

Homework 10 Sherman-Morrison-Woodbury Formula.

If the square nonsingular matrix A undergoes a rank-one update

$$\bar{A} = A + ab^T$$

where

$$A \in \mathbb{R}^{n \times n}$$

$$a, b \in \mathbb{R}^n$$

Show that \bar{A} is invertible only if

$$b^T A^{-1} a \neq -1$$

In this case the inverse of \bar{A} is given by:

$$\bar{A}^{-1} = A^{-1} - \frac{A^{-1} a b^T A^{-1}}{1 + b^T A^{-1} a}$$

Please prove the formula is correct. It is easy to verify it by multiplication of the definitions of \bar{A}^{-1} and \bar{A} together and check that they produce the identity.