Simulation Of Vortex Pinning in Two-Band Superconductors

Chad Sockwell

Florida State University

kcs12j@my.fsu.edu

February 5, 2016
About Me

- Chad Sockwell

- Undergrad in Physics and S.C. at FSU (Honors Thesis)

- Modeling Superconductivity With Max Gunzburger and Janet Peterson

- Currently Master’s student in S.C.

- Applying to DOE Fellowship

- Possible PhD in Physics or SC
Outline

- Background and Motivation
- Simulation of Superconductivity
- Challenges and Future Work
Background

- What is a Superconductor (SC)?
  - Zero Electrical Resistance
  - No Waste Heat

- Normal metals are penetrated by Mag. Fields

- Some SC are not (Meissner Effect)

- Some are penetrated only by tubes of flux (Vortices)

Figure: 3D SC with Vortices
Why Vortex Dynamics are Important

- Moving Vortices (Flux flow) creates Resistance
  \[ f \hat{x} = J \hat{y} \times B \hat{z} \]
  \[ E \hat{y} = B \hat{z} \times u \hat{x} \]

- Vortices (B) + Current (J) = Flux flow

- Flux Flow induces Electric Field (E) and Voltage (V)

- Resistance Now Exists (\[ R = \frac{V}{I} \])
Vortex Pinning Comes to the Rescue

- Immobilizing the **Vortices** Is Crucial

- **Non Superconducting Metal** $\rightarrow$ **Normal Metal** $\rightarrow$ Pinning Sites (Outlined in Black)

- **Vortices** "Stick" To Impurities

- Limited Increase In $J_c$
Applications of Large Scale Simulations

Large Scale Simulations Could Improve Technology:

- Efficient Current Carriers
- Powerful Magnets (by magnetization)
- MRI
- Efficient Mag Lev
 modeling Vortex Pinning with Normal Inclusions

- TD-Ginzburg-Landau model (Coupled System of NL PDES)
- \( \psi \in \mathbb{C} \), Like complex phase field order parameter
- \( \mathbf{A} \in \mathbb{R}^2 \) magnetic vector potential

\[
\Gamma \left( \frac{\partial \psi}{\partial t} - i \frac{\kappa}{\sigma} J y \right) + (|\psi|^2 - \tau)\psi + \left(\frac{i}{\kappa} \nabla - \mathbf{A}\right)^2 \psi = 0 \text{ in } \Omega
\]

\[
\sigma \frac{\partial \mathbf{A}}{\partial t} - \mathbf{J} + \nabla \times \nabla \times \mathbf{A} + \frac{i}{2\kappa} (\psi^* \nabla \psi - \psi \nabla \psi^*) + |\psi|^2 \mathbf{A} = \nabla \times \mathbf{H} \text{ in } \Omega
\]

\[
\nabla \times \mathbf{A} \times \mathbf{n} = (\mathbf{H} - J \hat{z} (x - L/2)) \times \mathbf{n} \text{ on } \partial \Omega
\]

\[
\mathbf{A} \cdot \mathbf{n} = 0 \text{ on } \partial \Omega
\]

\[
\nabla \psi \cdot \mathbf{n} = 0 \text{ on } \partial \Omega
\]

\[
\psi(t = 0) = \psi_0 ; \mathbf{A}(t = 0) = \mathbf{A}_0 ; \nabla \cdot \mathbf{A}(t = 0) = 0 \text{ in } \Omega
\]
Simulation of Magnesium Diboride (Two Band Model)

Need Material Parameters: $\kappa$, $\sigma$, $\Gamma$

Various Inputs: Field ($\mathbf{H}$), Applied Current ($\mathbf{J}$), Temperature ($\tau$)

Change Material Parameters in Normal Metals (Impurities)

$$\Gamma \left( \frac{\partial \psi}{\partial t} - i \frac{\kappa}{\sigma} J y \right) + (|\psi|^2 - \tau) \psi + \left( \frac{i}{\kappa} \nabla - \mathbf{A} \right)^2 \psi = 0$$

in $\Omega$

$$\sigma \frac{\partial \mathbf{A}}{\partial t} - \mathbf{J} + \nabla \times \nabla \times \mathbf{A} + \frac{i}{2\kappa} \left( \psi^* \nabla \psi - \psi \nabla \psi^* \right) + |\psi|^2 \mathbf{A} = \nabla \times \mathbf{H}$$

in $\Omega$
Numerical Methods

- FEM $\rightarrow$ Triangular Piecewise Parabolic Elements & Gauss Quadrature
- Newton’s Method, Full Jacobian
- Sparse Storage (CRS)
- Adaptive Backward Euler
- Parallel Solver (SUPERLU) (9/10 NL Time Cost)
- These methods were "Good Enough for Now"
Passing a Resistance Free Current

- Metal - Superconductor Interface

- $\psi \rightarrow 0$ in Vortices and Metals

- Flux Flow produces Resistive (or Normal) Current

- Impurities Outline In Black

- Normal Current $\rightarrow$ Resistance

- Did Pinning Prevent Flux Flow?
Computational Challenges For Practical Simulation

- Large Spatial and Time Scale
- Large Storage Costs → Distributed memory
- Long Solve Times → Parallel Iterative Solvers & Preconditioners
  - Trilinos Distributed Environment & Solver’s
  - ML & Hypre AMG Preconditioners?
  - Jacobian Free Newton-Kylov Methods?
- 3D-Modeling to Infinity
  - BEM for the Exterior?
- Finding Maximum Current
  - Optimization, Continuation?
Need Lots of Vortices