

## Home work 6 (Convergence of steepest descent Method)

Apply the steepest descent method with exact line search to 3-D quadratic function

$$f(x) = \frac{1}{2} x^T Q x - c^T x$$

with

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & \gamma^2 \end{pmatrix}$$

$$\text{and } c = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Here  $\gamma$  is a parameter that assumes values of 1, 10, and 100.

Use the starting point  $x_0 = (0, 0, 0)^T$

How do the results of your running the code for 20 iterations for each value of  $\gamma$  compare with results of convergence theory for S.D. method developed in class?

That means that you set up a Table with 2 columns and compare and print the values of

$$\frac{f(x_{k+1}) - f(x_*)}{f(x_k) - f(x_*)} = \frac{E(x_{k+1})}{E(x_k)}$$

and compare them to the bound:

$$\left[ \frac{\text{cond}(Q) - 1}{\text{cond}(Q) + 1} \right]^2$$

For example if  $\gamma=10$   $\text{cond}(Q)=100$  and the bound on the rate constant is 0.9607.

By the way

$$x_* = Q^{-1}c = \begin{pmatrix} 1 \\ \frac{1}{\gamma} \\ \frac{1}{\gamma^2} \end{pmatrix}$$

You should expect that the observed rate constant is close *but less* than the bound given in the theorem.