the GLSE. Finally, the vortex motion under an inhomogeneous potential in the GLSE is also studied.

Keywords: Ginzburg-Landau equation; nonlinear Schrödinger equation; complex Ginzburg-Landau equation; Ginzburg-Landau-Schrödinger equation; vortex state, reduced dynamic laws; vortex interaction.

AMS subject classification: 35Q55, 65T99, 65Z05, 65N12, 65N35, 81-08

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