The goal of this project is to complete our module for piecewise polynomials and test the module on a particular function.

1. Complete the module for piecewise interpolation by including routines for piecewise quadratic, cubic and cubic Hermite interpolation analogous to the ones we did for piecewise linear interpolation.

2. Consider the function

\[
\frac{x}{1 + x} \quad 0 \leq x \leq 10
\]

which we want to interpolate. Write a driver program to use your piecewise interpolation module to generate approximations to this function for piecewise linear, quadratic, cubic and cubic Hermite polynomials using 10, 20, 40, and 80 intervals. In each case calculate a Euclidean norm of the error (normalized by length of vector) and tabulate your results. Make 4 plots, one for each type of interpolation. On each plot graph the function and each of its four interpolants for 10, 20, 40, 80 intervals.

3. (graduates only) Another common type of interpolation is cubic spline. In this case, we require the piecewise interpolation be continuous and have continuous first and second derivatives. Cubic splines are usually preferable to cubic Hermite interpolants. Instead of writing your own routine for calculating cubic splines, I want you to go to netlib and download routine(s) for computing the coefficients and evaluating the polynomial at a point. Use these to repeat your calculations in (2) using cubic splines. Cubic splines have two extra conditions to satisfy so these are usually satisfied by imposing a condition at each end of the given interval. Typically one uses “natural splines” but you can use a code that implements any you desire. You do not have to call the routines using function overloading, simply call them.