Quasirandom Numbers

Thomas Britton

The Problem

Compute an integral with 360 dimensions

Curse of Dimensionality

For a uniform grid you will have to place 2^d points to find your answer

$2^{360} \sim 2.35 * 10^{108}$

Randomness

Pros

- Cheap
- Easy
- Law of Large Numbers

Cons

- Can’t guarantee accuracy
- Law of Large Numbers

What makes something random?

1) Unpredictable - no rule for their selection

2) Independent - knowing one won’t help you know another

3) Uniformly Distributed

How do the types of random stack up?

	Unpredictable	Independent	Uniform
True Random	Yes	Yes	Yes
Pseudorandom	No	Yes	Yes
Quasirandom	No	No	Yes
Uniform Grid	No	No	No?

van der Corput sequence

Integer	Base 2	Reverse	Fraction	Decimal
0	0	0	.0	0
1	1	1	.1	1/2
2	10	01	.01	1/4
3	11	11	.11	3/4
4	100	001	.001	1/8
5	101	101	.101	5/8
6	110	011	.011	3/8
7	111	111	.111	7/8

Erros

True Random/Pseudorandom

$1 / (N^{1/2})$

Quasirandom

$(\log N)^d / N$

N	10	100	1000
TR	.316	.1	.0316
QR	.1	.02	.003

$$(\log N)^d / N$$

Why bother?

Terribly slow convergence!

With d = 360 the error will be astronomical

Nonetheless Paskov and Traub tried it

And it worked! How?

Dimensionality Reduction

The tool wasn’t better than others, the problem was just easier than it appeared

The power in Quasirandom numbers is you don’t have to tell it which variables can be ignored