MID TERM EXAM FOR NUMERICAL OPTIMIZATION

MAD 5420

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Applications

Please solve one out of the following 2 numerical questions:

1. Use the Quasi-Newton method of Davidon-Powell Fletcher to minimize the function

$$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$$

From the starting point:

$$x_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Using Davidon Fletcher Powell algorithm with:

$$H_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Use minimizing step length α by minimizing:

$f(x+\alpha d)$ with respect to α .

Due to the function being quadratic you should converge in 2 full iterations.

2. Consider Newton's method:

$$\nabla^2 f(x_k)p = -\nabla f(x_k)$$

For the quadratic function:

$$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1 x_2 + x_2^2$$

By taking as starting point:

$$x_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Calculate one iteration using

$$x_{k+1} = x_k - [\nabla^2 f(x_k)]^{-1} \nabla f(x_k)$$

Show that indeed we converged in one iteration i.e. that

$$g_2 = \nabla f(x_2) = \begin{bmatrix} \partial f / \partial x_1 \\ \partial f / \partial x_2 \end{bmatrix}_{x_2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Theory

Please solve one out of the following 2 theoretical questions:

1(a). Describe the Davidon-Fletcher-Powell quasi-Newton algorithm.

1(b). Prove the hereditary positive definiteness of DFP, i.e., if H_k is positive definite then so is H_{k+1}.

Recall that DFP quasi-Newton rank 2 update is given by:

$$H_{k+1} = H_k + \frac{p_k p_k^T}{p_k^T y_k} - \frac{H_k y_k y_k^t H_k}{y_k^t H_k y_k}$$

2. Prove the finite step convergence of DP Quasi Newton when the function f is quadratic with constant Hessian Q.

I.e. that DFP produces direction vectors p_k that are Q-orthogonal and that if the method is carried n steps then $H_n = Q^{-1}$

The theorem to prove says that if f is quadratic with positive definite Hessian Q, then for the Davidon-Fletcher-Powell method

$$p_i^T Q p_j = 0 \quad 0 \le i < j \le k$$

$$H_{k+1} Q p_i = p_i \text{ for } 0 \le i \le k$$