Maximum Likelihood Ensemble Filter: Exploiting dynamic localization of Lyapunov vectors

Milija Zupanski[1], Steven J. Fletcher[1], I. Michael Navon[2], Bahri Uzunoglu[2], Ross P. Heikes[3], Tod D. Ringler[3], and David A. Randall[3]

[1] Cooperative Institute for Research in the Atmosphere / Colorado State University, Fort Collins CO
[2] School of Computational Science and Information Technology / Florida State University, Tallahassee, FL
[3] Department of Atmospheric Science / Colorado State University, Fort Collins CO

Background

Maximum Likelihood Ensemble Filter (MLEF)

- Deterministic square-root filter
- Posed as a control theory problem: Minimize nonlinear cost-function in ensemble-spanned subspace
- Founded on the ideas from variational methods, Iterative Kalman filters and Ensemble Transform Kalman Filter (ETKF)

Cold-start initialization of ensemble perturbations

- Random perturbations
- Fourier expansion (waves)
- Can have a significant impact on the algorithm performance

Goal: Improve the MLEF performance by proper initialization of ensembles

Analysis error covariance preconditioning

- Make the ensemble algorithm converge faster
- Initial ensemble perturbation should correspond to the analysis error covariance

$$P_f^{1/2} = \begin{pmatrix} b_1 & b_2 & \cdot & \cdot & b_S \end{pmatrix} \qquad b_i = M(x_a^{k-1} + p_i) - M(x_a^{k-1})$$

Issues

- Randomness – perturbation location
- Correlations – dynamical structure
- Spatially localized structure

A solution

- Use the Kardar-Parisi-Zhang (KPZ) equation to create uncorrelated random, spatially localized ensemble perturbations
- Use compactly-supported error covariance modeling operator to force desired correlations

Methodology

Kardar-Parisi-Zhang equation

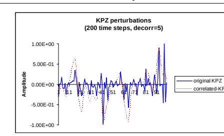
$$\frac{\partial h}{\partial t} = \nabla^2 h + (\nabla h)^2 + \xi$$

- Used to explain the dynamics of interfaces moving through random media
- A general class of equations used to represent Lyapunov vectors as the exponential of roughened interface
- Produces spatially localized random perturbations

Compactly-supported error covariance

- Commonly used in variational and ensemble data assimilation methods
- Applied to KPZ perturbations to force prescribed correlation length-scale

Typical structure of correlated and uncorrelated initial ensemble perturbations using KPZ



Algorithmoc details

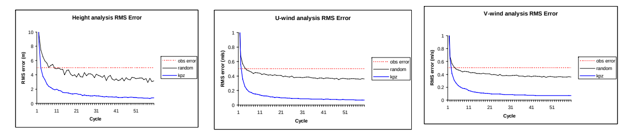
- Prescribe variance and correlation length-scale
- Use an empirical formula to define the length of KPZ integration: *Time=ε*Length_scale*
- Smooth the noisy KPZ perturbations by applying the compactly-supported error covariance operator

Experimental design

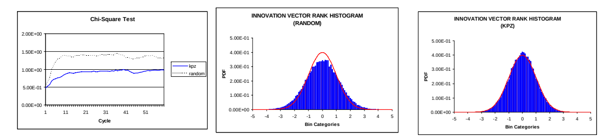
- CSU global shallow-water model on a geodesic grid
- Approximate model resolution: 4.5 degrees
- Initial conditions: Zonal flow over an isolated mountain
- Identical twin experiment: Observations created by adding random perturbations to the true model run
- Assimilation period: 6-hour intervals, up to 15 days
- 1025 height and wind observations, 1000 ensembles

Results

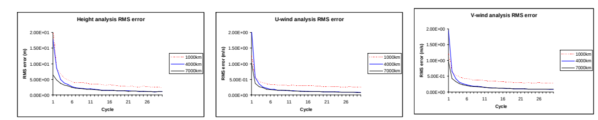
Analysis RMS error



Innovation vector statistics



Impact of prescribed decorrelation length scale



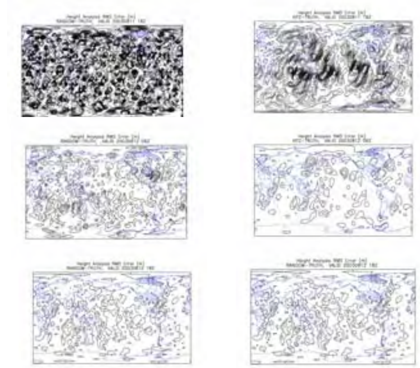
Height Analysis Increments

Uncorrelated random | Correlated-KPZ

Cycle 1

Cycle 2

Cycle 3



Future Plans:

- Evaluate and improve the robustness of the method
- Employ directly the KPZ with *correlated* noise

References:
Heikes, R., and D.A. Randall, 1995: Numerical integration of the shallow-water equations on a twisted icosahedral grid. Part I: Basic design and results of tests. *Mon. Wea. Rev.*, **123**, 1862-1880.
Pikovsky, A., and A. Politi, 1998: Dynamic localization of Lyapunov vectors in spacetime chaos. *Nonlinearity*, **11**, 1049-1062.
Zupanski, M., 2005: Maximum Likelihood Ensemble Filter: Theoretical Aspects. *Mon.Wea.Rev.*, in print.

Acknowledgements:
We would like to thank the National Centers for Atmospheric Research for providing supercomputing facilities. This work was supported by the National Science Foundation Collaboration in Mathematical Geosciences grant # 0327651.