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An International Journal
computers & mathematics with applications

Computers and Mathematics with Applications 52 (2006) xiii–xv
www.elsevier.com/locate/camwa

Preface

"All forecast models, whether they represent the state of the weather, the spread of a disease, or levels of economic activity, contain unknown parameters. These parameters may be the model's initial conditions, its boundary conditions, or other tunable parameters which have to be found for a realistic result", see [1].

Estimation of the state or evolution of the atmosphere using the information provided by a numerical weather prediction (NWP) model and observations of the atmosphere is carried out by data assimilation (DA), see [2]. Current DA methods are based on either using results from estimation theory (such as application of Kalman, extended Kalman or ensemble Kalman filtering) or variational methods related to optimal control of distributed parameter systems being then governed typically by partial differential equations. The objective of variational DA is to determine a model trajectory (by adjusting initial conditions used for model integration which serve as control variables) that satisfies the model equations as a (strong or weak) constraint while simultaneously minimizing the lack of fit between model predictions and heterogeneous observations in a least-squares sense as represented in an adequately formulated cost functional whose minimum with respect to the control variables we seek, see e.g., [3–5] for further details. Gradient based efficient large-scale minimization algorithms require availability of gradient of the said cost functional with respect to the control variables (provided efficiently by adjoint methods, which are integrated backwards in time) are used for this purpose. See [6–9]. The impact of different discretization techniques for the advection term(s) in the framework of inverse problems and problems related to DA have not been extensively tested, except for work by Vukicevic *et al.*. [10] and Thuburn and Haine [11] and for use of high accuracy schemes see recent work of Akella and Navon [12]. Vukicevic *et al.* [10] indicate that more accurate advection schemes should be used to solve both, forward and adjoint models in time to achieve higher accuracy regarding recovery of initial conditions for data assimilation.

Four-dimensional variational data assimilation (4D-Var) is a method of estimating a set of parameters by optimizing the fit between the solution of the model and a set of observations which the model is meant to predict. 4D-Var is a simple generalization of 3D-Var for observations that are distributed in both space and time. In 3D-Var no proper account is made of the time at which an observation is made. In 4D-Var formulation, the observation operators are generalized to include different types of observations in the form of covariance matrices that will allow a comparison between the model state and the observations at the appropriate time, see e.g., [4,13–15]. Inclusion of background error covariance term [16] as well as model error terms characterizes modern cost functionals used for purpose of 4D-VAR data assimilation.

For weather forecasting, the method of 4D-Var that is computationally intensive has been adopted by some leading NWP operational centers, as it is flexible enough to allow a range of atmospheric observations of many different types to be digested within a framework of a numerical model of the atmosphere and has been shown to improve weather forecasts.

0898-1221/06/$ - see front matter. Published by Elsevier Ltd.
doi:10.1016/j.camwa.2006.11.001

Over a given time interval, under the assumption that the model is perfect, with the same input data, the 4D-Var analysis at the end of the time interval is equivalent to the Kalman filter analysis at the same time for simple linear models [17]. The recent trend in data assimilation is to combine the advantages of 4D-Var and the Kalman filter techniques, see e.g., [13].

This special issue contains several papers on this topic related to some aspects of the research activities of Prof. I. M. Navon in 4D variational data assimilation during the last 16 years. The first paper by Hoffman *et al.* uses 4D-Var for calculating optimal perturbations for weather modification. The iterative convergence error of goal functional is considered (Alekseev) for solving the steady problem by time iterations. The functional error is calculated using the adjoint error correction method and time derivative. Adjoint *a-posteriori* error measures for anisotropic mesh optimization (Power *et al.*) are a first step toward being able to 'predict' the flow's future movement, and adapt the mesh to satisfy error reduction in goal functionals accordingly. Adjoint models can be derived with either of the following two approaches: the continuous approach based on the Euler-Lagrange equations followed by discretization, or the discrete approach in which the discrete representation of the nonlinear problem is differentiated. The two approaches do not commute and it is only the latter that is completely consistent with the discretized representation of the forward equations, and as such is the one implemented operationally using automatic differentiation.

Gejadze and Copeland develop the adjoint sensitivities to the free-surface barotropic Navier-Stokes equations in order to allow for the assimilation of measurements of currents and free surface elevations into an unsteady flow solution by open boundary control. A particular application is to the construction of a fully three dimensional coastal ocean model that allows assimilation of tidal elevation and current data.

Gejadze *et al.* have also developed the DA procedure in order to allow for the assimilation of measurements of currents and free surface elevations into an unsteady flow solution. Estimation technique using the Kalman filter finite element method is presented by Suga and Kawahara and is demonstrated in the analysis of the Onjuku coast. Another paper by the Kawahara Lab members is on optimal control of water level considering time delay system. Sakthivel *et al.*. present the theory of controllability and observability of certain integro-differential equations. For nonlinear problem they use the Kakutani fixed-point theorem.

The last three papers address high order methods. Giraldo discusses hybrid Eulerian- Lagrangian semi-implicit time-integrators. Although explicit time-integrators are the easiest methods to implement, their main disadvantage is that small time steps must be observed in order to maintain stability. In atmospheric modeling, the reason for this prohibitively small time-step is due to the presence of fast moving gravity waves. These waves require a small time-step while only carrying a very small percent of the total energy in the system. It turns out that discretizing the gravity wave terms implicitly in time is the more effective way of increasing the time-step. After the gravity wave terms have been successfully discretized the next set of terms responsible for controlling the maximum time-step are the Rossby waves (advection terms). By rewriting the equations in terms of the Lagrangian derivative the troublesome advection terms are absorbed into the Lagrangian derivative.

Li and Liu develop a sixth-order compact scheme coupled with alternating direction implicit (ADI) methods and apply it to parabolic equations in both 2-D and 3-D.

Steppeler developed a direct method suitable for partial and fully implicit time integration of the primitive equations meteorological models. Cao *et al.* show that the proper orthogonal decomposition constitutes an efficient model reduction technique for simulating physical processes governed by partial differential equations, in this instance an ocean model and have in a subsequent work in progress generalized their approach to adaptively reduced 4D-Var data assimilation.

I wish to thank all the authors for their contribution in honor of Professor I. Michael Navon 65^{th} birthday and all the referees for their careful reading of the manuscripts and their comments

and suggestions. I want also to thank Professor Ervin Y. Rodin, Editor- in-Chief of Computers & Mathematics with Applications, for his invitation to edit this special issue.

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Beny Neta
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