Homework 18 (Augmented Lagrangian)

Consider the problem:

Minimize f(x) = $\frac{1}{2}\left[x_1^2 + \frac{1}{3}x_2^2\right]$

Subject to $x_1 + x_2 = 1$

Use an unconstrained minimization method such as the conjugate gradient algorithm or the BFGS Q-N one to write a small code in order to compare method of augmented Lagrangian with that of quadratic penalty where the augmented Lagrangian is given by:

$$L(x, c_k, \lambda_k) = \frac{1}{2}\left[x_1^2 + \frac{1}{3}x_2^2\right] + \lambda_k(x_1 + x_2 - 1) + \frac{1}{2}c_k((x_1 + x_2 - 1)^2$$

In the quadratic penalty method λ_k =0 for all k.

Use the following updating in the Augmented Lagrangian for the multipliers:

$$\lambda_{k+1} = \lambda_k + c_k(x_1^{(k)} + x_2^{(k)} - 1)$$

$$\lambda_0 = 0$$

For the penalty coefficients try 2 options:

$$c_{k+1} = 10c_k$$
$$c_0 = 0.2$$

Or try

$$c_k = 0.1 \times 2^k$$
$$c_k = 0.1 \times 4^k$$
$$c_k = 0.1 \times 8^k$$

Put the computational results in a form of table for k, $x_1^{(k)}, x_2^{(k)}$ for about 10-15 iterations for both methods and the different update variants.

Using first order conditions minimization of $L(x, c_k, \lambda_k)$ yields:

$$x_1^{(k)} = \frac{c_k - \lambda_k}{1 + 4c_k} \qquad x_2^{(k)} = \frac{3(c_k - \lambda_k)}{1 + 4c_k}$$

So it is evident that the optimal solution is:

$$x^* = (0.25, 0.75)$$
$$\lambda^* = -0.25$$

Your results should show (if your coding is correct) that method of augmented Lagrangian requires a smaller number of minimizations to obtain the solution and that the number of minimizations for both penalty and A-L methods decreases when the penalty parameter is increased at a faster rate.

However effects of ill-conditioning are felt more under these circumstances when the unconstrained minimization is carried out numerically.