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FSU-GSM Forecast Error Sensitivity to Initial Conditions: Application to Indian Summer Monsoon

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With 17 Figures

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Summary

The full-physics adjoint of the FSU Global Spectral Model of version T42L12 is applied to carry out sensitivity analysis of the localized model forecast error to the initial conditions for a case test occurring on June 8, 1988 during the Indian summer monsoon. The results show that adjoint sensitivity based on ECMWF analysis can be used to identify regions with large analysis uncertainties. The conclusion is that more observations are required over the northern Bay of Bengal to improve the quality of analyses so as to ameliorate the model forecast skill.

1. Introduction

Sensitivity analysis deals with the calculation of the gradients of a model forecast aspect with respect to the model parameters. The model parameters might be model initial conditions, boundary conditions or other parameters. The adjoint method is an efficient approach to carry out sensitivity analysis. This method allows us to calculate the gradients of any forcast aspect with respect to all of the model input variables and parameters with only one integration of the forward nonlinear model and one backward integration of its adjoint model. The use of adjoint in sensitivity studies was initiated by the early work of Cacuci (1981a, b), who introduced a general sensitivity theory for nonlinear systems. Hall et al. (1982) applied the theory successfully to sensitivity of a climate radiative-convective model to some parameters. An in-depth review of the entire range of applications of sensitivity theory has been presented by Cacuci (1988). Later, Errico and Vukicevic (1992) indicated that the adjoint fields quantify the previous conditions that most affect a specified forecast aspect. Rabier et al. (1992) used the adjoint of a global primitive equation model to investigate the following question: to which aspects of the initial conditions is cyclogenesis most sensitive in a simple idealized situation? Zou et al. (1993c) examined the sensitivity of a blocking index in a two-layer isentropic model using a response functional depending on both space and time.

One of the applications of adjoint sensitivity is to trace back the geographical regions where large forecast errors originate. Since the numerical weather prediction model forecasts are generally sensitive to the small errors in the initial conditions, the errors in analyses might amplify rapidly in model forecasts, leading to large forecast errors. Some studies have been carried out recently applying adjoint sensitivity to targeted or adaptive observations. For instance, Morss et al. (1998) examined adaptive observation strategies using a multilevel quasi-geostrophic channel model and a realistic data assimilation scheme. Pu et al. (1998) applied the quasi-inverse linear and adjoint methods to targeted observations during FASTEX. Both of their results indicated that the adjoint method was useful in determining the locations for adaptive observations.

In this study, a sensitivity experiment using the adjoint method is carried out for a case on June 8, 1988 occurring during the Indian summer monsoon. We will explore the sensitivity of the 1-day forecast error over a localized region of interest with respect to the initial conditions, which will be taken as a diagnostic tool to identify possible regions where analysis problems are leading to large forecast errors, and we expect that the sensitivity analysis will provide us with an indication as to the placement of adaptive observations in the locations where they are most needed, i.e., adding observations in the areas of large uncertainty (Lorenz and Emanuel, 1998).

2. Experimental Setup

The model used in this study is a T42L12 version of the FSU Global Spectral Model (GSM) developed by Krishnamurti's lab (Krishnamurti et al., 1988), i.e., the horizontal resolution is of a triangular truncation type with a total wavenumber of 42 and 12 levels in the vertical. The full physical processes are applied for both the forecast model and the full-physics adjoint model (Zhu et al., 1997), including planetary boundary layer processes, vertical diffusion, dry adjustment, large-scale condensation and evaporation, deep cumulus condensation, horizontal diffusion and radiation processes. In the FSU GSM, the model state variables are vorticity, divergence, virtual temperature, logarithm of the surface pressure and the dewpoint depression. The adjoint integration is performed in the vicinity of a basic trajectory derived from the forward nonlinear FSU GSM starting from an ECMWF analysis valid 24 hours before the verification time. The gradients of the 1-day forecast error with respect to the initial conditions are called sensitivity patterns.

Let us denote by $\mathbf{X}$ the state vector of the model at time t and $J(\mathbf{X}(t_1))$ the forecast aspect. Suppose the time evolution of the atmosphere is governed by the equation

$$\frac{d\mathbf{X}}{dt} = \mathbf{F}(\mathbf{X}), \tag{1}$$

whose corresponding discretized tangent linear model is as follows

$$\delta\mathbf{X}(t_1) = \mathbf{P}(t_1, t_0)\delta\mathbf{X}(t_0), \tag{2}$$

where t_0 and t_1 denote the initial time and verification time, respectively. $\delta\mathbf{X}$ is the vector of perturbation variable. The adjoint of the tangent linear model is

$$\delta\mathbf{X}'(t_0) = \mathbf{P}^T(t_1, t_0)\delta\mathbf{X}'(t_1), \tag{3}$$

where $\delta\mathbf{X}'$ is the vector of adjoint variable. As shown in Rabier (1992) the gradient of J with respect to $\mathbf{X}(t_0)$ is equal to

$$\nabla_{\mathbf{X}(t_0)}J = \mathbf{P}^T(t_1, t_0)\nabla_{\mathbf{X}(t_1)}J, \tag{4}$$

where the operator $\mathbf{P}^T$ is the adjoint of the tangent linear operator $\mathbf{P}$. Since the adjoint sensitivity in this study consists of the gradient of J with respect to the initial conditions, it can be computed by integrating the adjoint model backward in time. A small perturbation or a small analysis error $\delta\mathbf{X}(t_0)$ in the initial conditions $\mathbf{X}(t_0)$ will result in a change in the forecast error J given by $\delta J = \langle\nabla_{\mathbf{X}(t_0)}J, \delta\mathbf{X}(t_0)\rangle$. Hence, in the geographical areas with a large (small) gradient value, a change in the initial conditions has a large (small) impact upon the forecast error in the direction of $\delta\mathbf{X}(t_0)$.

We studied the sensitivity of 1-day forecast error integrated from 12UTC June 7, 1988 over a limited area domain, namely the Indian Monsoon area, with respect to the initial conditions from ECMWF analysis data. The forecast aspect is defined as the square norm of the differences between the model 1-day forecasts and the verifying analysis over the limited area. The limited area or region of interest is defined to be the area between 60E and 100E in longitude, equator and 30N in latitude. A projection operator (masking operator) is applied to obtain the localized model forecast error over the limited area domain. On June 8, 1988, the Indian summer monsoon entered its active stage. A cross-equatorial flow set in, both the Arabian Sea and the Bay of Bengal branches were established, with depressions over the east central Arabian Sea and over the northern Bay of Bengal. Figures 1 and 2 display the geopotential height fields at 500 hPa at 12 UTC June 7 and June 8, 1988 and the model 1-day forecast, respectively. We observe that the depression over the northern

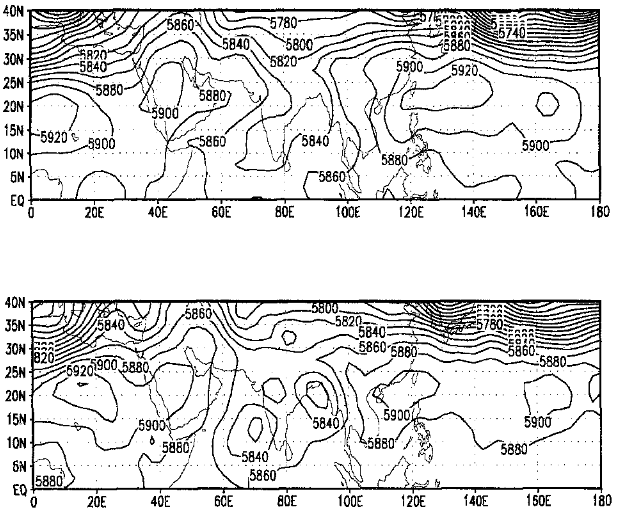

Fig. 1. The geopotential height field at 500 hPa for 12 UTC June 7 (upper panel) and June 8 (bottom panel), 1988

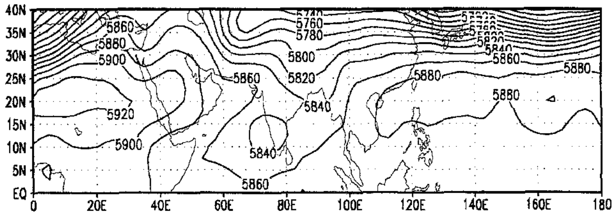

Fig. 2. The geopotential height field at 500 hPa of the model 1-day forecast

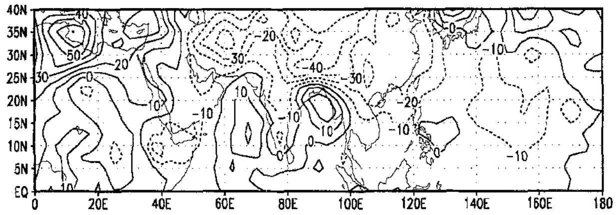

Fig. 3. The difference field of the geopotential height field at 500 hPa between the model 1-day forecast and the verifying analysis

Bay of Bengal dose not fully develop in the model 1-day forecast. The difference field of the geopotential height field at 500 hPa between the model 1-day forecast and the verifying analysis is displayed in Fig. 3. The differences are found to be rather large over the northern Bay of Bengal around 17.5 N.

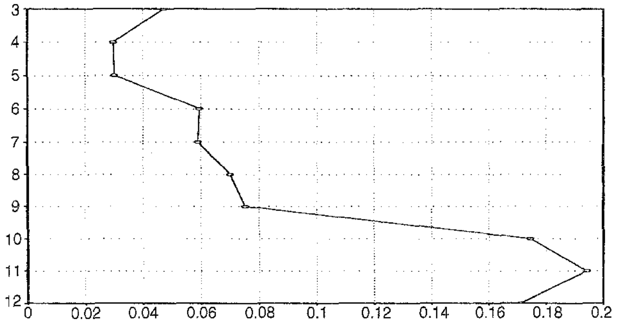

Fig. 4. The squared sum of sensitivities with respect to the initial analysis of dewpoint depression for each model vertical level

3. Results of the Numerical Experiment

The objective of this study is to find out the geographical areas to which the forecast aspect is the most sensitive. The gradients of J, i.e., the sensitivity patterns, are evaluated with respect to the model state variables.

It is known that the analysis of moisture field is usually unreliable over the tropics due to the lack of sufficient observations, i.e., there is a large uncertainty in this analysis. Figure 4 presents the squared sum of sensitivities with respect to the initial analysis of dewpoint depression for each model vertical level. The striking feature is that the forecast error is very sensitive to the initial analyses of dewpoint depression at the lowest three model vertical levels, while the sensitivities to the upper model levels are small. In order to provide a closer look at a single model vertical level, the sensitivity patterns with respect to the dewpoint depression at the lowest three model levels, i.e., model vertical levels 12, 11 and 10, are presented in Figs. 5–7, respectively. A large positive maximum center located upstream of the region with large forecast errors over the northern Bay was observed for both of the lowest two model levels, but a large negative maximum center is more pronounced at the third lowest level. The

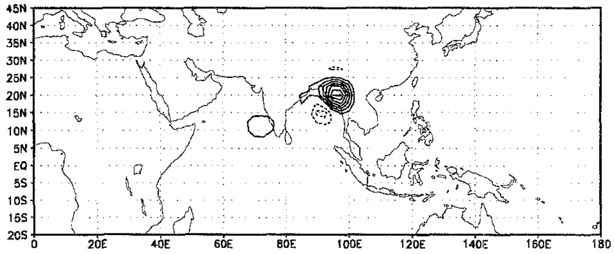

Fig. 5. The sensitivities with respect to the dewpoint depression at the lowest model level. Isoline interval is $1\,K^{-1}$

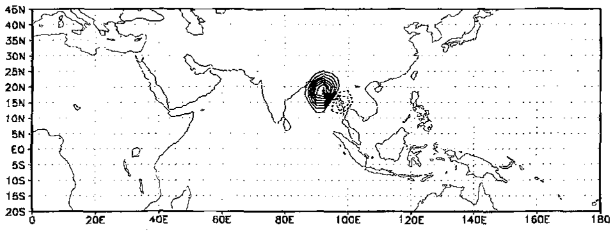

Fig. 6. The sensitivities with respect to the initial analysis of dewpoint depression at the second lowest model level. Isoline interval is 1 K^{-1}

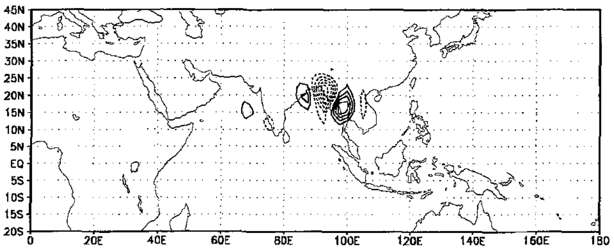

Fig. 7. The sensitivities with respect to the initial analysis of dewpoint depression at the third lowest model level. Isoline interval is 1 K^{-1}

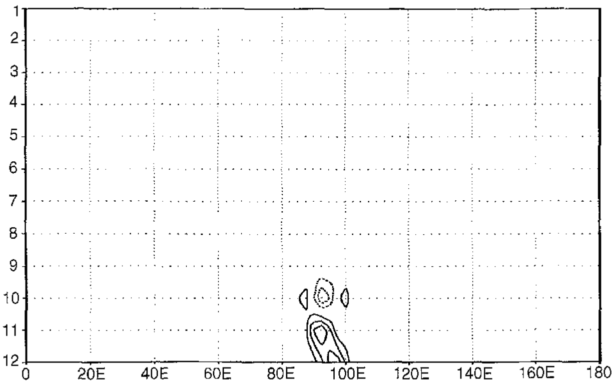

Fig. 8. The vertical cross-section at 20 N for the sensitivity with respect to the dewpoint depression at time t_0. Isoline interval is 2 K^{-1}

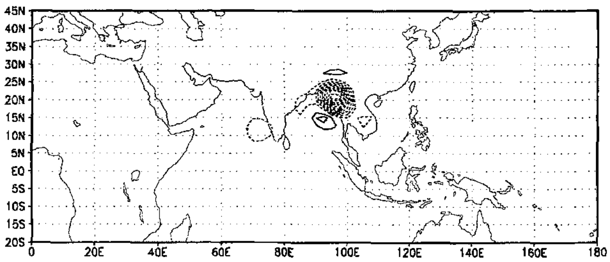

Fig. 9. The sensitivity pattern with respect to the initial analysis of virtual temperature at model vertical level 12. Isoline interval is 1 K^{-1}

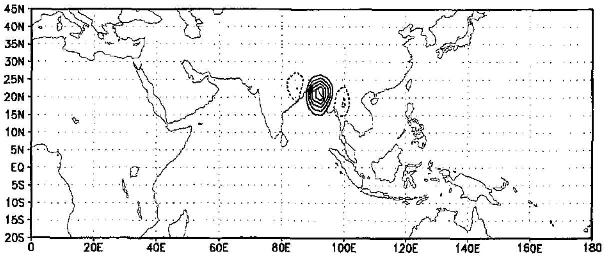

Fig. 10. The sensitivity pattern with respect to the initial analysis of virtual temperature at model vertical level 10. Isoline interval is 2 K^{-1}

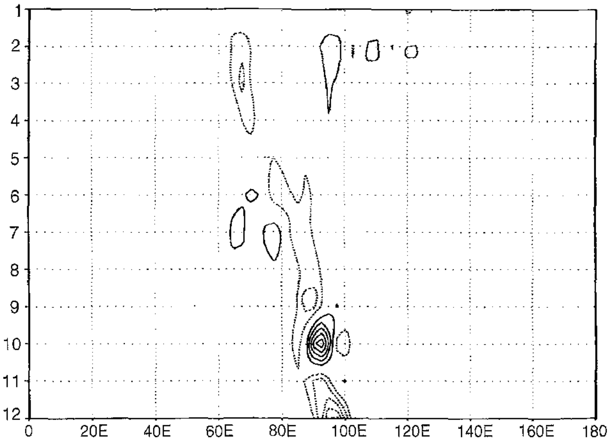

Fig. 11. The vertical cross-section at 20 N for the sensitivity pattern with respect to virtual temperature at time t_0. Isoline interval is 2 K^{-1}

analyses of dewpoint depression at time t_0 are diagnosed to be too dry over the northern Bay of Bengal at the lowest two model vertical levels. The results obtained also show that the model 1-day forecast error is most sensitive to the analysis errors in the dewpoint depression around 90 E, 20 N. Additional observations around this point are expected to improve the model 1-day forecast. The vertical cross-section at 20 N for the sensitivity with respect to the dewpoint depression at time t_0 is displayed in Fig. 8. The pattern is tilted in the vertical to the west, which indicates that further growth of the depression is sensitive to baroclinic perturbations at the initial time.

The sensitivity patterns with respect to the initial analysis of virtual temperature at model vertical levels 12 and 10 (Figs. 9 and 10) also indicate the locations of the geographical regions where the analysis problems lie in. The analyses of virtual temperature over the northern Bay of Bengal are diagnosed to be too low at model vertical level 12 and too high at model vertical level 10. The vertical cross-section at 20 N is displayed in Fig. 11.

The calculation of the squared sum of sensitivities with respect to the initial analysis of vorticity for each model vertical level indicates that the model 1-day forecast error is sensitive to the uncertainties in the analysis at model vertical levels 11 and 7, which are approximately located above the surface and at 700 hPa, respectively. The sensitivity pattern with respect to the initial analyses of vorticity at model vertical levels 11 and 7 are displayed in Fig. 12 and Fig. 13, respectively. Two important areas with opposite signs are observed for both sensitivity patterns. The vertical cross-section at 20 N for the sensitivity pattern with respect to vorticity at time t_0 (Fig. 14) exhibits two large centers with opposite signs, both of which were located in the lower troposphere around 90 E, 20 N. This indicated that the forecast error was very sensitive to the vorticity analysis uncertainties in the lower atmosphere. One maximum center, which is located at model vertical level 7, is also observed in the vertical cross-section at 10 N for the sensitivity pattern with respect to the initial analysis of vorticity (Fig. 15), and a westward-tilting of the vertical structure is not observed. The analysis uncertainties at model vertical level 7 are mainly distributed over the eastern Arabian Sea, while the analysis uncertainties at model vertical level 11 are mainly located around 90 E, 20 N.

The sensitivity signal is also calculated in order to pinpoint the overall location of the analysis uncertainties. It is represented by the sum of squares of the sensitivity patterns throughout the whole range of vertical levels. The sensitivity signals for vorticity and dewpoint depression are displayed in Figs. 16 and 17, respectively. It is apparent that the model 1-day forecast error is most sensitive to the analysis

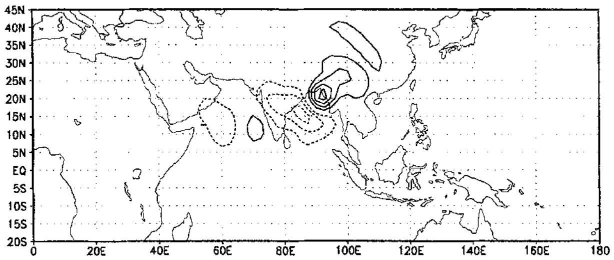

Fig. 12. The sensitivity pattern with respect to the initial analysis of vorticity at model vertical level 11. Isoline interval is 200000s

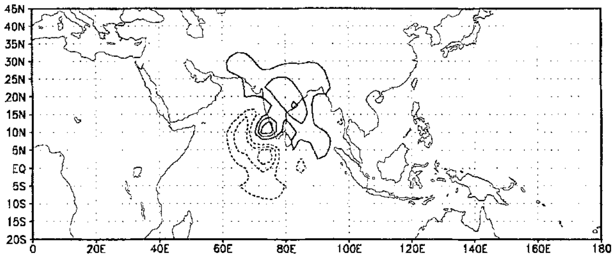

Fig. 13. The sensitivity pattern with respect to the initial analysis of vorticity at model vertical level 7. Isoline interval is 200000s

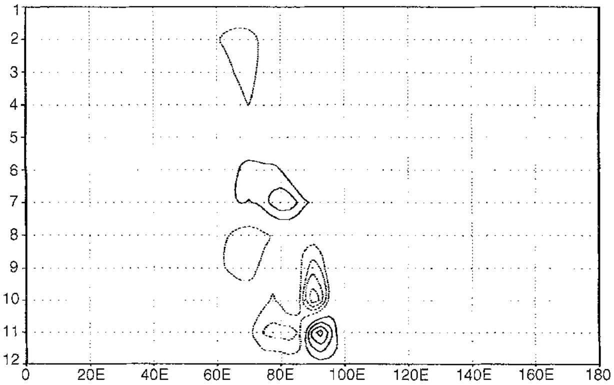

Fig. 14. The vertical cross-section at 20 N for the sensitivity pattern with respect to vorticity at time t_0. Isoline interval is 200000s

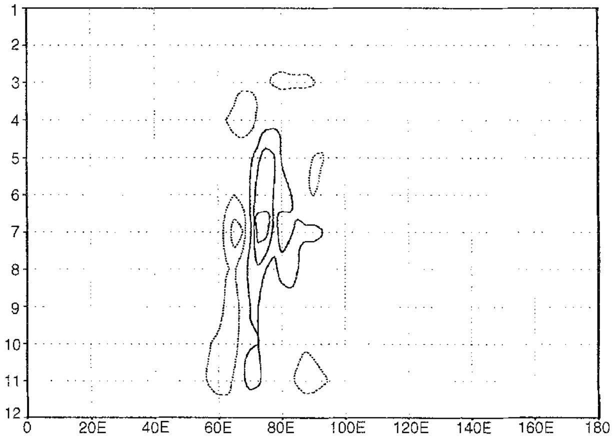

Fig. 15. The vertical cross-section at 10 N for the sensitivity pattern with respect to vorticity at time t_0. Isoline interval is 200000s

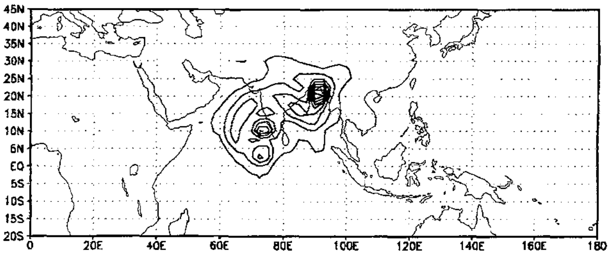

Fig. 16. The sensitivity signal for vorticity at time t_0

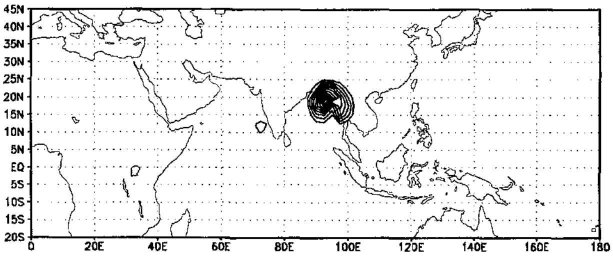

Fig. 17. The sensitivity signal for dewpoint depression at time t_0

errors located at around 90 E, 20 N over the northern Bay of Bengal.

4. Conclusions

In this study, the sensitivity of the model 1-day forecast error to the initial conditions for an Indian summer monsoon case is applied to localize regions with large analysis uncertainties. Our results show that the model 1-day forecast error is most sensitive to errors in the analyses of the lower troposphere, especially over the northern Bay of Bengal around 90 E, 20 N. More moisture and wind field observations are required over this region to improve the quality of the analyses in order to ameliorate the model forecast skill.

However, this sensitivity study is performed in "a posteriori" diagnostic way in this study. For practical problems, large forecast uncertainties can also be identified using the ensemble forecast system (see Kalnay and Toth, 1996). The forecast difference may be obtained by subtracting one member of the ensemble from another, then the localized forecast errors may be applied to the adjoint model in order to obtain the adjoint sensitivity.

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