A NOVEL NORMAL INCLUSION MODELING STRATEGY FOR VORTEX PINNING IN TWO-BAND, HIGH-TEMPERATURE SUPERCONDUCTORS

K. CHAD SOCKWELL*

Abstract. New high-temperature superconductors give great promise to a new generation of superconducting electronics, free of electrical resistance, which is made feasible by higher critical temperature values. Modeling and simulating a process known as vortex pinning is crucial in preserving the resistance free properties of these materials. Unfortunately, many of these materials are what is known as two-band superconductors making the conventional modeling strategy for vortex pinning unsuitable. In this paper, a novel strategy to model vortex pinning in two-band superconductors is presented and simulations of the material magnesium diboride are shown to validate this strategy.

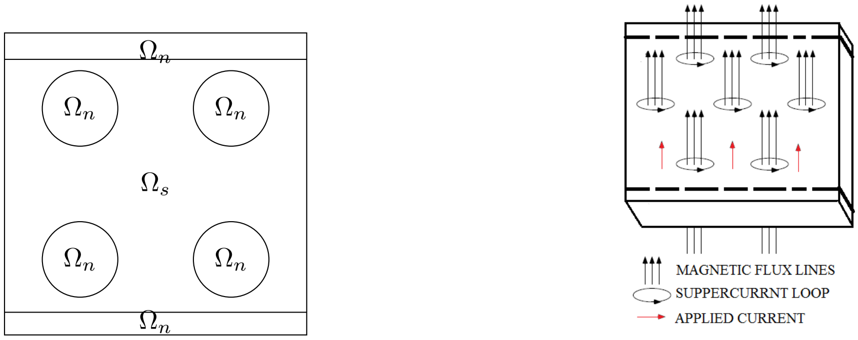
1. Introduction. Superconductivity is a phenomenon that only occurs below a critical temperature T_c. It is characterized by two properties, resistance free electrical conduction and the Meissner effect, which prevents magnetic fields from completely penetrating a superconductor. A more thorough discussion on superconductivity can be found in [4]. These two properties give great promise to a new generation of resistance free electronics and powerful magnets. However, the typical T_c value is close to 0 K, requiring expensive liquid helium as a coolant, therefore complicating possible superconducting technology. Thankfully, in recent years high-temperature superconductors (HTS) have been discovered, allowing the use of more inexpensive coolants. Every year new HTS are discovered bringing the highest critical temperature closer to room temperature than ever before.

Although HTS are bringing the dream of superconducting technology to the forefront of reality, they are accompanied by properties that complicate the modeling process. In particular, magnesium diboride (MgB_2) and iron-pnictides, both HTS materials, possess two superconducting electron bands that give rise to unconventional physical effects through Josephson like coupling between the bands, [5]. In [2], a two-band Ginzburg-Landau (G-L) model was found to give rise to these composite effects in MgB_2, verifying this fact in MgB_2. Moreover, all HTS are Type-II superconductors, which means they are penetrated by lattice forming tubes of magnetic of flux, known as vortices, when placed in a magnetic field. This fact is very important when passing a resistance free applied current through a superconductor. The applied current accelerates and distorts the vortex lattice, a process know as flux flow. This movement in the lattice induces a resistance in the superconductor, thereby diminishing the material's resistance free property. Fortunately, vortices can be immobilized and pinned to impurities in the sample (normal inclusions), thereby preventing flux flow. Since the integrity of the superconductor depends on vortex pinning, modeling and simulating this process is an extremely important aspect in producing future superconducting technology. Currently there are no models that provide normal inclusion pinning in two-band superconductors, which comprise a large set of HTS. This is because the modeling strategy for the one-band case, in [1], is not suitable for the two-band case due to the coupling effects in two-band materials. In this work, we fill this gap by providing a new modeling strategy for vortex pinning by normal inclusions in two-band superconductors. This is done by developing a composite model that is suitable for two-band superconductors.

The paper is organized as follows. In section 2, a Ginzburg-Landau (G-L) model is presented that is capable of modeling vortex pinning, by normal inclusions, in two-band superconductors is presented. The model is novel in its treatment of normal inclusions in two-band materials. In section 3, a candidate material, MgB_2, is chosen to present a simulation of vortex pinning using the new model. First the numerical methods and material parameters are presented. Then a simulation is shown to verify that the novel modeling strategy indeed produces vortex pinning by normal inclusions. This is done by quantifying the amount of resistance in a material with an applied current which contains normal inclusions and comparing that resistance to the amount of resistance in a material with no normal inclusions.

2. Modified Two-Band Time-Dependent Ginzburg-Landau Model (M2B-TDGL). In this section the composite model to describe pinning effects in two-band superconductors in revealed. In order to demonstrate vortex pinning in two-band superconductors, the new composite model is capable of modeling features such as normal inclusions, and applied electrical currents in the two-band formalism. It is composed of the normal inclusion G-L model [1] and the Two-band TDGL model, which was used to model MgB_2 in [2], and has a novel pinning strategy to ensure vortex pinning in both superconducting bands. As previously mentioned, coupling between the bands prevents the modeling strategy in [1] from producing normal inclusions in both bands. The domain is two dimensional, $\Omega \in \mathbb{R}^2$, on the x-y plane. It contains both superconducting regions Ω_s and normal inclusion regions Ω_n, as shown in Fig. 1a. The

*Department of Scientific Computing, Florida State University (kcs12j@my.fsu.edu, http://people.sc.fsu.edu/~kcs12j/).



(a) A superconducting sample geometry with 4 circular normal inclusions and two metal bands at the top and bottom that serve as metal leads where an applied current enters and leaves the sample.

(b) The setup of the superconducting sample for the simulations. The external magnetic field **H** is in the z-direction and the applied current density **J** is in the y-direction. The normal metal - superconducting metal interfaces are illustrated by the dashed lines.

domain also contains two metal strips that act as current leads, where the current enters and leaves. We can consider the domain $\Omega = \Omega_s \cup \Omega_n$ as a two-dimensional cutout of a three dimensional superconductor, placed in a magnetic field **H** and with an electrical current density **J** applied to it, as shown in Fig. 1b.

The Modified Two-Band Time-Dependent Ginzburg-Landau model (M2B-TDGL) requires several physical quantities. It consists of 3 variables, the two order parameters ψ_1 and ψ_2 which represent the density of superconducting electrons in the metal. The third variable is the magnetic vector potential **A**, whose curl is the magnetic field produced by the vortices in the superconductor. Three quantities are that necessary for the model (any G-L model in fact) are the two characteristic lengths λ_j, ξ_j and the G-L parameters κ_j, for the j^{th} band. Since we are modeling a two-band superconductor, each band will have its own characteristic lengths and G-L parameter. Three more quantities that will become relevant in a later section are the (resistive) normal current density $\mathbf{J}_n$, the (resistance-free) super current density $\mathbf{J}_s$, and the total current density $\mathbf{J}_{total}$. They are given in the model as

$$\mathbf{J}_n = \sigma \mathbf{E} = (-\sigma \frac{\partial \mathbf{A}}{\partial t} + \mathbf{J}) \ , \ \mathbf{J}_s = -\sum_{j=1}^{2} \frac{i}{2\kappa_j} \left(\psi_j^* \nabla \psi_j - \psi_j \nabla \psi_j^* \right) + \mathbf{A}|\psi_j|^2 \ , \ \mathbf{J}_{total} = \mathbf{J}_n + \mathbf{J}_s \ . \tag{1}$$

These quantities will be particularly important when we try to determine if the new modeling strategy for vortex pinning is effective. Now that we have defined several necessary quantities for the model, we can define the equations. The equations are scaled and non-dimensionalized, using the relations in [3], to make the equations suitable for computations. Currently we only wish to show that the model produces vortex pinning in a two-band material. Since this is our goal we can simplify some of the model parameters that are obtained from the normal inclusion model in [1]. We can assume that the superconducting metal and normal metal have similar effective electron masses, permeabilities, normal conductivities σ , and relaxation constants Γ. These values can be easily modified to represent more realistic normal metals in future work. The equations for the M2B-TDGL model are given by

$$\left(\frac{\partial \psi_1}{\partial t} - i \frac{Jy}{\sigma} \kappa_1 \psi_1 \right) - \alpha_1(x,y)\psi_1 + b(x,y)|\psi_1|^2 \psi_1 + (-i \frac{1}{\kappa_1} \nabla - \mathbf{A})^2 \psi_1 + \eta(x,y)\psi_2 = 0 \ , \tag{2}$$

$$\Gamma \left(\frac{\partial \psi_2}{\partial t} - i \frac{Jy}{\sigma} \kappa_1 \psi_2 \right) - \alpha_2(x,y)\psi_2 + b(x,y)|\psi_2|^2 \psi_2 + (-i \frac{1}{\kappa_2} \nabla - \nu \mathbf{A})^2 \psi_2 + \eta(x,y)\nu^2 \psi_1 = 0 \ , \tag{3}$$

$$\sigma \frac{\partial \mathbf{A}}{\partial t} + \nabla \times \nabla \times \mathbf{A} + \sum_{j=1}^{2} \frac{i}{2\tilde{\kappa_j}} \left(\psi_j^* \nabla \psi_j - \psi_j \nabla \psi_j^* \right) + \mathbf{A}|\psi_j|^2 = \nabla \times \mathbf{H} + \mathbf{J} \ , \tag{4}$$

where $\tilde{\kappa_1} = \kappa_1$ and $\tilde{\kappa_2} = \kappa_2 \nu$ with $\nu = \frac{\lambda_2 \xi_2}{\lambda_1 \xi_1}$, and J is the magnitude of **J**. The boundary conditions on $\partial \Omega_s$ and $\forall t > 0$ are

$$\mathbf{A} \cdot \mathbf{n} = 0 \ , \ \nabla \psi_1 \cdot \mathbf{n} = 0 \ , \ \nabla \psi_2 \cdot \mathbf{n} = 0 \ , \ (\nabla \times \mathbf{A}) \times \mathbf{n} = \left(\mathbf{H} - \left(x - \frac{L}{2} \right) J \hat{\mathbf{z}} \right) \times \mathbf{n} \ , \tag{5}$$

where L is the length of the square domain. The initial conditions in $\Omega_s \cup \Omega_n$ for $\psi_j\,(x,y,t)$ and $\mathbf{A}\,(x,y,t)$ are such that the superconductor is in thermal equilibrium

$$\nabla \cdot \mathbf{A}\,(x,y,0) = 0 \ , \ \psi_1\,(x,y,0) = \psi_{1,0}\,(x,y) \ , \ \psi_2\,(x,y,0) = \psi_{2,0}\,(x,y) \ , \ \mathbf{A}\,(x,y,0) = \mathbf{A}_0\,(x,y) \ . \tag{6}$$

To implement normal inclusions in the superconducting sample, four spatial domain dependent functions have been introduced $\alpha_j(x,y)$, $b(x,y)$, and $\eta(x,y)$. If (x,y) is in Ω_s, we have $b(x,y)=1$ and $\eta(x,y)=\eta$, while in Ω_n we have $b(x,y)=0$ and $\eta(x,y)=0$. As for $\alpha_j(x,y)$, we have

$$\alpha_j(x,y)|_{\Omega_s} = \Big(1-\frac{T}{T_{c,j}}\Big)\ \ ,\ \ \alpha_j(x,y)|_{\Omega_n} = \alpha_0 < \min\{\Big(1-\frac{T}{T_{c,1}}\Big),\Big(1-\frac{T}{T_{c,2}}\Big)\} < 0\ .$$

These values ensure that normal inclusions are treated as normal metals. The rationale for the $\alpha(x,y)$ and $b(x,y)$ values in the normal metals can be found in [1], except in the one band case $\alpha_0=-1$. However our choice in $\alpha(x,y)$ reflects the fact that one of the bands in the two-band model can have $\Big(1-\frac{T}{T_{c,j}}\Big)<0$, which is not possible in the one-band case.

3. Modeling Vortex Pinning in MgB_2. Now that we have defined the M2B-TDGL model, we would like to investigate if the new pinning strategy truly produces vortex pinning in simulations of a two-band superconductor. In order to do this, we must first discuss the numerical methods used in the simulation and the input parameters. To fully exemplify all the features of the model, a two-band material possessing impurities would be an ideal candidate. MgB_2 possesses two superconducting bands, anisotropies, and is typically dirty in that the material contains impurities and thus normal inclusions. Although this model does not consider the effects of anisotropy, it can easily be modified to do such as in [3]. Also, since MgB_2 is the most well studied two-band superconductor, this leads one to believe the material parameters are readily available and reliable.

The M2B-TDGL model consists of a non-linear time-dependent system of PDEs. The spatial domain is discretized using the finite element method (FEM) with parabolic Lagrangian triangular elements. An adaptive backward Euler scheme in time is chosen for stability and to take advantage of the dynamical time scales in the G-L system. The equations are linearized using Newton's method to gain quadratic convergence. All simulations in this work use 151,050 degrees of freedom. Finally, the resulting linear system is solved with GMRES with a multi-grid preconditioner provided in Trilinos.

The next step to producing a simulation is defining the input parameters that will go into the model. The material parameters for MgB_2 that are necessary for the model are shown in Table 1. All these parameters were either found in literature or derived from other parameters. The parameters consist of the characteristic lengths and G-L parameter for each band, relaxation constants σ and Γ, the coupling parameter η, the critical temperature for each band, and the material's composite critical temperature T_c. As previously mentioned, the critical temperatures for each band produce a peculiar situation for a simulation temperature of $T=30$ K.

The temperature dependent term in Eq. (3), $(1-T_{c,2}/30\text{ K})=-1.542$, having a value less than 0 would normally signify that superconductivity cannot persist in this band at 30 K. Yet due to Josephson like coupling effects it does persists. In order to produce non-superconducting behavior in the normal inclusions we set $\alpha_0=-2$ and $\eta(x,y)=0$ in Ω_n. The non-material parameters needed for the simulation are the magnetic field $\mathbf{H}$, applied current $\mathbf{J}$ and the temperature T. These parameters will be used for all simulations in this work, but they are non-dimensionalized before being entered into the model.

Table 1: The material parameters for the material MgB_2 and the model parameters for the simulation.

ξ_1=13.0 nm	ξ_2=51.0 nm	λ_1=47.8 nm	λ_2=33.6 nm
κ_1=3.62	κ_2=0.66	Γ=236.8685	$\sigma=4.8680$
T_{c1}=35.6 K	T_{c2}=11.8 K	T_c=39.0 K	$\eta=-0.1701$
$\mathbf{H}$=0.2648 T $\hat{z}$	$\mathbf{J}$=0.0843 MA cm^2 $\hat{y}$	L=500 nm	$T=30$ K

Now we can finally present vortex pinning simulations for the material MgB_2 using the M2B-TDGL model. The simulation set up uses the normal inclusion and metal band arrangement shown in Fig. 1a, with $N=4$ normal inclusions inside the superconductor. The simulations are run with the magnetic field $\mathbf{H}$ until a steady state vortex lattice is found. Then the lattice is used as the initial conditions for the next simulation that introduces the current density $\mathbf{J}$.

The order parameters from the vortex pinning simulation are shown in Fig. 2. The magnitude of the order parameters, $|\psi_j|\to 0$, in the location of a vortex or normal material. This can be confusing, however Fig. 1a allows us to distinguish the normal material from the vortices. Fig. 2 (left pair) shows the order parameters after one time step with the applied current. The vortex lattice retains a similar shape as the initial conditions. The four large normal inclusions (where $\psi_j\to 0$) contain three vortices each, while six other vortices lie outside of the normal inclusions. Fig. 2 (right pair) shows the order parameters after 100 time steps, At this point the current has induced flux flow, distorting the lattice and producing a resistive current. However the vortices pinned inside the normal inclusion have retained their position. The y-component of the quantities in Eq. (1), averaged in the x-direction, are shown at the same time step in Fig. 3 (left). As expected the applied current enters and leaves the sample as a normal current through the metal leads. However, since some of the vortices are pinned, a significant portion of the applied current passes through the superconductor as a resistance-free current.

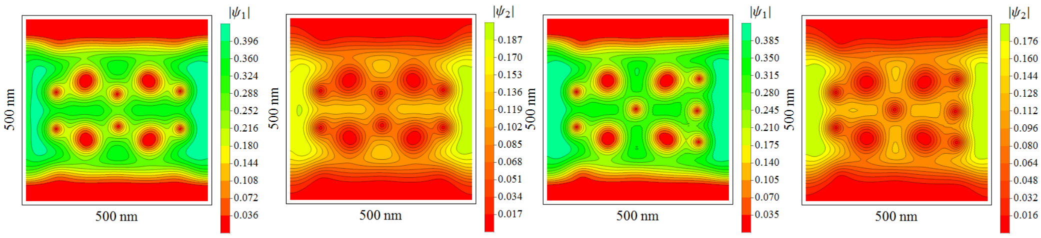


Fig. 2: The order parameters at the first time step for the MgB_2 parameters given in Table 1 (left pair). The order parameters after 100 time steps (right pair). The applied current has induced flux flow and distorted the vortex lattice outside of the normal inclusions.

Although the results from Fig. 2 seem to show that the new pinning strategy indeed works, it would be more insightful to quantify the resistance in the sample. Then we can say that the normal inclusions lessen resistance in the superconductor and improve its integrity. In order to quantify the resistance in the sample, the change in voltage can be used as a proxy since it is proportional to the resistance. The voltage in the direction of the applied current is given by $V(y) = V(0) - \int_0^y E_{y,avg}(y')y'$, where $\mathbf{E}$ is the electric field from Eq. (1) and $E_{y,avg}$ is its y-component averaged over the x-direction. Fig. 3 shows the voltage for the average electric field in the y-direction after 10,000 time steps. This is done for the previous sample with $N = 4$ normal inclusions and a sample with $N = 0$ for comparison. Clearly the sample with 4 normal inclusions has a smaller change in voltage inside the superconductor, and thus less resistance. This shows that the normal inclusions indeed lessen resistance in the superconductor and this strongly suggest that the new pinning strategy does model vortex pinning correctly.

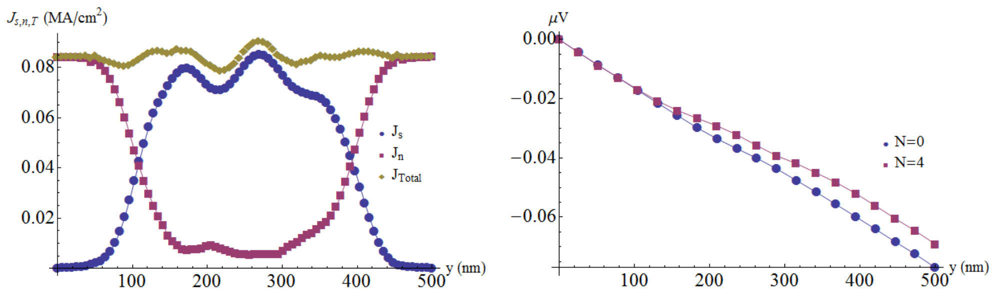


Fig. 3: The normal current, super current, and total current for the simulation after 100 time steps (left). The voltage after 10,000 time steps for the simulation with $N = 4$ normal inclusions and for a simulation with $N = 0$ normal inclusions. The simulation with no normal inclusions yields a larger voltage change over the domain, signifying this sample has more resistance.

4. Conclusions. In this paper, a new composite Ginzburg-Landau model was presented, with a novel pinning strategy suitable for two-band superconductors. A simulation of vortex pinning and an applied current was shown using the model for the material MgB_2. It was found that the normal inclusions lessened resistance incurred by flux flow, strongly suggesting the novel pinning strategy works.

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