Homework No 10 (BFGS Quasi-Newton Method)

Consider the BFGS quasi Newton method:

$$
\begin{aligned}
H_{k+1} &= (I - \rho_k s_k y_k^T) H_k ((I - \rho_k y_k s_k^T) + \rho_k s_k s_k^T \\
s_k &= x_{k+1} - x_k \\
y_k &= \nabla f_{k+1} - \nabla f_k \qquad\qquad \text{(BFGS)} \\
\rho_k &= \frac{1}{y_k^T s_k}
\end{aligned}
$$

for the inverse Hessian approximation H_k.

For the direct Hessian approximation B_k we have via Sherman Morrison – Woodbury formula:

$$B_{k+1} = B_k - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k} + \frac{y_k y_k^T}{y_k^T s_k} \qquad \text{(BFGS)}$$

Use the attached code conminf.f or write equivalent code by yourself and test it for the difficult Woods function:

$$
\begin{aligned}
f(x) &= 100(x_2 - x_1^2)^2 + (1 - x_1)^2 + 90(x_4 - x_3^2)^2 + (1 - x_3)^2 \\
&+ 10.1[(x_2 - 1)^2 + (x_4 - 1)^2] + 19.8(x_2 - 1)(x_4 - 1)
\end{aligned}
$$

a) Determine the Hessian of Woods function and obtain its eigenvalues and condition number for the point $x = (-3, -1, -3, -1)^T$

b) Use accuracy $\varepsilon = 10^{-3}$ and $\varepsilon = 10^{-2}$ and compare results for starting points:

c) $x = (-3, -1, -3, -1)^T$ and $x = (-1.2, 1, 1.2, 1)^T$

d) Plot in semi log function and gradient norm vs. the number of iterations

e) Flowchart code of Conminf.f and explain its line-search method.