

## Homework 21 (The NEOS Portfolio Optimization problem)

Please visit the page:

<http://www-fp.mcs.anl.gov/otc/Guide/CaseStudies/port/>

at the NEOS web location.

After having read the attached papers:

1. Joseph Czyzyk, Timothy Wisniewski and Stephen J. Wright, 1999: Optimization Case Studies in the NEOS Guide. SIAM Review
2. **Perold:** Large Scale Portfolio Optimization, 1984. Management Science Vol 30

Please describe in some detail the portfolio optimization problem related to the fact that given a wide range of possible investments with possibly correlated risks and rewards – how does an investor go about selecting a portfolio of stocks and securities that maximizes expected return while meeting an acceptable standard of risk?

We use a mathematical model based on the fact that we want to choose allocation vector  $w$  so as to keep the expected return large while keeping the variance small.

We define a risk tolerance parameter  $k$  to quantify the relative weighting of the two above mentioned aims and solve the following optimization problem:

Min

$$\frac{k}{2} w^T Q w - \mu^T w$$

$$e^T w = 1, w \geq 0$$

Run the following portfolio optimizations constrained minimization on the NEOS server after studying the concepts of the Efficient Frontier, utility functions and mathematical formulations and choose 21 stocks out of the 30 possible and compare NEOS runs for the levels of risk:

1)  $k=0.1$ , 2)  $k=1.0$ , 3)  $k=10$  and 4)  $k=100$

Explain the constrained optimization and the underlying mathematics that guide the optimization of the portfolio.

Show and discuss the results obtained in terms of the mix of stocks and bonds for each one of the cases going from low risk to high risk tolerance.