Math. Mod. Meth. Appl. Sci., **15** (2005), pp. 1863-1896

Dynamics of the Ground State and Central Vortex States in Bose-Einstein Condensation[1]

Weizhu Bao
Department of Computational Science
National University of Singapore, Singapore 117543

Yanzhi Zhang
Department of Computational Science
National University of Singapore, Singapore 117543

Abstract

In this paper, we study dynamics of the ground state and central vortex states in Bose-Einstein condensation (BEC) analytically and numerically. We show how to define the energy of the Thomas-Fermi (TF) approximation, prove that the ground state is a global minimizer of the energy functional over the unit sphere and all excited states are saddle points in linear case, derive a second order ordinary differential equation (ODE) which shows time-evolution of the condensate width is a periodic function with/without a perturbation by using the variance identity, prove that the angular momentum expectation is conserved in two-dimensions (2D) with a radial symmetric trap and 3D with a cylindrical symmetric trap for any initial data, and study numerically stability of central vortex states as well as interaction between a few central vortices with winding numbers ± 1 by a fourth-order time-splitting sine-pseudospectral (TSSP) method. The merit of the numerical method is that it is explicit, unconditionally stable, time reversible and time transverse invariant. Moreover, it conserves the position density, performs spectral accuracy for spatial derivatives and fourth-order accuracy for time derivative, and possesses 'optimal' spatial/temporal resolution in the semiclassical regime. Finally we find numerically the critical angular frequency for single vortex cycling from the ground state under a far-blue detuned Gaussian laser stirrer in strong repulsive interaction regime and compare our numerical results with those in the literatures.

Keywords: Bose-Einstein condensation; Gross-Pitaevskii equation; time-splitting sine pseudospectral method; variance identity; ground state; central vortex state.

AMS subject classification: 81Q05, 65M70, 65N35, 65N25, 35B40

[1]The authors acknowledge support by the National University of Singapore grant No. R-151-000-030-112. W. B. thanks helpful discussions with Qiang Du and Robert Pego. The authors thank the referees for their valuable comments and suggestions for improving the paper.