Simulation Of Vortex Pinning in Two-Band Superconductors

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About Me

- Chad Sockwell
- Undergrad in Physics and S.C. at FSU (Honors Thesis)
- Modeling Superconductivity With Max Gunzburger and Janet Peterson
- Currently Master's student in S.C.
- Applying to DOE Fellowship
- Possible PhD in Physics or SC

Outline

- Background and Motivation
- Simulation of Superconductivity
- Challenges and Future Work

Background

- What is a Superconductor (SC)?
  - Zero Electrical Resistance
  - No Waste Heat
- Normal metals are penetrated by Mag. Fields
- Some SC are not (Meissner Effect)
- Some are penetrated only by tubes of flux (Vortices)

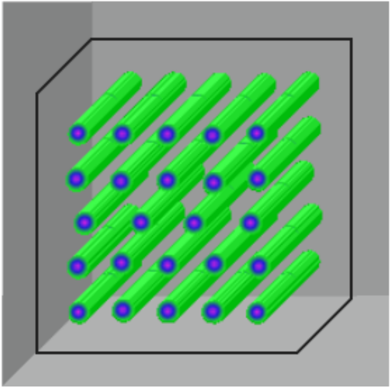


Figure : 3D SC with Vortices

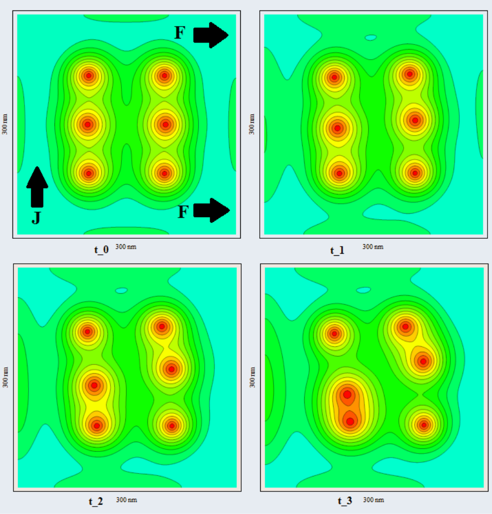
Why Vortex Dynamics are Important

- Moving Vortices (Flux flow) creates Resistance

$$f\ \hat{\mathbf{x}} = J\ \hat{\mathbf{y}} \times B\ \hat{\mathbf{z}}$$

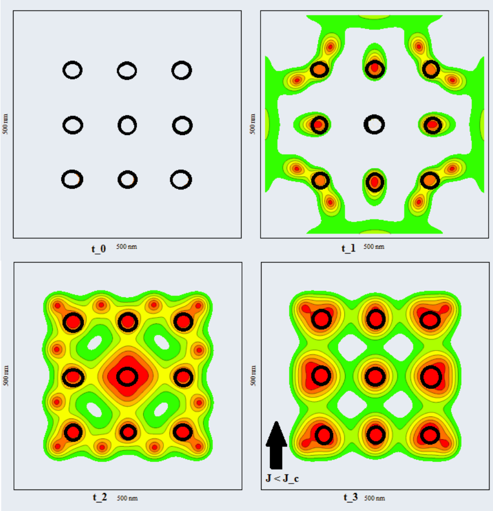
$$E\ \hat{\mathbf{y}} = B\ \hat{\mathbf{z}} \times u\ \hat{\mathbf{x}}$$

- Vortices (B) + Current (J)= Flux flow
- Flux Flow induces Electric Field (E) and Voltage (V)
- Resistance Now Exists ($\frac{V}{I}$=R)



Vortex Pinning Comes to the Rescue

- Immobilizing the Vortices Is Crucial
- Non Superconducting Metal= Normal Metal= Pinning Sites (Outlined in Black)
- Vortices "Stick" To Impurities
- Limited Increase In J_c



Applications of Large Scale Simulations

Large Scale Simulations Could Improve Technology:

- Efficient Current Carriers
- Powerful Magnets (by magnetization)
- MRI
- Efficient Mag Lev

Modeling Vortex Pinning with Normal Inclusions

- TD-Ginzburg-Landau model (Coupled System of NL PDES)
- $\psi \in \mathbb{C}$, Like complex phase field order parameter
- $\mathbf{A} \in \mathbf{R}^2$ magnetic vector potential

$$\Gamma(\frac{\partial \psi}{\partial t} - i\frac{\kappa}{\sigma}Jy) + (|\psi|^2 - \tau)\psi + (\frac{i}{\kappa}\nabla - \mathbf{A})^2\psi = 0 \text{ in } \Omega$$

$$\sigma\frac{\partial \mathbf{A}}{\partial t} - \mathbf{J} + \nabla \times \nabla \times \mathbf{A} + \frac{i}{2\kappa}(\psi^*\nabla\psi - \psi\nabla\psi^*) + |\psi|^2\mathbf{A} = \nabla \times \mathbf{H} \text{ in } \Omega$$

$$\nabla \times \mathbf{A} \times \mathbf{n} = (\mathbf{H} - J\hat{\mathbf{z}}(x - L/2)) \times \mathbf{n} \text{ on } \partial\Omega$$

$$\mathbf{A} \cdot \mathbf{n} = 0 \text{ on } \partial\Omega$$

$$\nabla\psi \cdot \mathbf{n} = 0 \text{ on } \partial\Omega$$

$$\psi(t=0) = \psi_0 \;;\; \mathbf{A}(t=0) = \mathbf{A}_0 \;;\; \nabla \cdot \mathbf{A}(t=0) = 0 \text{ in } \Omega$$

Model To Simulation

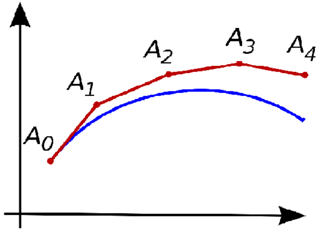
- Simulation of Magnesium Diboride (Two Band Model)
- Need Material Parameters: κ, σ, Γ
- Various Inputs: Field ($\mathbf{H}$), Applied Current ($\mathbf{J}$), Temperature (τ)
- Change Material Parameters in Normal Metals (Impurities)

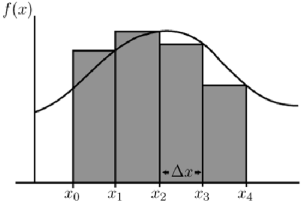
$$\Gamma(\frac{\partial \psi}{\partial t} - i\frac{\kappa}{\sigma}Jy) + (|\psi|^2 - \tau)\psi + (\frac{i}{\kappa}\nabla - \mathbf{A})^2\psi = 0 \text{ in } \Omega$$

$$\sigma\frac{\partial \mathbf{A}}{\partial t} - \mathbf{J} + \nabla \times \nabla \times \mathbf{A} + \frac{i}{2\kappa}(\psi^*\nabla\psi - \psi\nabla\psi^*) + |\psi|^2\mathbf{A} = \nabla \times \mathbf{H} \text{ in } \Omega$$

Numerical Methods

- FEM →Triangular Piecewise Parabolic Elements & Gauss Quadrature

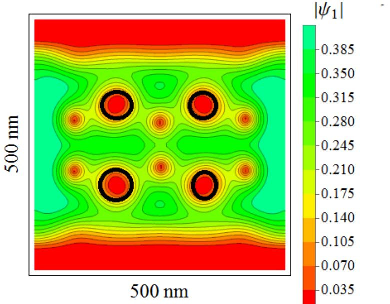


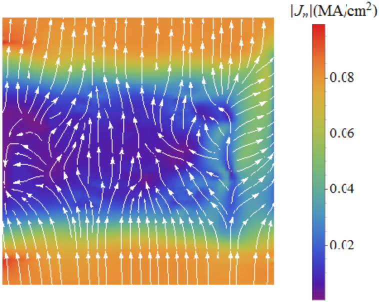


- Newton's Method, Full Jacobian
- Sparse Storage (CRS)
- Adaptive Backward Euler
- Parallel Solver (SUPERLU) (9/10 NL Time Cost)
- These methods were "Good Enough for Now"

Passing a Resistance Free Current

- Metal - Superconductor Interface
- $\psi \rightarrow 0$ in Vortices and Metals
- Flux Flow produces Resistive (or Normal) Current
- Impurities Outline In Black
- Normal Current$\rightarrow$Resistance
- Did Pinning Prevent Flux Flow?





Computational Challenges For Practical Simulation

- Large Spatial and Time Scale
- Large Storage Costs→ Distributed memory
- Long Solve Times → Parallel Iterative Solvers & Preconditioners
  - Trilinos Distributed Environment & Solver's
  - ML & Hypre AMG Preconditioners?
  - Jacobian Free Newton-Kylov Methods?
- 3D-Modeling to Infinity
  - BEM for the Exterior?
- Finding Maximum Current
  - Optimization, Continuation?

Need Lots of Vortices