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Energy and Chemical Potential Asymptotics for the Ground State of Bose-Einstein Condensates in the Semiclassical Regime[1]

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Abstract

Asymptotic approximations for the energy and chemical potential of the ground state in Bose-Einstein condensates are presented in the semiclassical regime with several typical trapping potentials. As preparatory steps, we begin with the three dimensional (3D) Gross-Pitaevskii equation (GPE), review several typical external trapping potentials, scale the 3D GPE and show how to reduce it to 1D and 2D GPEs in certain limiting trapping frequency regime. For the 1D box potential, we derive asymptotic approximations up to $o(1)$ in term of the scaled interacting parameter β_d for energy and chemical potential of the ground and all excited states in both weakly interacting regime, i.e. $\beta_d \to 0$, and strongly repulsive interacting regime, i.e. $\beta_d \to \infty$, respectively. For the 1D harmonic oscillator, double well and optical lattice potentials, as well as a more general external potential in high dimensions, we get asymptotic approximations up to $o(1)$ in term of the scaled interacting parameter β_d for the energy and chemical potential of the ground state in semiclassical regime, i.e. $\beta_d \to \infty$. Our extensive numerical results confirm all our asymptotic approximations, provide convergence rate and suggest several very interesting conclusions.

Keywords: Bose-Einstein condensation; Gross-Pitaevskii equation; box potential; energy; chemical potential; harmonic oscillator potential; double well potential; optical lattice potential; ground state; excited state; semiclassical regime.

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