Sparse Grids

Sparse grids can be used for interpolation or quadrature of smooth functions in high dimensions; they can provide accurate estimates at a cost that does not increase exponentially in the spatial dimension.

The desired accuracy is the black diagonal. A single product grid can only cover this line by covering the entire square. The excess accuracy is reflected in the high number of points. By combining lower order product rules, a sparse grid reaches the same desired accuracy at a lower cost in points.

On a budget of $10^6$ points, we can’t afford a 2 point product rule beyond dimension 20. But even to dimension 100, a sparse grid can give precision of level 7.

Mixed Families

Each spatial dimension of a sparse grid can use a separate indexed family of 1D quadrature rules. Here, we use Gauss-Hermite in X and Clenshaw-Curtis in Y.

Growth Rules

The default growth rule for Clenshaw Curtis is exponential. But the user can select a slower growth rule that is also nested, and just as precise. On the left, the “slow growth” rule uses few points from the 5th level on.

The precision plots for regular and slow growth Clenshaw Curtis show how the slow growth rule does a better job of reaching the precision line (black diagonal) without exceeding it so much.

Anisotropy

The user can specify anisotropic weights for each dimension. The sparse grid increases its precision in the preferred directions.

Reference

Nobile, Tempone, Webster,
An Anisotropic Sparse Grid Stochastic Collocation Method for Partial Differential Equations with Random Input Data,

Acknowledgement

A grant from Sandia National Laboratories supported this research.