Abstract. Commonly held assumptions about the natural numbers would lead an observer to assume that matters such as the value of the largest integer are well known. But it is here demonstrated that every candidate for the largest integer is flawed. Moreover, a second argument shows that the largest integer must be very large indeed. Implications for the proposed printing of all integers in a single book are discussed.

Key words. natural numbers, greatest integer question, the big book

1. Introduction. Since ancient times, counting has been a preoccupation of the shepherd, the dancer, the owner of beans, and many other independent researchers. Recently, professional counters have appeared, and counting has been put on a rational footing. The natural numbers have been discovered, named, and copyrighted. A question that arises on many occasions, though, is the “extent” of the natural numbers. We recall the words of Socrates at the Euphrates:

I heard a man say that twenty seemed like quite a large number, since it counted his fingers and toes. Another said perhaps forty was the largest. With such wisdom all around us, who needs philosophers?

We can formulate the underlying question more precisely once we have clarified what we mean by largest integer. We therefore make the following definition:

**Definition 1.1 (The Largest Integer).** Suppose that \( n \) is a very large integer. We say that \( n \) is the **largest integer** if, for any integer \( m \), it must be the case that \( m \leq n \).

Using this definition, it is clear that, for instance, 20 cannot be the largest integer, since 40 is bigger. But we will now show, by explicit construction, that 40 itself cannot be the largest integer. To do so, we will employ the following algorithm:

1. Set \( n = 1 \);
2. Set \( n = n + 1 \);
3. If \( n \leq 40 \) go to step 2.
4. Print “\( n \) is bigger than 40”.
5. Stop.

Using this algorithm, the authors discovered the number 41, which is demonstrably bigger than 40. It may be the largest integer. However, the authors believe that, with a few simple changes, the algorithm shown above may also be applied to the case of 41. Whether this is so, and the nature of the number that might be discovered in that case, are open questions at this point.

However, no paper is complete with a table of results. And here, we present the results of a poll asking for the favorite number. As can be seen, more than \( \frac{1}{3} \) of the votes were for 1, ordinarily thought of as “the loneliest number”!

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**Table 1.1**

Favorite integers, and percentage popularity.

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Other researchers [1, 2] have suggested that simply because numbers like 40, and our newly discovered 41, are not popular, they should not be further investigated. However, our attitude is that deep thought is appropriate to such issues precisely because they currently hover at the edges of the human sensibility. The aim of our work, ultimately, is to efficiently determine statistical information about the random field $u = u(t, x; \omega)$ from numerical approximations of white noise:

$$\frac{\partial u}{\partial t} = Au - \gamma N(u) + g + \epsilon \frac{\partial W}{\partial t}. \quad (1.1)$$

Here $D \subset \mathbb{R}^N$ is a convex, bounded and polygonal spatial domain, $(\Omega, \mathcal{F}, P)$ is a probability space described in section 2, and $A$ is a linear second-order elliptic operator with deterministic coefficients, defined on a space of functions satisfying certain boundary conditions, $N(u)$ is a nonlinear function of the random process $u$, $g$ represents a deterministic function and $W$ denotes an infinite dimensional Brownian motion or Wiener process. The additive noise that appears in (1.1) is in the form of space-time Brownian white noise.

2. Preliminaries. We begin by recalling the mathematical formulation of a probability space $(\Omega, \mathcal{F}, P)$, where $\Omega$, $\mathcal{F}$ and $P$ are the set of random events, the minimal $\sigma$-algebra of subsets of $\Omega$, and the probability measure, respectively.

2.1. Casting Out Hobbits. Given a set $\{\vec{w}_n\}^N_{n=1}$, it may be necessary to examine that set for Hobbits, and to cast them out. In this work, we will always assume that this has already been done. A typical network of Hobbits is displayed in Figure 2.1 and the obvious targets for further study appear vividly in the various symmetries.

2.2. Computational results. We ran the program many times, for lots of different starting points. We had some tables of results to show you, but now I can’t find them. You’re probably better off.

3. Conclusions. It is the firm belief of the authors and their mothers that this investigation has laid to rest the burning issue of the largest integer. Both 20 and 40 have been shown to be inadequate, and the new candidate 41 has been discovered. Initial results suggest, though, that the very largest integer is approximately 42, and with that we can stop.

REFERENCES