# NUMERICAL OPTIMIZATION EXAM

Time- 1 Hour.

Solve 3 out of the 5 following questions:

1. Use the steepest descent method to minimize the following function:

$$f(x_1,x_2)=x_1-x_2+2x_1^2+2x_1x_2+x_2^2$$

starting from the point **x=(0,0)**

**for 4 iterations.**

Hint: Search direction is : $s_i=-\nabla f_i$

and you still have to find step-length at each iteration.

2. Consider the following objective function:

$$f(x_1, x_2)=\frac{1}{2}(x_1^2+x_2^2)$$

this can be written as:

$$f(x)=\frac{1}{2}x^tAx$$

where

$$A=\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

Apply the Davidon- Fletcher-Powell Q-N method

$$H_{k+1}=H_k+\frac{p_k p_k^t}{p_k^t y_k} - \frac{H_k y_k y_k^t H_k}{y_k^t H_k y_k}$$

Start from $x_0=(4,4)$ and show 2 iterations.

Choose $H_0$ ,the initial estimate of the inverse Hessian matrix A to be the unit matrix:

$$A=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Show that the matrix $H_2$ obtained after 2 iterations of DFP is the approximate inverse of the Hessian matrix A.

3. Describe the Davidon-Fletcher-Powell Quasi-Newton algorithm.

b. Prove the hereditary positive definiteness of DFP i.e. if $H_k$ is positive definite so is $H_{k+1}$. I.e. show that

$$\mathbf{x}^{\mathbf{T}} H_{k+1} \mathbf{x} > \mathbf{0} \text{ for all } \mathbf{x} \neq 0$$

Hint: The DFP formula is :

$$H_{k+1} = H_k + \frac{p_k\, p_k^t}{p_k^t y_k} - \frac{H_k y_k y_k^t H_k}{y_k^t H_k y_k}$$

4. Prove the finite-step convergence Theorem of the Davidon-Fletcher-Powell Q-N method which states:

If f is a quadratic function with a constant Hessian G, then the DFP method produces direction vectors $p_k$ that are G orthogonal (conjugate) and if the method is carried out for n steps then:

$$H_n = G^{-1}$$

or:

$$p_i G p_j = 0 \quad \text{for } 0 \le i < j \le k$$

$$H_{k+1} G\, p_i = p_i \quad \text{for } 0 \le i \le k$$

Hint: Use induction and the fact that for the quadratic case:

$y_k = g_{k+1} - g_k = Gx_{k+1} - Gx_k = Gp_k$

5. Use a Fibonacci search to find the minimum of the unimodal function :

$$f(x) = -xe^{-x}$$

in the interval of uncertainty [0,3] with $\varepsilon = 0.1$.

Use the usual Fibonacci method to derive :

$$L_2 = L_1 \frac{F_{n-1}}{F_n} + \frac{(-1)^n}{F_n} \varepsilon$$

Conduct 4 steps of Fibonacci using your calculator and summarize results in the form of a table:

| k | $x_k$ | $f(x_k)=-x_k e^{-x_k}$ |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |

6 .Use the Newton minimization method for multivariate minimization to minimize the function:

$f(x_1,x_2)=2x_1^2+ x_2^2 +x_1x_2-x_1-3x_2.$

Calculate gradient and Hessian and use formula :

$x^{k+1}=x^k - H^{-1}(x^k).g(x^k)$

from

$x_0=(1,1)^T$ for 2 iterations.

*GOOD LUCK  !!!!*