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# Dynamics of Rotating Bose-Einstein Condensates and Their Efficient and Accurate Numerical Computation[1]

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**Abstract**

In this paper, we study the dynamics of rotating Bose-Einstein condensates (BEC) based on the Gross-Pitaevskii equation (GPE) with an angular momentum rotation term and present an efficient and accurate algorithm for numerical simulations. We examine the conservation of the angular momentum expectation and the condensate width and analyze the dynamics of a stationary state with a shift in its center. By formulating the equation in either the two-dimensional polar coordinate system or the three-dimensional cylindrical coordinate system, the angular momentum rotation term becomes a term with constant coefficients. This allows us to develop an efficient time-splitting method which is time reversible, unconditionally stable, efficient, and accurate for the problem. Moreover, it conserves the position density. We also apply the numerical method to study issues such as the stability of central vortex states and the quantized vortex lattice dynamics in rotating BEC.

**Keywords:** rotating Bose-Einstein condensation; Gross-Pitaevskii equation; angular momentum rotation; time-splitting; ground state; central vortex state; energy; condensate width; angular momentum expectation.

**AMS subject classification:** 35Q55, 65T99, 65Z05, 65N12, 65N35, 81-08

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