The Britney Spears Problem

Why getting it *almost* right is OK
and
Why scrambling the data may help

Oops I made it again…

I’m Popular because I’m Popular

- With respect to the internet, answering:

Which *of these* is the most popular web search?

is a much much easier question than answering:

Wha **ch?**

Straightforward Approach

- Let's assume Google received their engine-search requests via one long data stream that they could read-in in real time…
- The straightforward solution would be to append new words to an array containing all words that have already been encountered and update a corresponding counter
- …, "Yo dog", "Girls gone wild", "Dog ate chocolate", …

{yo=1, dog=2, girls=1, gone=1, wild=1, ate=1, chocolate=1}

Need for a constant-space algorithm

- Deciding whether to append the new word or increment a past counter might require an expensive search through the array
- But **more importantly**, the size of the array would be *astronomical* with no maximum cap on memory

Image credit: The very Google servers pictured above (trippy right?)

Majority Rule

- Imagine if the English language was dumbed down to a few words, or better yet… the integers 1 to 9
- Also, assume that one number (let’s say 4) had the *majority* of the number instances. (This means >50% of the numbers are actually 4)
- With the “majority rule” method we would have two pieces of memory:

1) the most common number up to that point (maj)
2) a ‘counter’ that we associate with that number (count)

Majority Rule

- The rule is that we increment when we stream across the number stored in memory, and decrement otherwise. Example:

  … 4
  maj=4
  count=1

  … 4 4
  maj=4
  count=2

  … 2 4 4
  maj=4
  count=1

  … 1 2 4 4
  maj=4
  count=0

  3 1 2 4 4
  maj = 3
  count=1

Majority Rule

- In this case, if 4 had actually been the majority, maj would have =4 when the stream was complete.
- Method is guaranteed to find the majority *if there is one*, but the number stored in memory at algorithm completion is *not* guaranteed to be a number with >50% of the occurrences
- Extend this to use an *m* number of maj variables to find the n/(m+1) frequency. Example: use m=99 to find if a word appears in 1% of web search queries. Actually pretty robust!

Almost Right

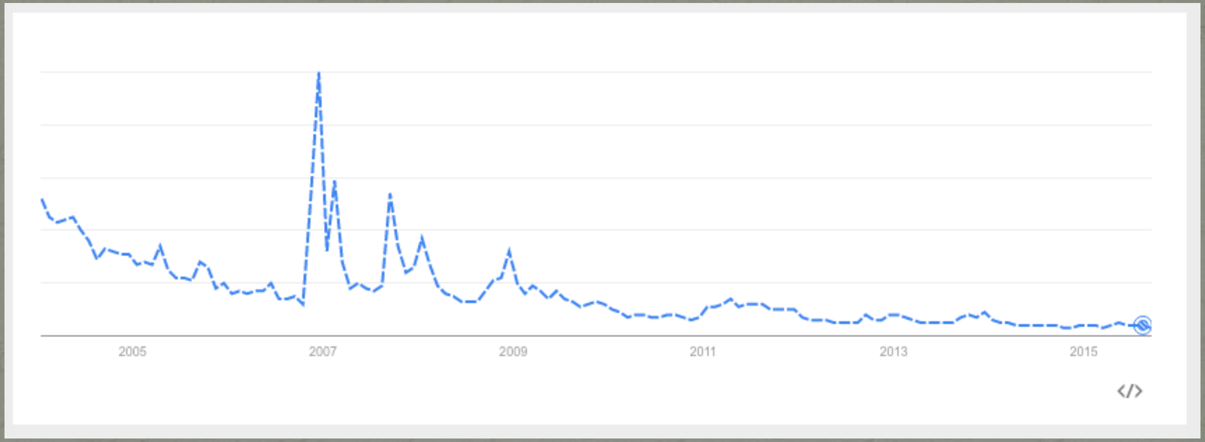
- Going back to the original straightforward method of appending to a huge array… what if we just removed the most infrequent elements every once in a while?
- This solution gives very good results, but we still have the unbounded space problem.
- This (along with Majority Rule) illustrates that we will not get the correct answer 100% of the time if we must obey the constant-space rule.
- But is that really all that bad?

Making a Hash

- A uniform random distribution actually has expected statistical properties (much like the standard normal distribution)
- A method used in computer science called “hashing” essentially bins and scrambles values that come from a unpredictable distribution to make them appear as if they are uniformly distributed.
- The bins can then be analyzed statistically to make generalizations about the data stream

Thanks, Britney!

You’ll always be Number 1 in my book, even though the 90’s misses you.



Reference:
Hayes, Brian. “The Britney Spears problem.” *American Scientist* 96.4 (2008): 274.