NUMERICAL OPTIMIZATION

MAD 5420-01

Spring 2004

Term: Spring 2004
Instructor: Prof .I.M.Navon
Hours: 1;25: 2:15 MWF
Room: 107 LOV

SYLLABUS

- Introduction
- The NCP problem
- Characterization of single variable unconstrained minimum
- Necessary and sufficient first and second order conditions for an unconstrained minimum.
- Examples
- **Unconstrained minimization methods**
- Properties of a quadratic function
- Convex and concave functions
- Method of bisection
- Newton's method
- The secant method
- Regula -falsi method
- Univariate minimization
- Unimodal functions
- Fibonnacci search for univariate minimum and algorithm
- Golden section method and algorithm
- Quadratic fit-polynomial interpolation for univariate minimum
- Cubic fit polynomial interpolation
- Methods for minimization of multivariate smooth functions
- Direction of descent
- Global convergence of descent methods
- Computing step-length along direction of descent
- Armijo rule for step length

• Goldstein Armijo test for step length
• The Wolfe test for step length
• Step length for a quadratic function
• Computation of good search directions
• Method of steepest descent
• Examples
• Newton's method for multivariate functions
• Problems with Newton's method
• Modification of Newton's method for indefinite Hessian
• Methods based on Choleski decomposition
• **Quasi-Newton Methods**
• Matrix outer products and rank one matrices
• Symmetry and positive-definiteness in Hessian approximations with Q-N method.
• Algorithmic implementation of Quasi-Newton methods
• Davidon Fletcher Powell (DFP) Quasi-Newton (Q-N) Algorithm
• Examples
• Positive definiteness of DFP Q-N update to the Hessian.
• Finite-step convergence of DFP Q-N method
• Davidon Fletcher Powell Q-N method with cubic interpolation for the step-length.
• Q-N method for positive definite quadratic form
• Variable metric form of the Quasi-Newton descent methods.
• Broyden-Fletcher-Goldfarb-Shanno (BFGS) Q-N method
• Use of Sherman Morrisson Inverse matrix formula
• Variational metric properties of DFP and BFGS Q-N Methods.
• Scaling issues related to Quasi-Newton method
• Eigenvalues of Hessian and the interlocking eigenvalues Theorem.
• Choice of appropriate scale factors and self scaling Q-N algorithm.
• Limited Memory Quasi-Newton Methods
• **Conjugate Gradient algorithms**
• Conjugate directions Theorem
• Expanding Subspace Theorem
• Geometric 2-D interpretation of conjugacy concept and polarity.
• Conjugate gradient (C-G) algorithm and examples.
• C-G Theorem

- Fletcher -Reeves C-G algorithm for nonlinear functions.
- C-G methods viewed as an optimal process.
- Convergence rate of the C-G method
- Hessian with clustered eigenvalues and preconditioning
- Examples
- Different C-G algorithms with various line-search methods
- Polak-Ribiere C-G algorithm
- Restart methods for nonlinear C-G algorithm
- Powell's (1977) restart C-G method using Beale's condition
- Memoryless Q-N like C-G methods: Codes and algorithms.
- Scaling and preconditioning in C-G methods
- Variable-storage Q-N like C-G Methods : Codes, Algorithms and Examples
- Truncated Newton methods and examples
- Use of scientific minimization software libraries. (NAG, IMSL, NEOS)
- **Constrained Optimization**
- First and second order conditions : theory
- Karush- Kuhn Tucker (KKT) Theorem
- Linear and nonlinear constraints and the active set approach
- Penalty and barrier functions methods
- Reduced gradient and gradient projection methods
- Exact penalty methods
- Augmented Lagrangian Methods
- Sequential Quadratic Programming (SQP)
- Reduced Gradient methods (GRG)
- Rate of convergence of constrained minimization methods
- Nondifferentiable constrained minimization methods
- The subgradient bundle algorithm for nondifferentiable minimization
- **Global Minimization methods**
- Simulated annealing methods theory and applications
- Genetic algorithms-Theory and applications
- Other global search methods
- Experience with simulated annealing and genetic algorithms