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Numerical Simulation of Vortex Dynamics in Ginzburg-Landau-Schrödinger Equation[1]

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Abstract

The rich dynamics of quantized vortices governed by the Ginzburg-Landau-Schrödinger equation (GLSE) is an interesting problem studied in many application fields. Although recent mathematical analysis and numerical simulations have led to a much better understanding of such dynamics, many important questions remain open. In this article, we consider numerical simulations of the GLSE in two dimensions with non-zero far-field conditions. Using two dimensional polar coordinates, transversely highly oscillating far-field conditions can be efficiently resolved in the phase space, thus giving rise to an unconditionally stable, efficient and accurate time-splitting method for the problem under consideration. This method is also time reversible for the case of the non-linear Schröodinger equation. By applying this numerical method to the GLSE, we obtain some conclusive experimental findings on issues such as the stability of quantized vortex, interaction of two vortices, dynamics of the quantized vortex lattice and the motion of vortex with an inhomogeneous external potential. Discussions on these simulation results and the recent theoretical studies are made to provide further understanding of the vortex stability and vortex dynamics described by the GLSE.

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