Passing Resistance Free Transport Currents Through Two Band-Superconductors

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Introduction

Superconductors provide a way to transport a resistance free electrical current. This novel property gives a promising future to the next generation of efficient electronics. Unfortunately superconductors must operate below a critical temperature T_c and can only carry a limited amount of resistance free transport current J_c. Numerical simulations of superconducting materials can give insight into their properties and help design better superconducting technology. A Simulation using the Modified Two Band Ginzburg-Landau model is presented that can investigate the current carrying properties of two band superconductors.

Flux Tube Vortices

When placed in a magnetic field, superconductors are penetrated by the magnetic field in the form of flux tubes, known as vortices. This is opposed to non-superconducting materials which are fully penetrated by magnetic fields. Simulations of vortex dynamics are critical because the interaction between the applied current and vortices dictate J_c. The electrical current causes the vortices to move (flux flow) and creates electrical resistance. J_c can be increased by placing impurities in the superconductor, which immobilize the vortices in the sample. Vortices can be modeled by an order parameter ψ which goes to 0 where a vortex or an impurity is located in the superconductor.

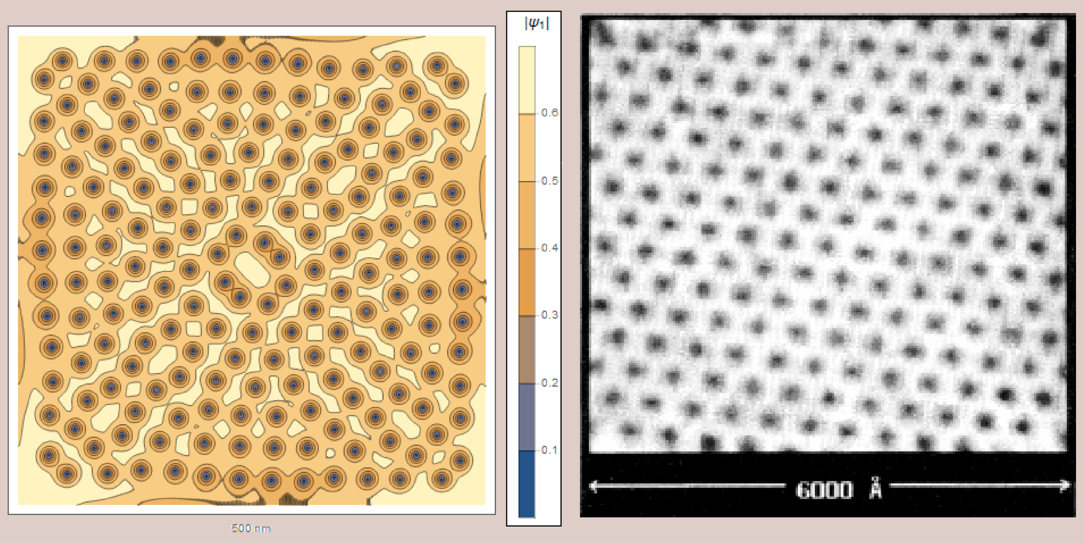


Figure 1: Simulation (left) and SEM image (right)

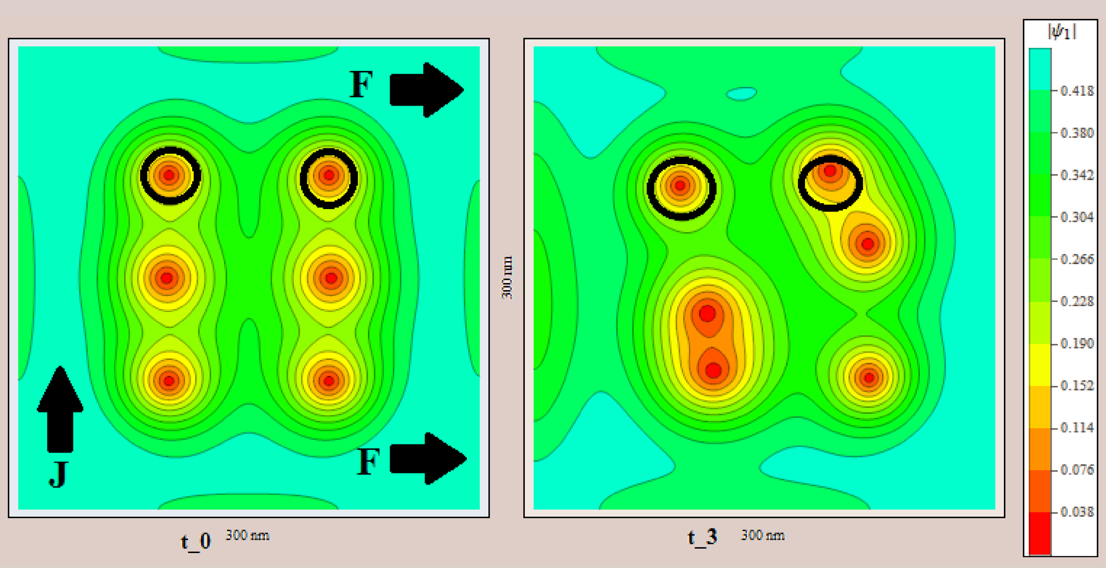


Figure 2: Time series of a superconductor with an applied current and two impurities outlined in black. As time increase the vortices move, generating resistance. However, the ones in the impurities remain immobilized.

Magnesium Diboride

Magnesium Diboride (MgB_2) is a superconducting material that is a good candidate for transporting resistance free current due to its ductility and relatively high critical temperature (39 K). It possesses two superconducting bands and is described by two coupled order parameters through the Two Band Ginzburg-Landau Model. Each band is essentially a pathway in the material for the superconducting electrons.

Modified Two Band Ginzburg-Landau Model

The Modified Two Band Ginzburg-Landau Model describes two band superconductors with normal metal impurities in the sample. The model consists of a coupled non-linear system of PDEs. Consider a sample sheet in the $x-y$ plane with a magnetic field $\mathbf{H}$ in the $\hat{\mathbf{z}}$ direction and an applied current density $\mathbf{J}$ in the $\hat{\mathbf{y}}$ direction. To model impurities the functions $a(x,y)$, $b(x,y)$, and $\eta(x,y)$ change value to represent impurities in the sample. The order parameters $\psi_{1,2} \in \mathbb{C}$ and the magnetic vector potential $\mathbf{A} \in \mathbf{R}^2$ are the variables of interest. The rest of the parameters are material specific.

$$\left(\frac{\partial \psi_1}{\partial t} - i\frac{Jy}{\sigma}\kappa_1\psi_1\right) - \alpha_1(x,y)\psi_1 + b(x,y)|\psi_1|^2\psi_1 + \left(-i\frac{\xi_{1,ab}}{x_0}\nabla - \frac{x_0}{\lambda_{1,ab}}\mathbf{A}\right)^2\psi_1 + \eta(x,y)\psi_2 = 0$$

$$\Gamma\left(\frac{\partial \psi_2}{\partial t} - i\frac{Jy}{\sigma}\kappa_1\psi_2\right) - \alpha_2(x,y)\psi_2 + b(x,y)|\psi_2|^2\psi_2 + \left(-i\frac{\xi_{2,ab}}{x_0}\nabla - \nu\frac{x_0}{\lambda_{2,ab}}\mathbf{A}\right)^2\psi_2 + \eta(x,y)\nu^2\psi_1 = 0$$

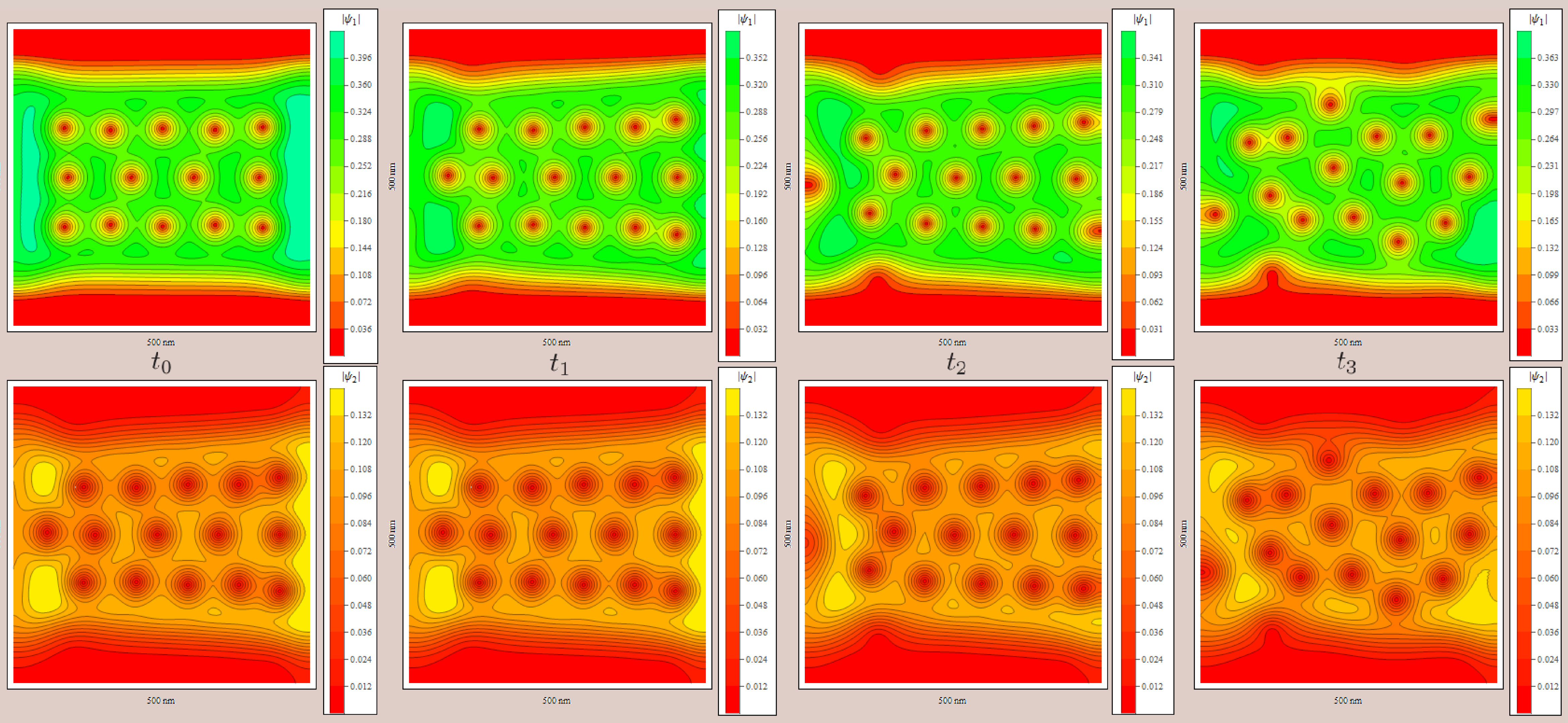
$$\nabla\times\mathbf{H} + \mathbf{J} = \sigma\frac{\partial \mathbf{A}}{\partial t} + \nabla\times\nabla\times\mathbf{A} + [\frac{i}{2\kappa_1}\left(\psi_1^*\nabla\psi_1 - \psi_1\nabla\psi_1^*\right) + \frac{x_0^2}{\lambda_{1,ab}^2}\mathbf{A}|\psi_1|^2]$$
$$+[\frac{i}{2\nu\kappa_2}\left(\psi_2^*\nabla\psi_2 - \psi_2\nabla\psi_2^*\right) + \frac{x_0^2}{\lambda_{2,ab}^2}\mathbf{A}|\psi_2|^2]$$

With Boundary Conditions

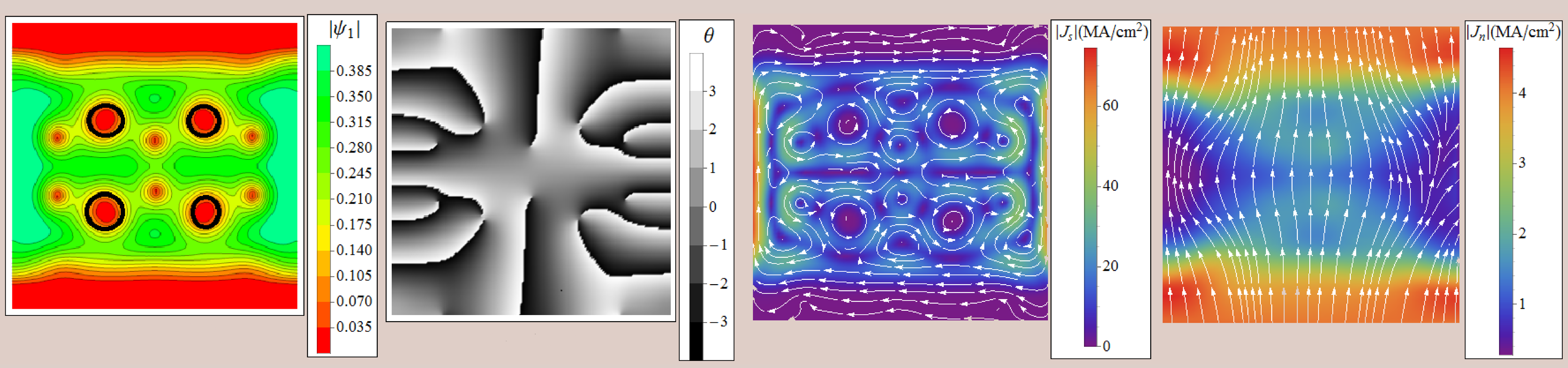
$$\nabla\psi_1\cdot\mathbf{n} = 0, \quad \nabla\psi_2\cdot\mathbf{n} = 0, \quad \mathbf{A}\cdot\mathbf{n} = 0, \quad (\nabla\times\mathbf{A})\times\mathbf{n} = \left(\mathbf{H} - \left(x - \frac{L}{2}\right)J\hat{\mathbf{z}}\right)\times\mathbf{n}$$

Simulation Results

Once the material parameters for MgB_2 are entered into the model, various numerical methods are needed to produce the simulation. For ease of implementation of the boundary conditions, the finite element method is chosen with triangular parabolic elements and Gauss quadrature. Newton's method linearizes the equations and backward Euler is used for time. The main challenge in this work is producing large scale simulations for practical use. For future work, Trilinos has been implemented to size up the scale of the simulation.



The first example (above) is a time series of the order parameters as a current is passed in the y direction. There are normal metal leads on the top and bottom to introduce the applied current. The conditions are $T = 30$K, $\mathbf{H}$= 0.2648 T, and $\mathbf{J}$= 33.717 $\hat{\mathbf{y}}$ MA cm^{-2}. Since there are no pinning sites, flux flow is seen in the sample as time increases, leading to a high resistance in the sample.



The Second example (above) shows the first order parameter with four normal metal sites (large red circles outlined in black). They provide pinning locations for the vorctices. The conditions are $\mathbf{H}$=0.2648 T, $J = 4.2147$ $\hat{\mathbf{y}}$ MA cm^{-2}, and $T = 39$ K. The complex phase of ψ_1, θ shows the location of the vortices where it jumps from 0 to 2π. The super current density J_s is the resistance free competent of the current, it encircles the sample to cancel the magnetic field from the interior of the sample. The normal current density J_n is the resistive component of the current and drops sharply in the superconducting region in the middle. The pinning sites provide in increase in J_c by immobilizing the vortices.