

## Homework 14 (Equality Constrained Minimization)

1. Find a rectangle of sides  $a$  and  $b$  respectively that has the greatest possible area subject to a prescribed perimeter  $2(a+b)$ .
2. Find the dimensions  $x$ ,  $y$  and  $z$  of a box to maximize the volume of the box, given a fixed amount of cardboard of surface  $72$  sq. ft.
3. Consider an equality constrained problem with 2 equality constraints:

$$h_1(x) = x_1^2 + x_2^2 + x_3^2 - 3 = 0$$

$$h_2(x) = 2x_1 - 4x_2 + x_3^2 + 1 = 0$$

At the feasible point:

$$x_* = (1, 1, 1)^T$$

Calculate gradients of the constraints at  $x_*$  and show that they are linearly independent, so  $x_*$  is a regular point.

4. Consider the problem :

Minimize:

$$-(x_1x_2 + x_2x_3 + x_1x_3)$$

Subject to  $x_1 + x_2 + x_3 = 3$

Write first order conditions and you find

$$x_1^* = x_2^* = x_3^* = 1$$

$$\lambda^* = 2$$

Write the Hessian of the Lagrangian  $\nabla_{xx}^2 L(x^*, \lambda^*) = \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$

Show that second order sufficiency conditions apply i.e.

$$y \in V = \{y \mid \nabla h(x^*)^T y = 0\} = \{y \mid y_1 + y_2 + y_3 = 0\}$$

With  $y \neq 0$

$y^T \nabla_{xx}^2 L(x^*, \lambda^*) y > 0$  and sufficient conditions for second order equality constrained minimum are satisfied and  $x^*$  is a strict local minimum.