Homework 15 Constrained optimization

a) Consider the following problem:

Min $f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 + 40x_1 + 20x_2 - 3000$

$$\text{Subject to } \begin{aligned} g_1(x_1, x_2, x_3) &= x_1 - 50 \geq 0 \\ g_2(x_1, x_2, x_3) &= x_1 + x_2 - 100 \geq 0 \\ g_3(x_1, x_2, x_3) &= x_1 + x_2 + x_3 - 150 \geq 0 \end{aligned}$$

Write the KKT conditions for the Lagrangian with conditions on the Lagrange multipliers

$\mu_1 \geq 0, \mu_2 \geq 0, \mu_3 \geq 0$ $\mu_1 \geq 0, \mu_2 \geq 0, \mu_3 \geq 0$

where

$L(x, \lambda, \mu) = f(x) - \mu_1 g_1 - \mu_2 g_2 - \mu_3 g_3$

And obtain 8 different conditions on combinations of $\mu_1 \geq 0, \mu_2 \geq 0, \mu_3 \geq 0$ and derivatives of the Lagrangian with respect to x.

Analyze all cases and show then what is the optimal x^* satisfying the constrained optimization case.

b) Consider a minimization problem with 2 equality constraints given by:

$g_1(x) = x_1^2 + x_2^2 + x_3^2 - 3 = 0$
$g_2(x) = 2x_1 - 4x_2 + x_3^2 + 1 = 0$

At the feasible point:

X=$(1,1,1)^T$

Show that the gradients of the constraints are linearly independent so that x^* is a regular point of the constraints

3) Consider the equality constrained problem:

Min f (x) $= 3x_1 + 4x_2$

Subject to:

$$(x_1 + 1)^2 + x_2^2 = 1$$
$$(x_1 - 1)^2 + x_2^2 = 1$$

Solution is $x^* = (0,0)^T$

Show that gradients of constraint at x^* are linearly dependent and the gradients of the Lagrangian give rise to an inconsistent system.