```
       PROGRAM D10R11
C     Driver for routine DFPMIN
      PARAMETER(NDIM=3,FTOL=1.0E-6,PIO2=1.5707963)
      DIMENSION P(NDIM)
      WRITE(*,'(/1X,A)') 'Program finds the minimum of a function'
      WRITE(*,'(1X,A)') 'with different trial starting vectors.'
      WRITE(*,'(1X,A)') 'True minimum is (0.5,0.5,0.5)'
      DO 11 K=0,4
C        ANGL=PIO2*K/4.0
        P(1)=2.0
        P(2)=2.0
C        P(3)=0.0
        WRITE(*,'(/1X,A,3(F6.4,A))') 'Starting vector: (',
     *      P(1),',',P(2),',',P(3),')'
        CALL DFPMIN(P,NDIM,FTOL,ITER,FRET)
        WRITE(*,'(1X,A,I3)') 'Iterations:',ITER
        WRITE(*,'(1X,A,3(F6.4,A))') 'Solution vector: (',
     *      P(1),',',P(2),',',P(3),')'
        WRITE(*,'(1X,A,E14.6)') 'Func. value at solution',FRET
11    CONTINUE
       PAUSE
      END

      FUNCTION FUNC(X)
      DIMENSION X(2)
      FUNC=0.
      FUNC=100.*(X(2)-X(1)*X(1))**2+(1.-X(1))**2+FUNC
      END

      SUBROUTINE DFUNC(X,DF)
      PARAMETER (NMAX=50)
      DIMENSION X(2),DF(NMAX)
      DF(1)=2.*(200.*X(1)*(X(1)*X(1)-X(2))-1.+X(1))
      DF(2)=200.*(X(2)-X(1)*X(1))
C     DF(3)=BESSJ0(X(1)-0.5)*BESSJ0(X(2)-0.5)*BESSJ1(X(3)-0.5)
      RETURN
      END
      SUBROUTINE DFPMIN(P,N,FTOL,ITER,FRET)
      PARAMETER (NMAX=50,ITMAX=200,EPS=1.E-10)
      DIMENSION P(N),HESSIN(NMAX,NMAX),XI(NMAX),G(NMAX),DG(NMAX),
     *HDG(NMAX)
      FP=FUNC(P)
      CALL DFUNC(P,G)
      DO 12 I=1,N
        DO 11 J=1,N
          HESSIN(I,J)=0.
11      CONTINUE
        HESSIN(I,I)=1.
        XI(I)=-G(I)
12    CONTINUE
      DO 24 ITS=1,ITMAX
        ITER=ITS
        CALL LINMIN(P,XI,N,FRET)
        IF(2.*ABS(FRET-FP).LE.FTOL*(ABS(FRET)+ABS(FP)+EPS))RETURN
        FP=FRET
        DO 13 I=1,N
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        DG(I)=G(I)
13      CONTINUE
        FRET=FUNC(P)
        CALL DFUNC(P,G)
        DO 14 I=1,N
          DG(I)=G(I)-DG(I)
14      CONTINUE
        DO 16 I=1,N
          HDG(I)=0.
          DO 15 J=1,N
            HDG(I)=HDG(I)+HESSIN(I,J)*DG(J)
15        CONTINUE
16      CONTINUE
        FAC=0.
        FAE=0.
        DO 17 I=1,N
          FAC=FAC+DG(I)*XI(I)
          FAE=FAE+DG(I)*HDG(I)
17      CONTINUE
        FAC=1./FAC
        FAD=1./FAE
        DO 18 I=1,N
          DG(I)=FAC*XI(I)-FAD*HDG(I)
18      CONTINUE
        DO 21 I=1,N
          DO 19 J=1,N
            HESSIN(I,J)=HESSIN(I,J)+FAC*XI(I)*XI(J)
     *        -FAD*HDG(I)*HDG(J)+FAE*DG(I)*DG(J)
19        CONTINUE
21      CONTINUE
        DO 23 I=1,N
          XI(I)=0.
          DO 22 J=1,N
            XI(I)=XI(I)-HESSIN(I,J)*G(J)
22        CONTINUE
23      CONTINUE
24    CONTINUE
      PAUSE 'too many iterations in DFPMIN'
      RETURN
      END
       FUNCTION BESSJ0(X)
      REAL*8 Y,P1,P2,P3,P4,P5,Q1,Q2,Q3,Q4,Q5,R1,R2,R3,R4,R5,R6,
     *    S1,S2,S3,S4,S5,S6
      DATA P1,P2,P3,P4,P5/1.D0,-.1098628627D-2,.2734510407D-4,
     *    -.2073370639D-5,.2093887211D-6/, Q1,Q2,Q3,Q4,Q5/-.1562499995D-
     *1,
     *    .1430488765D-3,-.6911147651D-5,.7621095161D-6,-.934945152D-7/
      DATA R1,R2,R3,R4,R5,R6/57568490574.D0,-13362590354.D0,651619640.7D
     *0,
     *    -11214424.18D0,77392.33017D0,-184.9052456D0/,
     *    S1,S2,S3,S4,S5,S6/57568490411.D0,1029532985.D0,
     *    9494680.718D0,59272.64853D0,267.8532712D0,1.D0/
      IF(ABS(X).LT.8.)THEN
        Y=X**2
        BESSJ0=(R1+Y*(R2+Y*(R3+Y*(R4+Y*(R5+Y*R6)))))
     *      /(S1+Y*(S2+Y*(S3+Y*(S4+Y*(S5+Y*S6)))))
      ELSE
```

```
        AX=ABS(X)
        Z=8./AX
        Y=Z**2
        XX=AX-.785398164
        BESSJ0=SQRT(.636619772/AX)*(COS(XX)*(P1+Y*(P2+Y*(P3+Y*(P4+Y
     *      *P5))))-Z*SIN(XX)*(Q1+Y*(Q2+Y*(Q3+Y*(Q4+Y*Q5)))))
       ENDIF
       RETURN
       END
        FUNCTION BESSJ1(X)
       REAL*8 Y,P1,P2,P3,P4,P5,Q1,Q2,Q3,Q4,Q5,R1,R2,R3,R4,R5,R6,
     *    S1,S2,S3,S4,S5,S6
       DATA R1,R2,R3,R4,R5,R6/72362614232.D0,-7895059235.D0,242396853.1D0
     *,
     *    -2972611.439D0,15704.48260D0,-30.16036606D0/,
     *    S1,S2,S3,S4,S5,S6/144725228442.D0,2300535178.D0,
     *    18583304.74D0,99447.43394D0,376.9991397D0,1.D0/
       DATA P1,P2,P3,P4,P5/1.D0,.183105D-2,-.3516396496D-4,.2457520174D-5
     *,
     *    -.240337019D-6/, Q1,Q2,Q3,Q4,Q5/.04687499995D0,-.2002690873D-3
     *,
     *    .8449199096D-5,-.88228987D-6,.105787412D-6/
       IF(ABS(X).LT.8.)THEN
        Y=X**2
        BESSJ1=X*(R1+Y*(R2+Y*(R3+Y*(R4+Y*(R5+Y*R6)))))
     *      /(S1+Y*(S2+Y*(S3+Y*(S4+Y*(S5+Y*S6)))))
       ELSE
        AX=ABS(X)
        Z=8./AX
        Y=Z**2
        XX=AX-2.356194491
        BESSJ1=SQRT(.636619772/AX)*(COS(XX)*(P1+Y*(P2+Y*(P3+Y*(P4+Y
     *      *P5))))-Z*SIN(XX)*(Q1+Y*(Q2+Y*(Q3+Y*(Q4+Y*Q5)))))
     *      *SIGN(1.,X)
       ENDIF
       RETURN
       END
       SUBROUTINE LINMIN(P,XI,N,FRET)
       PARAMETER (NMAX=50,TOL=1.E-4)
       EXTERNAL F1DIM
       DIMENSION P(N),XI(N)
       COMMON /F1COM/ NCOM,PCOM(NMAX),XICOM(NMAX)
       NCOM=N
       DO 11 J=1,N
        PCOM(J)=P(J)
        XICOM(J)=XI(J)
11     CONTINUE
       AX=0.
       XX=1.
       BX=2.
       CALL MNBRAK(AX,XX,BX,FA,FX,FB,F1DIM)
       FRET=BRENT(AX,XX,BX,F1DIM,TOL,XMIN)
       DO 12 J=1,N
        XI(J)=XMIN*XI(J)
        P(J)=P(J)+XI(J)
12     CONTINUE
       RETURN
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      END
      SUBROUTINE MNBRAK(AX,BX,CX,FA,FB,FC,FUNC)
      PARAMETER (GOLD=1.618034, GLIMIT=100., TINY=1.E-20)
      FA=FUNC(AX)
      FB=FUNC(BX)
      IF(FB.GT.FA)THEN
        DUM=AX
        AX=BX
        BX=DUM
        DUM=FB
        FB=FA
        FA=DUM
      ENDIF
      CX=BX+GOLD*(BX-AX)
      FC=FUNC(CX)
1     IF(FB.GE.FC)THEN
        R=(BX-AX)*(FB-FC)
        Q=(BX-CX)*(FB-FA)
        U=BX-((BX-CX)*Q-(BX-AX)*R)/(2.*SIGN(MAX(ABS(Q-R),TINY),Q-R))
        ULIM=BX+GLIMIT*(CX-BX)
        IF((BX-U)*(U-CX).GT.0.)THEN
          FU=FUNC(U)
          IF(FU.LT.FC)THEN
            AX=BX
            FA=FB
            BX=U
            FB=FU
            GO TO 1
          ELSE IF(FU.GT.FB)THEN
            CX=U
            FC=FU
            GO TO 1
          ENDIF
          U=CX+GOLD*(CX-BX)
          FU=FUNC(U)
        ELSE IF((CX-U)*(U-ULIM).GT.0.)THEN
          FU=FUNC(U)
          IF(FU.LT.FC)THEN
            BX=CX
            CX=U
            U=CX+GOLD*(CX-BX)
            FB=FC
            FC=FU
            FU=FUNC(U)
          ENDIF
        ELSE IF((U-ULIM)*(ULIM-CX).GE.0.)THEN
          U=ULIM
          FU=FUNC(U)
        ELSE
          U=CX+GOLD*(CX-BX)
          FU=FUNC(U)
        ENDIF
        AX=BX
        BX=CX
        CX=U
        FA=FB
        FB=FC
```

```
        FC=FU
        GO TO 1
      ENDIF
      RETURN
      END
            FUNCTION BRENT(AX,BX,CX,F,TOL,XMIN)
      PARAMETER (ITMAX=100,CGOLD=.3819660,ZEPS=1.0E-10)
      A=MIN(AX,CX)
      B=MAX(AX,CX)
      V=BX
      W=V
      X=V
      E=0.
      FX=F(X)
      FV=FX
      FW=FX
      DO 11 ITER=1,ITMAX
        XM=0.5*(A+B)
        TOL1=TOL*ABS(X)+ZEPS
        TOL2=2.*TOL1
        IF(ABS(X-XM).LE.(TOL2-.5*(B-A))) GOTO 3
        IF(ABS(E).GT.TOL1) THEN
          R=(X-W)*(FX-FV)
          Q=(X-V)*(FX-FW)
          P=(X-V)*Q-(X-W)*R
          Q=2.*(Q-R)
          IF(Q.GT.0.) P=-P
          Q=ABS(Q)
          ETEMP=E
          E=D
          IF(ABS(P).GE.ABS(.5*Q*ETEMP).OR.P.LE.Q*(A-X).OR.
     *         P.GE.Q*(B-X)) GOTO 1
          D=P/Q
          U=X+D
          IF(U-A.LT.TOL2 .OR. B-U.LT.TOL2) D=SIGN(TOL1,XM-X)
          GOTO 2
        ENDIF
1       IF(X.GE.XM) THEN
          E=A-X
        ELSE
          E=B-X
        ENDIF
        D=CGOLD*E
2       IF(ABS(D).GE.TOL1) THEN
          U=X+D
        ELSE
          U=X+SIGN(TOL1,D)
        ENDIF
        FU=F(U)
        IF(FU.LE.FX) THEN
          IF(U.GE.X) THEN
            A=X
          ELSE
            B=X
          ENDIF
          V=W
          FV=FW
```

```
          W=X
          FW=FX
          X=U
          FX=FU
        ELSE
          IF(U.LT.X) THEN
            A=U
          ELSE
            B=U
          ENDIF
          IF(FU.LE.FW .OR. W.EQ.X) THEN
            V=W
            FV=FW
            W=U
            FW=FU
          ELSE IF(FU.LE.FV .OR. V.EQ.X .OR. V.EQ.W) THEN
            V=U
            FV=FU
          ENDIF
        ENDIF
11    CONTINUE
      PAUSE 'Brent exceed maximum iterations.'
3     XMIN=X
      BRENT=FX
      RETURN
      END
            FUNCTION DF1DIM(X)
      PARAMETER (NMAX=50)
      COMMON /F1COM/ NCOM,PCOM(NMAX),XICOM(NMAX)
      DIMENSION XT(NMAX),DF(NMAX)
      DO 11 J=1,NCOM
        XT(J)=PCOM(J)+X*XICOM(J)
11    CONTINUE
      CALL DFUNC(XT,DF)
      DF1DIM=0.
      DO 12 J=1,NCOM
        DF1DIM=DF1DIM+DF(J)*XICOM(J)
12    CONTINUE
      RETURN
      END
       FUNCTION F1DIM(X)
      PARAMETER (NMAX=50)
      COMMON /F1COM/ NCOM,PCOM(NMAX),XICOM(NMAX)
      DIMENSION XT(NMAX)
      DO 11 J=1,NCOM
        XT(J)=PCOM(J)+X*XICOM(J)
11    CONTINUE
      F1DIM=FUNC(XT)
      RETURN
      END
```